Description Logics – Querying

Petr Křemen

petr.kremen@fel.cvut.cz

December 12, 2018

Outline

1. What if OWL is not enough?
2. Complex Queries
   - Evaluation of Conjunctive Queries in $\mathcal{ALC}$
3. Modeling Error Explanation
   - Black-box methods
   - Algorithms based on CS-trees
   - Algorithm based on Reiter's Algorithm
   - Algorithm based on Reiter's Algorithm

Problems

- What if OWL is not enough?
- What if more complex queries than consistency checking are necessary?
- What to do if an ontology is inconsistent?

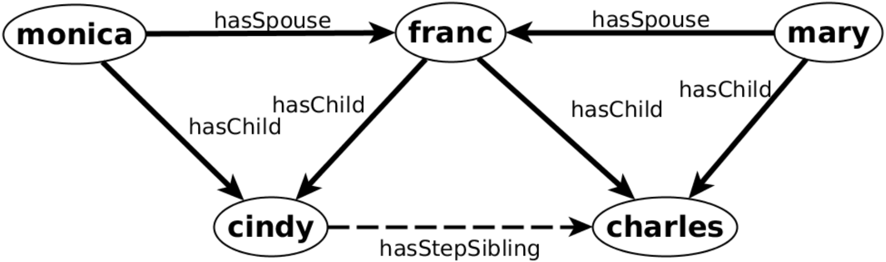
What if OWL is not enough?

SROIQ (OWL) Revision

$$Man \sqsubseteq Person$$
$$Man \sqsubseteq \neg Woman$$
$$Man \sqcap \exists hasChild \cdot Man \sqsubseteq FatherOfSons$$
$$hasSon \sqsubseteq hasChild$$
$$hasParent \circ hasBrother \sqsubseteq hasUncle$$
$$trans(hasDescendant)$$
$$sym(hasSpouse)$$
$$fun(hasMother)$$
$$hasWife \sqsubseteq hasHusband^{-}$$

How to express hasStepSibling?

How to express hasStepSibling?



How to express hasStepSibling?

$$\begin{aligned} \mathit{hasSpouse}(?m1, ?f), \mathit{hasSpouse}(?m2, ?f), \\ \mathit{hasChild}(?m1, ?c1), \mathit{hasChild}(?m2, ?c2), \\ \mathit{hasChild}(?f, ?c1), \mathit{hasChild}(?f, ?c2), ?c1! = ?c2 \\ \rightarrow \mathit{hasStepSibling}(?c1, ?c2) \end{aligned}$$

OWL2-DL + rules undecidable

... unless variables in rules are restricted to match named individuals only.

DL-safe Rules

A rule is DL-safe, if its variables are *distinguished*, i.e. thet can only match **named individuals** in the ontology. Consistency checking of OWL2-DL + DL-safe rules is decidable.

Complex Queries

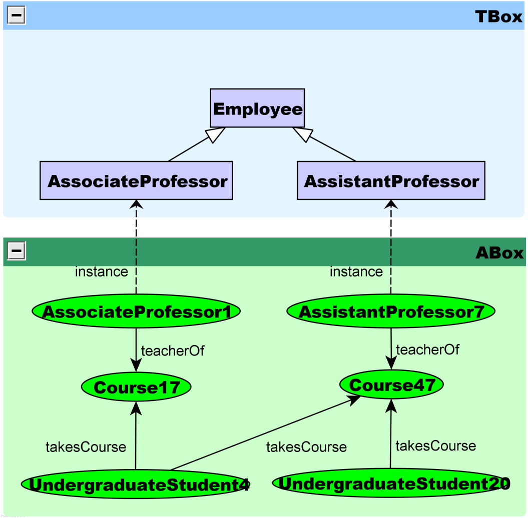
What if we need to answer a complex query?

- Consistency checking is not enough. What if we would like to ask more, e.g. ... **How many czech writers died in the Czech Republic according to DBPedia ?**

```
PREFIX dbo: <http://dbpedia.org/ontology/>
PREFIX dbr: <http://dbpedia.org/resource/>
PREFIX dcterms: <http://purl.org/dc/terms/>
SELECT COUNT(?x)
{
  ?x    dbo:deathPlace  dbr:Czech_Republic ;
        dcterms:subject dbr:Category:Czech_writers
}
```

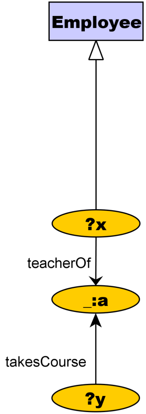
at the following endpoint:
`http://dbpedia-live.openlinksw.com/sparql/`

Conjunctive Queries

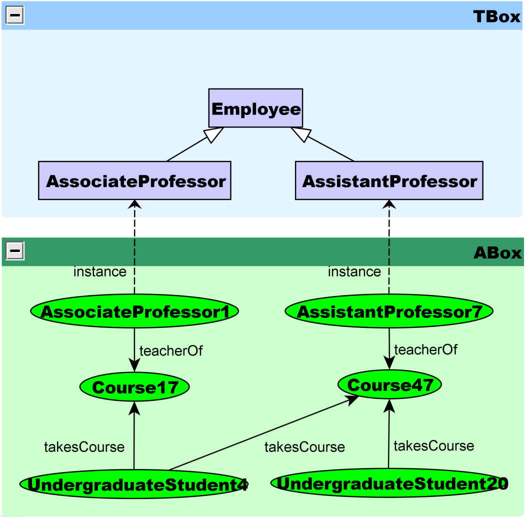


Conjunctive ABox Queries.

"Get all teachers and their students."



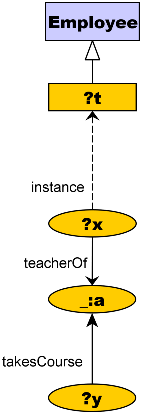
Metaqueries



Mixed TBox/ABox Queries.

"Get all teachers and their students."

"... together with the type of the teachers."



Query Types

Conjunctive (ABox) queries – queries asking for individual tuples complying with a graph-like pattern.

Query Types

Conjunctive (ABox) queries – queries asking for individual tuples complying with a graph-like pattern.

Example

"Find all mothers and their daughters having at least one brother." :

$$Q(?x, ?z) \quad \leftarrow \quad Woman(?x), hasChild(?x, ?y), hasChild(?x, ?z), Man(?y), Woman(?z)$$

Query Types

Conjunctive (ABox) queries – queries asking for individual tuples complying with a graph-like pattern.

