Logical reasoning and programming, lab session XII

(December 17, 2018)

XII.1 Prove the following formulae using tableaux

(a) $\exists X \forall Y r(X,Y) \rightarrow \forall Y \exists X r(X,Y)$,

(b) $\exists X(p(X) \rightarrow \forall X p(X))$,

(c) $\forall X \forall Y(p(X) \wedge p(Y)) \rightarrow \exists X \exists Y(p(X) \vee p(Y))$,

(d) $\forall X \forall Y(p(X) \wedge p(Y)) \rightarrow \forall X \forall Y(p(X) \vee p(Y))$,

(e) $\forall X \exists Y \forall Z \exists W(r(X,Y) \vee \neg r(W,Z))$,

(f) $\exists X(\forall Y \forall Z p(Y, f(X,Y,Z)) \rightarrow (\forall Y p(Y, f(X,Y,X)) \wedge \forall Y \exists Z p(g(Y), Z)))$.

XII.2 Try all the examples in the SMT-LIB Examples. You can use an online version of Z3, or you can install Z3 or CVC4 yourself. Another option is to use pySMT, a convenient way how to experiment with various SMT solvers in Python. If you want to learn a bit more about the Z3 prover, you should start with the tutorial.

XII.3 If we want to combine theories in SMT using the Nelson–Oppen method, we require that both of them are stably infinite. Assume two theories $\mathcal{T}_1$ with the language $\{f\}$ and $\mathcal{T}_2$ with the language $\{g\}$, where f and g are uninterpreted unary function symbols. Moreover, $\mathcal{T}_1$ has only models of size at most 2 (for example, it contains $\forall X \forall Y \forall Z(X = Y \vee X = Z)$ as an axiom). Show that the Nelson–Oppen method says that

$$f(x_1) \neq f(x_2) \wedge g(x_2) \neq g(x_3) \wedge g(x_1) \neq g(x_3).$$

is satisfiable in the union of $\mathcal{T}_1$ and $\mathcal{T}_2$, but this is clearly incorrect.