# Basics of Description Logic $\mathcal{ALC}$

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## 1 Understanding $\mathcal{ALC}$

Consider the following $\mathcal{ALC}$ theory $\mathcal{K} = (\mathcal{T}, \{\})$, where $\mathcal{T}$ contains the following axioms:

$$\begin{aligned} Man &\sqsubseteq Person \\ Woman &\sqsubseteq Person \sqcap \neg Man \\ Father &\equiv Man \sqcap \exists hasChild \cdot Person \\ GrandFather &\equiv \exists hasChild \cdot \exists hasChild \cdot \top \\ Sister &\equiv Person \sqcap \neg Man \sqcap \exists hasSibling \cdot Person \end{aligned}$$

**Ex. 1** — What is the meaning of these particular axioms ? Do they reflect your understanding of reality ? Formulate them in natural language.

**Answer (Ex. 1)** — For example, the third axiom defines a concept $Father$ as any $Man$ that has some $Person$ as a child. The fourth axiom is not well defined – it allows grandfathers to be women. More precise version of the fourth axiom might be e.g. $GrandFather \equiv Man \sqcap \exists hasChild \cdot \exists hasChild \cdot \top$.

**Ex. 2** — Rewrite the last axiom into the semantically equivalent FOPL formula.

**Answer (Ex. 2)** — Each TBox axiom corresponds to a universally closed FOPL formula. Notice that two different variables are enough for encoding of any $\mathcal{ALC}$ axiom. The encoding of the last axiom is:
$(\forall x)(Sister(x) \equiv Person(x) \wedge \neg Man(x) \wedge (\exists y)(hasSibling(x, y) \wedge Person(y)))$

**Ex. 3** — Consider the following interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$:

$$\begin{aligned} \Delta^{\mathcal{I}} &= Person^{\mathcal{I}} = \{B, A\} \\ Man^{\mathcal{I}} &= \{B\} \\ Woman^{\mathcal{I}} &= \{A\} \\ Father^{\mathcal{I}} &= GrandFather^{\mathcal{I}} = \{B\} \end{aligned}$$

$$\begin{aligned} hasChild^{\mathcal{I}} &= \{(B,B)\} \\ hasSibling^{\mathcal{I}} &= \{\} \\ Sister^{\mathcal{I}} &= \{B\} \end{aligned} \quad (1)$$

1. Is $\mathcal{I}$ a model $\mathcal{K}$ ? If yes, decide, whether $\mathcal{I}$ reflects reality.
2. We know that $\mathcal{ALC}$ has the *tree model property* and *finite model property.* In case $\mathcal{I}$ is a model, is $\mathcal{I}$ tree-shaped ? If not, find a model that is tree-shaped.

**Answer (Ex. 3)** — $\mathcal{I}$ is not a model of $\mathcal{K}$, as it does not satisfy the last axiom: $Sister^{\mathcal{I}} \neq Person^{\mathcal{I}} \cap (\Delta^{\mathcal{I}} \setminus Man^{\mathcal{I}}) \cap \{x \in \Delta^{\mathcal{I}} | (\exists y \in \Delta^{\mathcal{I}})((x,y) \in hasSibling^{\mathcal{I}} \wedge y \in Person^{\mathcal{I}})\}$

**Ex. 4** — How does the situation change when we consider the same $\mathcal{I}$, except that $Sister^{\mathcal{I}} = \{\}$ ?

**Answer (Ex. 4)** — Now, $\mathcal{I}$ is a model of $\mathcal{K}$ as it satisfies all axioms. However, it does not reflect the reality well, as it states that a person $B$ is his/her own child. This interpretation is finite, yet not tree-shaped. A tree-shaped model ensured by the *tree-model property* of $\mathcal{ALC}$ is e.g. the following infinite model $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$, where

$$\begin{aligned} \Delta^{\mathcal{I}} &= Person^{\mathcal{I}} = Man^{\mathcal{I}} = Father^{\mathcal{I}} = GrandFather^{\mathcal{I}} = \{A_1, A_2, \ldots\}_{i=1\ldots\infty} \\ Woman^{\mathcal{I}} &= Sister^{\mathcal{I}} = \{\} \\ hasChild^{\mathcal{I}} &= \{(A_i, A_{i+1})\}_{i=1\ldots\infty} \\ hasSibling^{\mathcal{I}} &= \{\} \end{aligned} \quad (2)$$

**Ex. 5** — Using the vocabulary from $\mathcal{K}$, define the concept "A father having just sons."

**Answer (Ex. 5)** — $FatherOfBoys \equiv Father \sqcap \forall hasChild \cdot Man$

**Ex. 6** — Using the vocabulary from $\mathcal{K}$, define the concept "A man who has no brother, but at least one sister with more than one child."

**Answer (Ex. 6)** — $HappyUncle \equiv Man \sqcap \exists hasSibling \cdot (Woman \sqcap \exists hasChild \cdot \top) \sqcap \forall hasSibling \cdot \neg Man$

**Ex. 7** — During knowledge modeling, it is often necessary to specify:

**global domain and range**of given role, i.e. statement of the type "By *hasChild* we always connect a *Person* (domain) with another *Person* (range)".

**local range**of given role, e.g. "Every father having only sons (domain) can be connected by *hasChild* (domain) just with a *Man* (range)".

Show, in which way it is possible to model global domain and range of these roles in $\mathcal{ALC}$.

**Answer (Ex. 7)** — Global domain and range can be modeled as:

$$\begin{aligned} \exists hasChild \cdot \top &\sqsubseteq Person \\ \top &\sqsubseteq \forall hasChild \cdot Person \end{aligned} \tag{3}$$

Local range is similar and only replaces the top concepts in the global range axiom:

$$\begin{aligned} \exists hasChild \cdot \top &\sqsubseteq Person \\ FatherOfSons &\sqsubseteq \forall hasChild \cdot Man \end{aligned} \tag{4}$$

## 2 Using Protégé

1. Go through the Protégé Crash Course on the tutorial web pages.
2. Create a new ontology in Protégé 4 and insert there all the definitions from Section 1. Verify correctness of your solution of the previous task (e.g. in the DL query tab).