Example

"Find all mothers and their daughters having at least one brother." :

$$\begin{aligned} Q(?x,?z) \quad \leftarrow \quad & Woman(?x), hasChild(?x,?y), hasChild(?x,?z), \\ & Man(?y), Woman(?z) \end{aligned}$$

Metaqueries – queries asking for individual/concept/role tuples. There are several languages for metaqueries, e.g. SPARQL-DL, OWL-SAIQL, etc.

Query Types

Conjunctive (ABox) queries – queries asking for individual tuples complying with a graph-like pattern.

Example

"Find all mothers and their daughters having at least one brother." :

$$\begin{aligned} Q(?x, ?z) \quad \leftarrow \quad & Woman(?x), hasChild(?x, ?y), hasChild(?x, ?z), \\ & Man(?y), Woman(?z) \end{aligned}$$

Metaqueries – queries asking for individual/concept/role tuples. There are several languages for metaqueries, e.g. SPARQL-DL, OWL-SAIQL, etc.

Example

"Find all people together with their type." in SPARQL-DL:

$$Q(?x, ?c) \leftarrow TYPE(?x, ?c), SUBCLASSOF(?c, Person)$$

Conjunctive (ABox) queries

Conjunctive (ABox) queries are analogous to database SELECT-PROJECT-JOIN queries.

Conjunctive Query

$$Q(?x_1, \dots, ?x_D) \leftarrow t_1, \dots t_T,$$

where each t_i is either

- $C(y_k)$ (where C is a concept)
- $R(y_k, y_l)$ (where R is a role)

and y_i is either (i) an individual, or (ii) variable from a new set V (variables will be differentiated from individuals by the prefix “?”). We need all $?x_i$ to be present also in one of t_i.

Conjunctive ABox Queries – Semantics

- Conjunctive queries of the form $Q()$ are called *boolean* – such queries only test existence of a relational structure in each model $\mathcal{I}$ of the ontology $\mathcal{K}$.
- Consider any interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$. *Evaluation* η is a function from the set of individuals and variables into $\Delta^{\mathcal{I}}$ that coincides with $\mathcal{I}$ on individuals.
- Then $\mathcal{I} \models_{\eta} Q()$, iff
  - $\eta(y_k) \in C^{\mathcal{I}}$ for each atom $C(y_k)$ from $Q()$ and
  - $\langle \eta(y_k), \eta(y_l) \rangle \in R^{\mathcal{I}}$ for each atom $R(y_k, y_l)$ from $Q()$
- Interpretation $\mathcal{I}$ is a model of $Q()$, iff $\mathcal{I} \models_{\eta} Q()$ for some η.
- Next, $\mathcal{K} \models Q()$ ($Q()$ is satisfiable in $\mathcal{K}$) iff $\mathcal{I} \models Q()$ whenever $\mathcal{I} \models \mathcal{K}$

Conjunctive ABox Queries – Variables

- Queries without variables are not practically interesting. For queries with variables we define semantics as follows. An N-tuple $\langle i_1, \ldots, i_n \rangle$ is a *solution* to $Q(?x_1, \ldots, ?x_n)$ in theory $\mathcal{K}$, whenever $\mathcal{K} \models Q'()$, for a boolean query Q' obtained from Q by replacing all occurences of $?x_1$ in all t_k by an individual i_1, etc.

Conjunctive ABox Queries – Variables

- Queries without variables are not practically interesting. For queries with variables we define semantics as follows. An N-tuple $\langle i_1, \ldots, i_n \rangle$ is a *solution* to $Q(?x_1, \ldots, ?x_n)$ in theory $\mathcal{K}$, whenever $\mathcal{K} \models Q'()$, for a boolean query Q' obtained from Q by replacing all occurences of $?x_1$ in all t_k by an individual i_1, etc.
- In conjunctive queries two types of variables can be defined:

Conjunctive ABox Queries – Variables

- Queries without variables are not practically interesting. For queries with variables we define semantics as follows. An N-tuple $\langle i_1, \ldots, i_n \rangle$ is a *solution* to $Q(?x_1, \ldots, ?x_n)$ in theory $\mathcal{K}$, whenever $\mathcal{K} \models Q'()$, for a boolean query Q' obtained from Q by replacing all occurences of $?x_1$ in all t_k by an individual i_1, etc.
- In conjunctive queries two types of variables can be defined:

distinguished occur in the query head as well as body, e.g. $?x, ?z$ in the previous example. These variables are evaluated as domain elements that are necessarily interpretations of some individual from $\mathcal{K}$. That individual is the binding to the distinguished variable in the query result.

Conjunctive ABox Queries – Variables

- Queries without variables are not practically interesting. For queries with variables we define semantics as follows. An N-tuple $\langle i_1, \ldots, i_n \rangle$ is a *solution* to $Q(?x_1, \ldots, ?x_n)$ in theory $\mathcal{K}$, whenever $\mathcal{K} \models Q'()$, for a boolean query Q' obtained from Q by replacing all occurences of $?x_1$ in all t_k by an individual i_1, etc.
- In conjunctive queries two types of variables can be defined:

  distinguished occur in the query head as well as body, e.g. $?x, ?z$ in the previous example. These variables are evaluated as domain elements that are necessarily interpretations of some individual from $\mathcal{K}$. That individual is the binding to the distinguished variable in the query result.

  undistinguished occur only in the query body, e.g. $?y$ in the previous example. Their can be interpretated as any domain elements.

Conjunctive Queries – Examples

Example

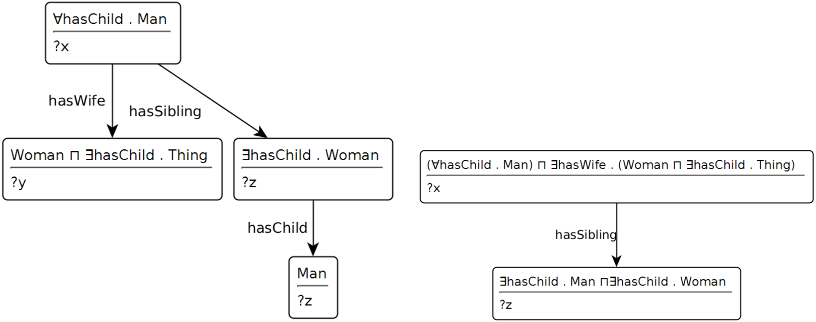
Let's have a theory $\mathcal{K}_4 = (\emptyset, \{(\exists R_1 \cdot C_1)(i_1), R_2(i_1, i_2), C_2(i_2)\})$.

