

Name:

Date:

**Written Exam — Logical Reasoning and Programming (sample)**

**Convention:** Variables are denoted by capital letters while other symbols by lower case letters.

1. The following Prolog program is given:

```

pass(S):-clever(S).
pass(S1):-next_to(S1,S2),pass(S2).
clever(trevor).
next_to(trevor,bob).
next_to(alice,carol).
next_to(X,Y):-next_to(Y,X).

```

(a) How many proofs are there for the query

```
?-pass(bob).
```

Explain your answer and give one of the proof trees. (3 points)

(b) Indicate all answers Prolog would find, and the order in which they are returned (duplicates included), for the query

```
?-pass(X).
```

(3 points)

(c) Discuss two possible adaptations to Prolog's search strategy, so that it would return all answers.

(2 points)

(d) Show that

```
pass(alice)
```

is not a logical consequence of the program. (3 points)

2. In the 3rd column, write the substitution under which the Prolog terms in the first two columns unify, or leave the cell empty if they do not unify.

|             |               |                                     |            |
|-------------|---------------|-------------------------------------|------------|
| $f([a,b])$  | $f([X,Y,Z])$  | $X = \quad , Y = \quad , Z = \quad$ | (4 points) |
| $[X _] + 1$ | $[1,2,3] + X$ | $X =$                               |            |
| $2 + 3$     | $X$           | $X =$                               |            |
| $[a X]$     | $[a,b,[c,d]]$ | $X =$                               |            |

3. You want to find a counter-example for a small problem using a MACE-style model finder like Paradox. What is roughly (in powers of ten) the maximal size (in the number of elements) of models you can obtain

- (a) 10      (b) 100      (c) 1,000      (d) 10,000      (e) 100,000      (f) 1,000,000

(2 points)

4. Show by resolution or tableaux that from

$$\forall X \forall Y \forall Z (r(X,Y) \wedge r(Y,Z) \rightarrow r(X,Z)),$$

$$\forall X \forall Y (r(X,Y) \vee r(Y,X))$$

all the following formulae are provable

- (a)  $\forall X(r(X, X))$ ,
- (b)  $\forall X\exists Y(r(X, Y) \vee r(Y, X))$ ,
- (c)  $\forall X\forall Y\forall Z\forall W(r(X, Y) \wedge r(Y, Z) \wedge r(Z, W) \rightarrow r(X, W))$ .

(10 points)

5. Write down all the pairs of formulae from the following list such that the first formula subsumes the second one

- (a)  $s(g(c)) \vee p(X, g(X))$ ,
- (b)  $p(A, g(A))$ ,
- (c)  $s(g(X)) \vee p(c, g(X))$ ,
- (d)  $s(a)$ ,
- (e)  $s(a) \vee p(X, g(a))$ .

(4 points)

6. Write down all the possible answer sets for the program

$$\begin{aligned}
 P = \{ & p(a). \\
 & q(a) \leftarrow p(a), \text{not } r(a). \\
 & r(a) \leftarrow p(a), \text{not } q(a). \}
 \end{aligned}$$

(4 points)

7. Why is it useful to delete learned clauses in CDCL? What can happen if we do not do that?

(2 points)

8. Describe the basic loop in a lazy SMT solver.

(4 points)

9. Let  $\varphi$  be a formula and  $\varphi'$  be a Skolem normal form of  $\varphi$ . Decide which of the following options are always true.

- (a)  $\{\varphi\} \models \varphi'$ ,
- (b)  $\{\varphi'\} \models \varphi$ ,
- (c)  $\{\varphi, \varphi'\}$  is satisfiable,
- (d)  $\varphi$  is unsatisfiable iff  $\varphi'$  is unsatisfiable.

(4 points)