GRAPHICAL MARKOV MODELS (WS2017)
6. SEMINAR

Assignment 1. Let K be a completely ordered finite set (you may assume K to be an interval of integers, $K \subset \mathbb{Z}$ where appropriate). Consider real valued functions on K^n.
a) Prove that a conical linear combination

$$f(\kappa) = \sum_{i=1}^{m} \alpha_i g_i(\kappa), \quad \alpha_i \geqslant 0 \, \forall i$$

of submodular functions $g_i\colon K^n \to \mathbb{R}$ is a submodular function.
b) Let f, g be submodular functions on K^n. Is their point-wise maximum a submodular function? And their minimum?
c) Consider the function $g\colon K^2 \to \mathbb{R}$ defined by

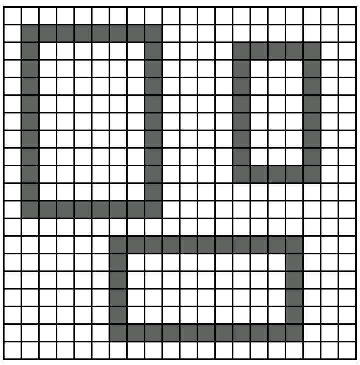
$$g(k, k') = \alpha(k - k')^2.$$

For which α is it submodular?
d) Prove that the truncated norm

$$g(k, k') = \min(M, |k - k'|)$$

is not submodular.

Assignment 2. Consider the language L of all b/w images $x\colon D \to \{b, w\}$ containing an arbitrary number of non-overlapping, non-nested and non-touching one pixel wide rectangular frames (see figure).



a) Prove that L is not expressible by a locally conjunctive predicate

$$x \in L \quad \text{if and only if} \quad f(x) = \bigwedge_{c \in \mathcal{C}} f_c(x_c) = 1$$

with predicates f_c, defined on image fragments x_c, where $c \subset D$ have bounded size $|c| < |D|$.

b) Show that L can be expressed by introducing a field $s\colon D \to K$ of non-terminal symbols, a locally conjunctive predicate on them and pixel-wise predicates g relating the non-terminal and terminal symbol in each pixel

$$x \in L \quad \text{if and only if} \quad \bigvee_{s \in K^D} \Big[\bigwedge_{c \in \mathcal{C}} f_c(s_c) \wedge \bigwedge_{i \in D} g(x_i, s_i) \Big] = 1$$

Find a suitable structure $\mathcal{C}$, an alphabet of non-terminal symbols K and predicates f_c, g.

Assignment 3. Given an undirected graph, the minimum vertex cover problem seeks to find the smallest subset of vertices, such that each edge of the graph is incident with at least one vertex from that set.
a) Formulate the problem as a $(\min, +)$ problem. Is it submodular?
b) Derive an algorithm for the case that the graph is a tree.