- Does $\mathcal{K} \models Q_1()$ hold for $Q_1() \leftarrow R_1(?x_1, ?x_2)$?
- What are the solutions of the query $Q_2(?x_1) \leftarrow R_1(?x_1, ?x_2)$ for $\mathcal{K}$?
- What are the solutions of the query $Q_3(?x_1, ?x_2) \leftarrow R_1(?x_1, ?x_2)$ for $\mathcal{K}$?

Evaluation of Conjunctive Queries in $\mathcal{ALC}$

Satisfiability of $\mathcal{ALC}$ Boolean Queries

- Satisfiability of the boolean query $Q()$ having a tree shape can be checked by means of the **rolling-up technique**.



Rolling-up Technique

- Each two atoms $C_1(y_k)$ and $C_2(y_k)$ can be replaced by a single query atom of the form $(C_1 \sqcap C_2)(y_k)$.

Rolling-up Technique

- Each two atoms $C_1(y_k)$ and $C_2(y_k)$ can be replaced by a single query atom of the form $(C_1 \sqcap C_2)(y_k)$.
- Each query atom of the form $R(y_k, y_l)$ can be replaced by the term $(\exists R \cdot X)(y_k)$, if y_l occurs in at most one other query atom of the form $C(y_l)$ (if there is no $C(y_l)$ atom in the query, consider w.l.o.g. that C is $\top$). X equals to

Rolling-up Technique

- Each two atoms $C_1(y_k)$ and $C_2(y_k)$ can be replaced by a single query atom of the form $(C_1 \sqcap C_2)(y_k)$.
- Each query atom of the form $R(y_k, y_l)$ can be replaced by the term $(\exists R \cdot X)(y_k)$, if y_l occurs in at most one other query atom of the form $C(y_l)$ (if there is no $C(y_l)$ atom in the query, consider w.l.o.g. that C is $\top$). X equals to
  - (i) C, whenever y_l is a variable,

Rolling-up Technique

- Each two atoms $C_1(y_k)$ and $C_2(y_k)$ can be replaced by a single query atom of the form $(C_1 \sqcap C_2)(y_k)$.
- Each query atom of the form $R(y_k, y_l)$ can be replaced by the term $(\exists R \cdot X)(y_k)$, if y_l occurs in at most one other query atom of the form $C(y_l)$ (if there is no $C(y_l)$ atom in the query, consider w.l.o.g. that C is $\top$). X equals to
  - (i) C, whenever y_l is a variable,
  - (ii)$C \sqcap Y_l$, whenever y_l is an individual. Y_l is a *representative concept* of individual y_l occuring neither in $\mathcal{K}$ nor in Q. For each y_l it is necessary to extend ABox of $\mathcal{K}$ with concept assertion $Y_l(y_l)$.

Satisfiability of $\mathcal{ALC}$ Boolean Queries (2)

... after rolling-up the query we obtain the query $Q()' \leftarrow C(y)$, that is satisfied in $\mathcal{K}$, iff $Q()$ is satisfied in $\mathcal{K}$:

- **If y is an individual, then $Q'()$ is satisfied, whenever $\mathcal{K} \models C(y)$ (i.e. $\mathcal{K} \cup \{(\neg C)(y)\}$ is inconsistent)**

Example

Consider a query $Q_4() \leftarrow R_1(?x_1, ?x_2), R_2(?x_1, ?x_3), C_2(?x_3)$. This query can be rolled-up into the query $Q_4' \leftarrow (\exists R_1 \cdot \top \sqcap \exists R_2 \cdot C_2)(?x_1)$. This query is satisfiable in $\mathcal{K}_4$, as $\mathcal{K}_4 \cup \{(\exists R_1 \cdot \top \sqcap \exists R_2 \cdot C_2) \sqsubseteq \bot\}$ is inconsistent.

Satisfiability of $\mathcal{ALC}$ Boolean Queries (2)

... after rolling-up the query we obtain the query $Q()' \leftarrow C(y)$, that is satisfied in $\mathcal{K}$, iff $Q()$ is satisfied in $\mathcal{K}$:

- **If y is an individual, then $Q'()$ is satisfied, whenever $\mathcal{K} \models C(y)$ (i.e. $\mathcal{K} \cup \{(\neg C)(y)\}$ is inconsistent)**
- **If y is a variable, then $Q'()$ is satisfied, whenever $\mathcal{K} \cup \{C \sqsubseteq \bot\}$ is inconsistent. Why ?**

Example

Consider a query $Q_4() \leftarrow R_1(?x_1, ?x_2), R_2(?x_1, ?x_3), C_2(?x_3)$. This query can be rolled-up into the query $Q_4' \leftarrow (\exists R_1 \cdot \top \sqcap \exists R_2 \cdot C_2)(?x_1)$. This query is satisfiable in $\mathcal{K}_4$, as $\mathcal{K}_4 \cup \{(\exists R_1 \cdot \top \sqcap \exists R_2 \cdot C_2) \sqsubseteq \bot\}$ is inconsistent.

Satisfiability of Boolean Queries in $\mathcal{ALC}$ (3)

... and what to do with queries with distinguished variables ?

- Let's consider just queries that form "connected component" and contain for some variable y_k at least two query atoms of the form $R_1(y_1, y_k)$ and $R_2(y_2, y_k)$.

Satisfiability of Boolean Queries in $\mathcal{ALC}$ (3)

... and what to do with queries with distinguished variables ?

- Let's consider just queries that form "connected component" and contain for some variable y_k at least two query atoms of the form $R_1(y_1, y_k)$ and $R_2(y_2, y_k)$.
- Question: *Why is it enough to take just one connected component?*

Satisfiability of Boolean Queries in $\mathcal{ALC}$ (3)

... and what to do with queries with distinguished variables ?

