Logical reasoning and programming, lab session X

(December 3, 2018)

X.1 Show that the resolution rule is correct.

X.2 Derive the empty clause $\Box$ using the resolution calculus from:

(a) $\{\{\neg p(X), \neg p(f(X))\}, \{p(f(X)), p(X)\}, \{\neg p(X), p(f(X))\}\}$

(b) $\{\{\neg p(X,a), \neg p(X,Y), \neg p(Y,X)\}, \{p(X,f(X)), p(X,a)\}, \{p(f(X),X), p(X,a)\}\}$

X.3 Prove using the resolution calculus that from

$$\begin{aligned}
&\forall X \forall Y (p(X,Y) \rightarrow p(Y,X)) \\
&\forall X \forall Y \forall Z ((p(X,Y) \wedge p(Y,Z)) \rightarrow p(X,Z)) \\
&\forall X \exists Y \, p(X,Y)
\end{aligned}$$

follows $\forall X p(X,X)$.

X.4 List all possible applications of the factoring rule on the clause

$$\{p(X,f(Y),Z), \neg s(Z,T), p(T,T,g(a)), p(f(b),S,g(W)), \neg s(c,d)\}.$$

If it is possible to use the factoring rule several times, then produce even these results.

X.5 Formulate the following problems in the TPTP language and (dis)prove them using the E prover. Assuming the following group axioms

$$\begin{aligned}
&e \cdot X = X, \\
&X^{-1} \cdot X = e, \\
&(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)
\end{aligned}$$

your task is to (dis)prove

(a) $X \cdot e = X$,

(b) $X \cdot X^{-1} = e$,

(c) $X \cdot Y = Y \cdot X$,

(d) $X \cdot Y = Y^{-1} \cdot X^{-1}$.