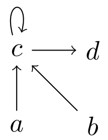
Logical reasoning and programming, lab session VIII

(November 19, 2018)

VIII.1 We have a language that contains only one binary predicate symbol $\in$ and we have an interpretation $\mathcal{M} = (D, i)$ such that $D = \{a, b, c, d\}$ and $i(\in)$ is given by the following diagram:



Meaning that $x \in y$ iff thery is an arrow from x to y. Decide whether the following formulae are valid in $\mathcal{M}$:

(a) $\exists X \forall Y (\neg(Y \in X))$,

(b) $\exists X \forall Y (Y \in X)$,

(c) $\exists X \forall Y (Y \in X \leftrightarrow Y \in Y)$,

(d) $\exists X \forall Y (Y \in X \leftrightarrow \neg(Y \in Y))$.

VIII.2 Show that the following formulae are valid and provide counter-examples for the opposite implications:

(a) $\forall X p(X) \vee \forall X q(X) \rightarrow \forall X (p(X) \vee q(X))$,

(b) $\exists X (p(X) \wedge q(X)) \rightarrow \exists X p(X) \wedge \exists X q(X)$,

(c) $\exists X \forall Y p(X, Y) \rightarrow \forall Y \exists X p(X, Y)$,

(d) $\forall X p(X) \rightarrow \exists X p(X)$.

VIII.3 Decide whether for any formula φ holds:

(a) $\varphi \equiv \forall \varphi$,

(b) $\varphi \equiv \exists \varphi$,

(c) $\models \varphi$ iff $\models \forall \varphi$,

(d) $\models \varphi$ iff $\models \exists \varphi$,

where $\forall \varphi$ ($\exists \varphi$) is the universal (existential) closure of φ. If not, does at least one implication hold?

VIII.4 Show that for any set of formulae Γ and a formula φ it holds, if $\Gamma \models \varphi$, then $\forall \Gamma \models \varphi$, where $\forall \Gamma = \{\forall \psi \colon \psi \in \Gamma\}$. Does the opposite direction hold?

VIII.5 Find a set of formulae Γ and a formula φ such that $\Gamma \models \varphi$ and $\Gamma \models \neg \varphi$.