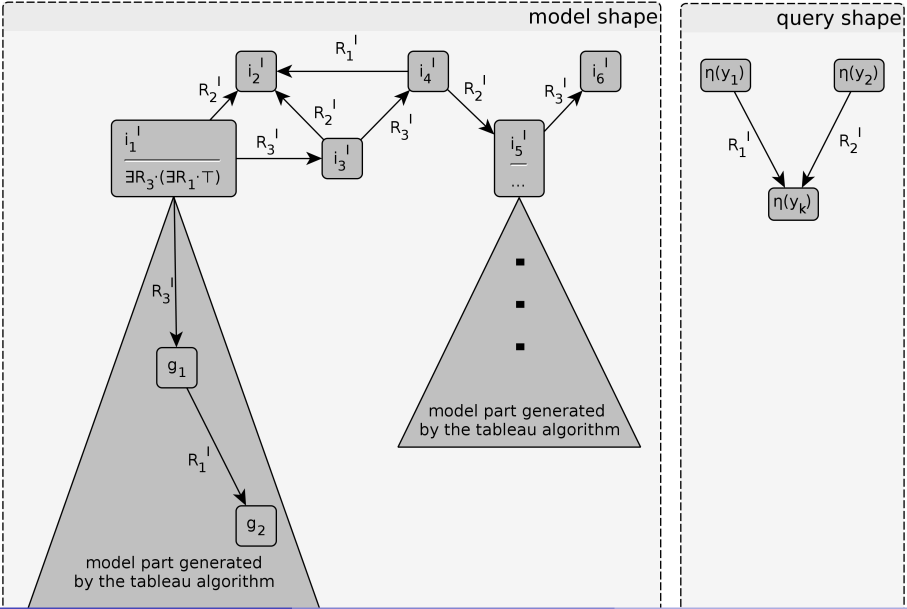
- Let's consider just queries that form "connected component" and contain for some variable y_k at least two query atoms of the form $R_1(y_1, y_k)$ and $R_2(y_2, y_k)$.
- Question: *Why is it enough to take just one connected component?*
- **Let's make use of the tree model property of $\mathcal{ALC}$. Each pair of atoms $R_1(y_1, y_k)$ and $R_2(y_2, y_k)$ can be satisfied only if y_k is interpreted as a domain element, that is an interpretation of an individual – y_k can be treated as distinguished. Why (see next slide) ?**

Satisfiability of Boolean Queries in $\mathcal{ALC}$ (3)

... and what to do with queries with distinguished variables ?

- Let's consider just queries that form "connected component" and contain for some variable y_k at least two query atoms of the form $R_1(y_1, y_k)$ and $R_2(y_2, y_k)$.
- Question: *Why is it enough to take just one connected component?*
- **Let's make use of the tree model property of $\mathcal{ALC}$. Each pair of atoms $R_1(y_1, y_k)$ and $R_2(y_2, y_k)$ can be satisfied only if y_k is interpreted as a domain element, that is an interpretation of an individual – y_k can be treated as distinguished. Why (see next slide) ?**
- For $\mathcal{SHOIN}$ and $\mathcal{SROIQ}$ there is no sound and complete decision procedure for general boolean queries.

$\mathcal{ALC}$ Model Example



Queries with Distinguished Variables – naive pruning

Consider arbitrary query $Q(?x_1, \ldots, ?x_D)$. How to evaluate it ?

- **naive way**: Replace each distinguished variable x_i with each individual occuring in $\mathcal{K}$. *Solutions* are those D-tuples $\langle i_1, \ldots, i_D \rangle$, for which a boolean query created from Q by replacing each x_k with i_k is satisfiable.

Queries with Distinguished Variables – naive pruning

Consider arbitrary query $Q(?x_1, \ldots, ?x_D)$. How to evaluate it ?

- **naive way**: Replace each distinguished variable x_i with each individual occuring in $\mathcal{K}$. *Solutions* are those D-tuples $\langle i_1, \ldots, i_D \rangle$, for which a boolean query created from Q by replacing each x_k with i_k is satisfiable.

Remind that $\mathcal{K}_4 = (\emptyset, \{(\exists R_1 \cdot C_1)(i_1), R_2(i_1, i_2), C_2(i_2)\})$. The query

$$Q_5(?x_1) \leftarrow R_1(?x_1, ?x_2), R_2(?x_1, ?x_3), C_2(?x_3)$$

has *solution* $\langle i_1 \rangle$ as

$$Q_5'() \leftarrow R_1(i_1, ?x_2), R_2(i_1, ?x_3), C_2(?x_3)$$

can be rolled into $Q_5''()$ for which $\mathcal{K}_4 \models Q_5''$:

$$Q_5''() \leftarrow (\exists R_1 \cdot \top \sqcap \exists R_2 \cdot C_2)(i_1)$$

Queries with Distinguished Variables – naive pruning

… another example

The query

$$Q_6(?x_1, ?x_3) \leftarrow R_1(?x_1, ?x_2), R_2(?x_1, ?x_3), C_2(?x_3)$$

has *solution* $\langle i_1, i_2 \rangle$ as

$$Q_6'() \leftarrow R_1(i_1, ?x_2), R_2(i_1, i_2), C_2(i_2)$$

can be rolled into Q_6'' for which $\mathcal{K}_4 \cup \{\mathbf{I_2(i_2)}\} \models Q_6''$.

$$Q_6''() \leftarrow (\exists R_1 \cdot \top \sqcap \exists R_2 \cdot (C_2 \sqcap I_2))(i_1).$$

Similarly $Q_7(?x_1, ?x_2) \leftarrow R_1(?x_1, ?x_2), R_2(?x_1, ?x_3), C_2(?x_3)$ has no solution.

Queries with Distinguished Variables – iterative pruning

- **... a bit more clever strategy than replacing all variables**: First, let's replace just the first variable $?x_1$ with each individual from $\mathcal{K}$, resulting in Q_2. If the subquery of Q_2 containing all query atoms from Q_2 without distinguished variables is not a logical consequence of $\mathcal{K}$, then we do not need to test potential bindings for other variables.

Queries with Distinguished Variables – iterative pruning

- **... a bit more clever strategy than replacing all variables**: First, let's replace just the first variable $?x_1$ with each individual from $\mathcal{K}$, resulting in Q_2. If the subquery of Q_2 containing all query atoms from Q_2 without distinguished variables is not a logical consequence of $\mathcal{K}$, then we do not need to test potential bindings for other variables.
- Many other optimizations are available.

