Logical reasoning and programming, lab session IX

(November 26, 2018)

IX.1 Produce equivalent formulae in prenex form:

(a) $\forall X(p(X) \rightarrow \forall Y(q(X,Y) \rightarrow \neg\forall Z r(Y,Z)))$,
(b) $\exists X p(X,Y) \rightarrow (q(X) \rightarrow \neg\forall Z p(X,Z))$,
(c) $\exists X p(X,Y) \rightarrow (q(X) \rightarrow \neg\exists Z p(X,Z))$,
(d) $p(X,Y) \rightarrow \exists Y(q(Y) \rightarrow (\exists X q(X) \rightarrow r(Y)))$,
(e) $\forall Y q(Y) \rightarrow (\forall X q(X) \rightarrow r(Z))$.

IX.2 In **IX.1** you could obtain in some cases various prefixes; the order of quantifiers can be different. Are all these variants correct?

IX.3 Can we produce a formula equivalent to **IX.1**e with just one quantifier?

IX.4 Produce Skolemized formulae equisatisfiable with those in **IX.1**. Try to produce as simple as possible Skolem functions.

IX.5 Skolemize the following formula

$$\forall X(p(a) \vee \exists Y(q(Y) \wedge \forall Z(p(Y,Z) \vee \exists U q(X,Y)))) \vee \exists W q(a,W).$$

Why is it possible in this particular case to do that without producing an equivalent formula in prenex form?

IX.6 Unify the following pairs of formulae:

(a) $\{p(X,Y) \doteq p(Y,f(Z))\}$,
(b) $\{p(a,Y,f(Y)) \doteq p(Z,Z,U)\}$,
(c) $\{p(X,g(X)) \doteq p(Y,Y)\}$,
(d) $\{p(X,g(X),Y) \doteq p(Z,U,g(U))\}$,
(e) $\{p(g(X),Y) \doteq p(Y,Y), p(Y,Y) \doteq p(U,f(W))\}$.

IX.7 What is the size of the maximal term that is produced when you try to unify

$$\{f(g(X_1,X_1),g(X_2,X_2),\ldots,g(X_{n-1},X_{n-1})) \doteq f(X_2,X_3,\ldots,X_n\}$$