Queries with Distinguished Variables – iterative pruning

For the query $Q_6(?x_1, ?x_3)$, the naive strategy needs to check four different bindings (resulting in four tableau algorithm runs)

$$\begin{gathered}\langle i_1, i_1\rangle,\\ \langle \mathbf{i_1}, \mathbf{i_2}\rangle,\\ \langle i_2, i_1\rangle,\\ \langle i_2, i_2\rangle.\end{gathered}$$

Out of them only $\langle i_1, i_2\rangle$ is a solution for Q_6. Consider only partial binding $\langle i_2\rangle$ for $?x_1$. Applying this binding to Q_6 we get $Q_7(?x_3) = R_1(i_2, ?x_2), R_2(i_2, ?x_3), C_2(?x_3)$. Its distinguished-variable-free subquery is $Q_7'() = R_1(i_2, ?x_2)$ and $\mathcal{K}_4 \nvDash Q_7'$. Because of **monotonicity** of $\mathcal{ALC}$, we do not need to check the two bindings for $?x_3$ in this case which saves us one tableau algorithm run.

Modeling Error Explanation

Petr Křemen

petr.kremen@fel.cvut.cz

December 12, 2018

Outline

1. What if OWL is not enough?

2. Complex Queries
   - Evaluation of Conjunctive Queries in $\mathcal{ALC}$

3. Modeling Error Explanation
   - Black-box methods
   - Algorithms based on CS-trees
   - Algorithm based on Reiter's Algorithm
   - Algorithm based on Reiter's Algorithm

Modeling Error Explanation

Motivation

- When an inference engine claims inconsistency of an ($\mathcal{ALC}$) theory/unsatisfiability of an ($\mathcal{ALC}$) concept, **what can we do with it ?**

Motivation

- When an inference engine claims inconsistency of an $(\mathcal{ALC})$ theory/unsatisfiability of an $(\mathcal{ALC})$ concept, **what can we do with it ?**
- We can start iterating through all axioms in the theory and look, "what went wrong".

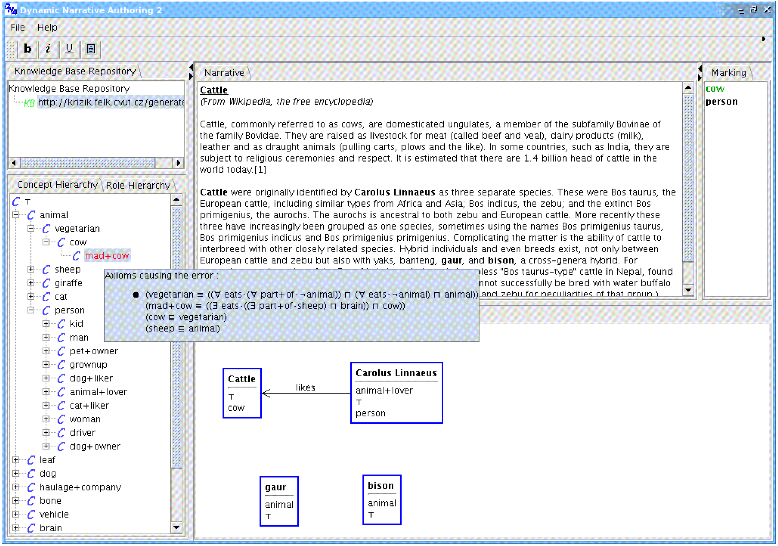
Motivation

- When an inference engine claims inconsistency of an ($\mathcal{ALC}$) theory/unsatisfiability of an ($\mathcal{ALC}$) concept, **what can we do with it ?**
- We can start iterating through all axioms in the theory and look, "what went wrong".
- ... but hardly in case we have **hundred thousand axioms**

Motivation

- When an inference engine claims inconsistency of an ($\mathcal{ALC}$) theory/unsatisfiability of an ($\mathcal{ALC}$) concept, **what can we do with it ?**
- We can start iterating through all axioms in the theory and look, “what went wrong”.
- ... but hardly in case we have **hundred thousand axioms**
- A solution might be to ask the computer to *localize the axioms causing the problem for us*.

DNA


Dynamic Narrative Authoring 2
File
Help
Knowledge Base Repository
Knowledge Base Repository
http://krizik.felk.cvut.cz/generate
Concept Hierarchy
Role Hierarchy
⊤
animal
vegetarian
cow
mad+cow
sheep
giraffe
cat
person
kid
man
pet+owner
grownup
dog+liker
animal+lover
cat+liker
woman
driver
dog+owner
leaf
dog
haulage+company
bone
vehicle
brain
Narrative
Cattle
(From Wikipedia, the free encyclopedia)
Cattle, commonly referred to as cows, are domesticated ungulates, a member of the subfamily Bovinae of the family Bovidae. They are raised as livestock for meat (called beef and veal), dairy products (milk), leather and as draught animals (pulling carts, plows and the like). In some countries, such as India, they are subject to religious ceremonies and respect. It is estimated that there are 1.4 billion head of cattle in the world today.[1]
Cattle were originally identified by Carolus Linnaeus as three separate species. These were Bos taurus, the European cattle, including similar types from Africa and Asia; Bos indicus, the zebu; and the extinct Bos primigenius, the aurochs. The aurochs is ancestral to both zebu and European cattle. More recently these three have increasingly been grouped as one species, sometimes using the names Bos primigenius taurus, Bos primigenius indicus and Bos primigenius primigenius. Complicating the matter is the ability of cattle to interbreed with other closely related species. Hybrid individuals and even breeds exist, not only between European cattle and zebu but also with yaks, banteng, gaur, and bison, a cross-genera hybrid. For
Axioms causing the error :
(vegetarian ≡ ((∀ eats·(∀ part+of·¬animal)) ⊓ (∀ eats·¬animal) ⊓ animal))
(mad+cow ≡ ((∃ eats·((∃ part+of·sheep) ⊓ brain)) ⊓ cow))
(cow ⊑ vegetarian)
(sheep ⊑ animal)
Marking
cow
person
Cattle
⊤
cow
likes
Carolus Linnaeus
animal+lover
⊤
person
gaur
animal
⊤
bison
animal
⊤

MUPS – example

Minimal unsatisfiability preserving subterminology (MUPS) is a minimal set of axioms responsible for concept unsatisfiability.

MUPS – example

Minimal unsatisfiability preserving subterminology (MUPS) is a minimal set of axioms responsible for concept unsatisfiability.

MUPS – example

Minimal unsatisfiability preserving subterminology (MUPS) is a minimal set of axioms responsible for concept unsatisfiability.

Example

Consider theory $\mathcal{K}_5 = (\{\alpha_1, \alpha_2, \alpha_3\}, \emptyset)$

$$
\begin{aligned}
\alpha_1 &: \mathit{Person} \sqsubseteq \exists \mathit{hasParent} \cdot (\mathit{Man} \sqcap \mathit{Woman}) \sqcap \forall \mathit{hasParent} \cdot \neg \mathit{Person}, \\
\alpha_2 &: \mathit{Man} \sqsubseteq \neg \mathit{Woman}, \\
\alpha_3 &: \mathit{Man} \sqcup \mathit{Woman} \sqsubseteq \mathit{Person}.
\end{aligned}
$$

Unsatisfiability of *Person* comes independently from two axiom sets (MUPSes), namely $\{\alpha_1, \alpha_2\}$ and $\{\alpha_1, \alpha_3\}$. Check it yourself !

MUPS

Currently two approaches exist for searching all MUPSes for given concept:

black-box methods perform many satisfiability tests using existing inference engine.

- ☺ flexible and easily reusable for another (description) logic
- ☹ time consuming

glass-box methods all integrated into an existing reasoning (typically tableau) algorithm.

- ☺ efficient
- ☹ hardly reusable for another (description) logic.

Glass-box methods

- For $\mathcal{ALC}$ there exists a complete algorithm with the following idea:
  - tableau algorithm for $\mathcal{ALC}$ is extended in such way that it "remembers which axioms were used during completion graph construction".
  - for each completion graph containing a clash, the axioms that were used during its construction can be transformed into a MUPS.
- Unfortunately, complete glass-box methods do not exist for OWL-DL and OWL2-DL. The same idea (tracking axioms used during completion graph construction) can be used also for these logics, but only as a preprocessing reducing the set of axioms used by a black-box algorithm.

Black-box methods

Task formulation

- Let's have *a set of axioms* X of given DL and *reasoner* R for given DL. We want to find MUPSes for :
  1. concept unsatisfiability, '
  2. theory (ontology) inconsistency,
  3. arbitrary entailment.
- It can be shown (see [0]) that w.l.o.g. we can deal only with *concept unsatisfiability*.
- **MUPS:** Let's denote $MUPS(C, Y)$ a minimal subset $MUPS(C, Y) \subseteq Y \subseteq X$ causing unsatisfiability of C.
- **Diagnose:** Let's denote $DIAG(C, Y)$ a minimal subset $DIAG(C, Y) \subseteq Y \subseteq X$, such that if $DIAG(C, Y)$ is removed from Y, the concept C becomes satisfiable.

Task formulation (2)

- Let's focus on concept C unsatisfiability. Denote

$$R(C, Y) = \left\{ \begin{array}{ll} \textit{true} & \text{iff} Y \not\models (C \sqsubseteq \bot) \\ \textit{false} & \text{iff} Y \models (C \sqsubseteq \bot)) \end{array} \right\}.$$

- There are many methods (see [0]). We introduce just two of them:
  - Algorithms based on CS-trees.
  - Algorithm for computing a single MUPS[0] + Reiter algorithm [0].

Algorithms based on CS-trees

CS-trees

- A naive solution: test for each set of axioms from $\mathcal{T} \cup \mathcal{A}$ for $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, whether the set causes unsatisfiability – minimal sets of this form are MUPSes.
- *Conflict-set trees (CS-trees)* systematize exploration of all these subsets of $\mathcal{T} \cup \mathcal{A}$. The main gist :

  > *If we found a set of axioms X that do not cause unsatisfiability of C (i.e. $X \not\models C \sqsubseteq \bot$), then we know (and thus can avoid asking reasoner) that $Y \not\models C \sqsubseteq \bot$ for each $Y \subseteq X$.*

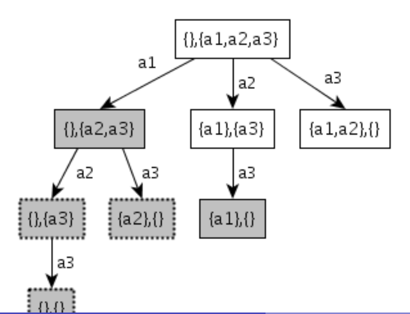
- CS-tree is a representation of the state space, where each state s has the form (D, P), where
  - D is a set of axioms that *necessarily has to be part of all MUPSes* found while exploring the subtree of s.
  - P is a set of axioms that *might be part of some MUPSes* found while exploring the subtree of s.

CS-tree Exploration – Example

Example

A CS-tree for unsatisfiability of *Person* (abbr. *Pe*, not to be mixed with the set P) in $\mathcal{K}_5 = \{\alpha_1, \alpha_2, \alpha_3\}$:

$$\underbrace{Pe \sqsubseteq \exists hP \cdot (M \sqcap W) \sqcap \forall hP \cdot \neg Pe}_{\alpha_1}, \quad \underbrace{M \sqsubseteq \neg W}_{\alpha_2}, \quad \underbrace{M \sqcup W \sqsubseteq Pe}_{\alpha_3}.$$



In gray states, the concept *Person* is satisfiable ($R(Pe, D \cup P) = true$). States with a dotted border are pruned by the algorithm.

CS-tree Exploration

The following algorithm is exponential in the number of tableau algorithm runs.

1 (Init) The root of the tree is an initial state $s_0 = (\emptyset, \mathcal{K})$ – apriori, we don't know any axiom being necessarily in a MUPS ($D_{s_0} = \emptyset$), but potentially all axioms can be there ($P_{s_0} = \mathcal{T} \cup \mathcal{A}$). Next, we define $Z = (s_0)$ and $R = \emptyset$

CS-tree Exploration

The following algorithm is exponential in the number of tableau algorithm runs.

1 (Init) The root of the tree is an initial state $s_0 = (\emptyset, \mathcal{K})$ – apriori, we don't know any axiom being necessarily in a MUPS ($D_{s_0} = \emptyset$), but potentially all axioms can be there ($P_{s_0} = \mathcal{T} \cup \mathcal{A}$). Next, we define $Z = (s_0)$ and $R = \emptyset$

2 (Depth First Search) If Z is empty, stop the exploration. Otherwise pop the first element s from Z.

CS-tree Exploration

The following algorithm is exponential in the number of tableau algorithm runs.

1 (Init) The root of the tree is an initial state $s_0 = (\emptyset, \mathcal{K})$ – apriori, we don't know any axiom being necessarily in a MUPS ($D_{s_0} = \emptyset$), but potentially all axioms can be there ($P_{s_0} = \mathcal{T} \cup \mathcal{A}$). Next, we define $Z = (s_0)$ and $R = \emptyset$

2 (Depth First Search) If Z is empty, stop the exploration. Otherwise pop the first element s from Z.

3 (Test) If $R(C, D_s \cup P_s) = \mathit{true}$ then no subset of $D_s \cup P_s$ can cause unsatisfiability – we continue with step 2.

CS-tree Exploration

The following algorithm is exponential in the number of tableau algorithm runs.

1. (Init) The root of the tree is an initial state $s_0 = (\emptyset, \mathcal{K})$ – apriori, we don't know any axiom being necessarily in a MUPS ($D_{s_0} = \emptyset$), but potentially all axioms can be there ($P_{s_0} = \mathcal{T} \cup \mathcal{A}$). Next, we define $Z = (s_0)$ and $R = \emptyset$
2. (Depth First Search) If Z is empty, stop the exploration. Otherwise pop the first element s from Z.
3. (Test) If $R(C, D_s \cup P_s) = \mathit{true}$ then no subset of $D_s \cup P_s$ can cause unsatisfiability – we continue with step 2.
4. (Finding an unsatisfiable set) We add $D_s \cup P_s$ into R and remove from R all $s' \in R$ such that $D_s \cup P_s \subseteq s'$. For $P_s = \alpha_1, \ldots, \alpha_N$ we push to Z a new state $(D_s \cup \{\alpha_1, \ldots, \alpha_{i-1}\}, P_s \setminus \{\alpha_1, \ldots, \alpha_i\})$ – we continue with step 2.

CS-tree Exploration (2)

- Soundness : Step 4 is important – here, we cover all possibilities. It always holds that $D_s \cup P_s$ differs to $D_s' \cup P_s'$ by just one element, where s' is a successor of s.

CS-tree Exploration (2)

- Soundness : Step 4 is important – here, we cover all possibilities. It always holds that $D_s \cup P_s$ differs to $D'_s \cup P'_s$ by just one element, where s' is a successor of s.
- Finiteness : Set $D_s \cup P_s$ is finite at the beginning and gets smaller with the tree depth. Furthermore, in step 4 we generate only finite number of states.

Algorithm based on Reiter's Algorithm

Another Approach – Reiter's Algorithm

There is an alternative to CS-trees:

1. Find a single (arbitrary) MUPS (*singleMUPS* in the next slides).
2. "remove the source of unsatisfiability provided by MUPS" (Reiter's algorithm in the next slides) from the set of axioms go explore the remaining axioms in the same manner.

Algorithm based on Reiter's Algorithm

Finding a single $MUPS(C, Y)$ – example

Example

The run of $singleMUPS(Person, \mathcal{K}_5)$ introduced next.

Finding a single $MUPS(C, Y)$ – example

Example

The run of $singleMUPS(Person, \mathcal{K}_5)$ introduced next.

1.PHASE :

$$\mathcal{K}_5 = \{\alpha_1, \alpha_2, \alpha_3\} \quad R(Person, \{\alpha_1\}) = true$$
$$S = \{\alpha_1\}$$

Finding a single $MUPS(C, Y)$ – example

Example

The run of $singleMUPS(Person, \mathcal{K}_5)$ introduced next.

1.PHASE :

$$\mathcal{K}_5 = \{\alpha_1, \alpha_2, \alpha_3\} \quad R(Person, \{\alpha_1, \alpha_2\}) = \mathit{false}$$
$$S = \{\alpha_1, \alpha_2\}$$

Finding a single $MUPS(C, Y)$ – example

Example

The run of $singleMUPS(Person, \mathcal{K}_5)$ introduced next.

1.PHASE :

$$\mathcal{K}_5 = \{\alpha_1, \alpha_2, \alpha_3\} \qquad R(Person, \{\alpha_1, \alpha_2\}) = \mathit{false}$$
$$S = \{\alpha_1, \alpha_2\}$$

2.PHASE :

$$S = \{\alpha_1, \alpha_2\} \qquad R(Person, \{\alpha_1, \alpha_2\} - \{\alpha_1\}) = \mathit{true}$$
$$K = \{\alpha_1\}$$

Finding a single $MUPS(C, Y)$ – example

Example

The run of $singleMUPS(Person, \mathcal{K}_5)$ introduced next.

1.PHASE :

$$\mathcal{K}_5 = \{\alpha_1, \alpha_2, \alpha_3\} \qquad R(Person, \{\alpha_1, \alpha_2\}) = false$$
$$S = \{\alpha_1, \alpha_2\}$$

2.PHASE :

$$S = \{\alpha_1, \alpha_2\} \qquad R(Person, \{\alpha_1, \alpha_2\} - \{\alpha_2\}) = true$$
$$K = \{\alpha_1, \alpha_2\}$$

singleMUPS(C, Y) – finding a single MUPS

The following algorithm is polynomial in the number of tableau algorithm applications – the computational complexity stems from the complexity of tableau algorithm itself.

1. (Initialization) Denote $S = \emptyset$, $K = \emptyset$

$singleMUPS(C, Y)$ – finding a single MUPS

The following algorithm is polynomial in the number of tableau algorithm applications – the computational complexity stems from the complexity of tableau algorithm itself.

1. (Initialization) Denote $S = \emptyset$, $K = \emptyset$
2. (Finding superset of MUPS) While $R(C, S) = \mathit{false}$, then $S = S \cup \{\alpha\}$ for some $\alpha \in Y \setminus S$.

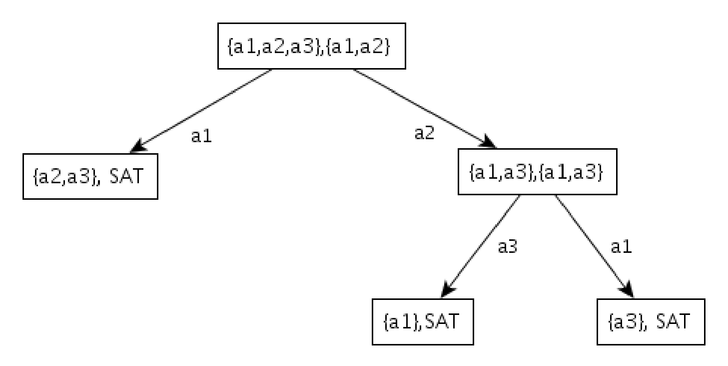
singleMUPS(C, Y) – finding a single MUPS

The following algorithm is polynomial in the number of tableau algorithm applications – the computational complexity stems from the complexity of tableau algorithm itself.

1. (Initialization) Denote $S = \emptyset$, $K = \emptyset$
2. (Finding superset of MUPS) While $R(C, S) = \mathit{false}$, then $S = S \cup \{\alpha\}$ for some $\alpha \in Y \setminus S$.
3. (Pruning found set) For each $\alpha \in S \setminus K$ evaluate $R(C, S \setminus \{\alpha\})$. If the result is *false*, then $K = K \cup \{\alpha\}$. The resulting K is itself a MUPS.

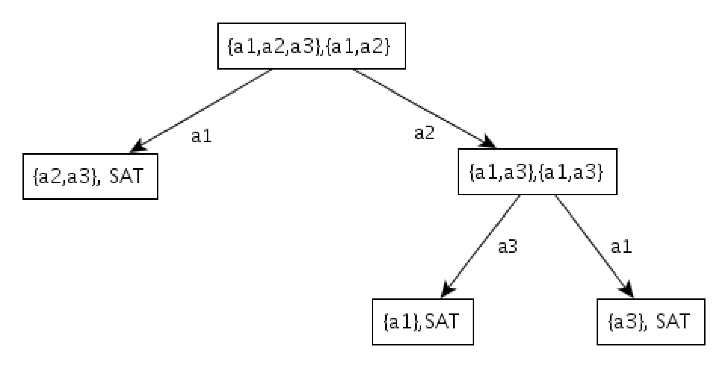
Finding all MUPSes – Reiter Algorithm, example

Example (continued)



Finding all MUPSes – Reiter Algorithm, example

Example (continued)



The algorithm ends up with two MUPSes $\{\alpha_1, \alpha_2\}$ a $\{\alpha_1, \alpha_3\}$. "For free" we got diagnoses $\{\alpha_1\}$ a $\{\alpha_2, \alpha_3\}$.

Finding all MUPSes – Reiter Algorithm

- Reiter algorithm runs $singleMUPS(C, Y)$ multiple times to construct so called "Hitting Set Tree", nodes of which are pairs $(\mathcal{K}_i, M_i)$, where $\mathcal{K}_i$ lacks some axioms comparing to $\mathcal{K}$ and $M_i = singleMUPS(C, \mathcal{K}_i)$, or $M_i =$ "SAT", if C is satisfiable w.r.t. $\mathcal{K}_i$.

Finding all MUPSes – Reiter Algorithm

- Reiter algorithm runs $singleMUPS(C, Y)$ multiple times to construct so called "Hitting Set Tree", nodes of which are pairs $(\mathcal{K}_i, M_i)$, where $\mathcal{K}_i$ lacks some axioms comparing to $\mathcal{K}$ and $M_i = singleMUPS(C, \mathcal{K}_i)$, or $M_i =$ "SAT", if C is satisfiable w.r.t. $\mathcal{K}_i$.
- Paths from the root to leaves build up *diagnoses* (i.e. minimal sets of axioms, each of which removed from $\mathcal{K}$ causes satisfiability of C).

Finding all MUPSes – Reiter Algorithm

- Reiter algorithm runs $singleMUPS(C, Y)$ multiple times to construct so called "Hitting Set Tree", nodes of which are pairs $(\mathcal{K}_i, M_i)$, where $\mathcal{K}_i$ lacks some axioms comparing to $\mathcal{K}$ and $M_i = singleMUPS(C, \mathcal{K}_i)$, or $M_i =$ "SAT'', if C is satisfiable w.r.t. $\mathcal{K}_i$.
- Paths from the root to leaves build up *diagnoses* (i.e. minimal sets of axioms, each of which removed from $\mathcal{K}$ causes satisfiability of C).
- Number of $singleMUPS(C, Y)$ calls is at most exponential w.r.t. the initial axioms count. Why ?

Finding all MUPSes – Reiter Algorithm (2)

1 (Initialization) Find a single MUPS for C in $\mathcal{K}$, and construct the root $s_0 = (\mathcal{K}, singleMUPS(C, \mathcal{K}))$ of the hitting set tree. Next, set $Z = (s_0)$.

Finding all MUPSes – Reiter Algorithm (2)

1 (Initialization) Find a single MUPS for C in $\mathcal{K}$, and construct the root $s_0 = (\mathcal{K}, singleMUPS(C, \mathcal{K}))$ of the hitting set tree. Next, set $Z = (s_0)$.

2 (Depth First Search) If Z is empty, STOP.

Finding all MUPSes – Reiter Algorithm (2)

1. (Initialization) Find a single MUPS for C in $\mathcal{K}$, and construct the root $s_0 = (\mathcal{K}, singleMUPS(C, \mathcal{K}))$ of the hitting set tree. Next, set $Z = (s_0)$.
2. (Depth First Search) If Z is empty, STOP.
3. (Test) Otherwise pop an element from Z and denote it as $s_i = (\mathcal{K}_i, M_i)$. If $M_i =$ "SAT", then go to step 2.

Finding all MUPSes – Reiter Algorithm (2)

1. (Initialization) Find a single MUPS for C in $\mathcal{K}$, and construct the root $s_0 = (\mathcal{K}, singleMUPS(C, \mathcal{K}))$ of the hitting set tree. Next, set $Z = (s_0)$.
2. (Depth First Search) If Z is empty, STOP.
3. (Test) Otherwise pop an element from Z and denote it as $s_i = (\mathcal{K}_i, M_i)$. If $M_i = \text{``}SAT''$, then go to step 2.
4. (Decomposition) For each $\alpha \in M_i$ insert into Z a new node $(\mathcal{K}_i \setminus \{\alpha\}, singleMUPS(\mathcal{K}_i \setminus \{\alpha\}, C))$. Go to step 2.

Modeling Error Explanation – Summary

- finding MUPSes is the most common way for explaining modeling errors.
- black-box vs. glass box methods. Other methods involve e.g. incremental methods [0].
- the goal is to find MUPSes (and diagnoses) – what to do in order to solve a modeling problem (unsatisfiability,inconsistency).
- above mentioned methods are quite universal – they can be used for many other problems that are not related with description logics.