



**DCGI**

**KATEDRA POČÍTAČOVÉ GRAFIKY A INTERAKCE**

# Visibility Algorithms

Jiří Bittner

# Outline

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- Visibility in graphics
- Depth Buffer
- Ray Casting
- Painter's algorithm
- BSP Trees
- Warnock's Algorithm
- Specialized Visibility Algorithms

MPG – chapter 11

# Visibility – What is that?

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[Premysl – panoramio.com]



[Julian Barker – flickr.com]

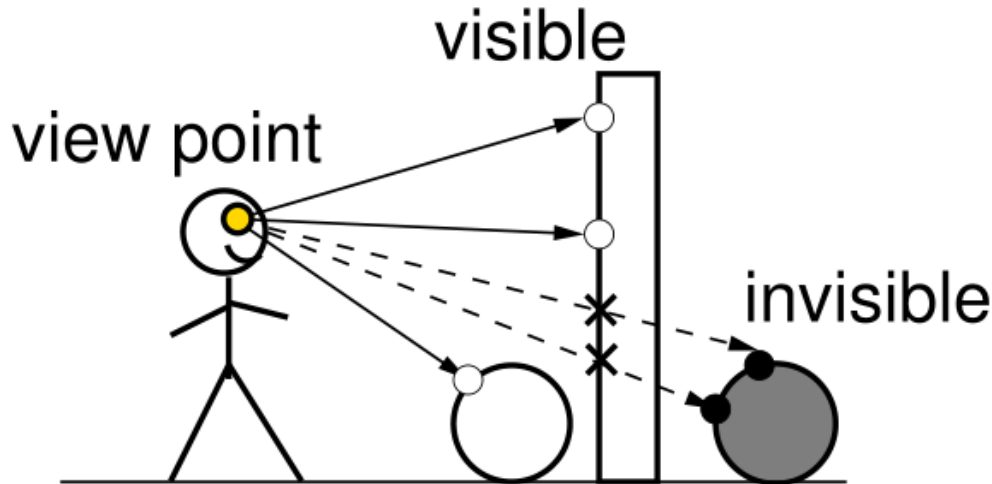


[Tim Bow – flickr.com]

# Visibility - Introduction

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- Points A,B visible  $\Leftrightarrow$  line segment AB does not intersect opaque object
- Example: visibility from a view point



# Visibility in Computer Graphics

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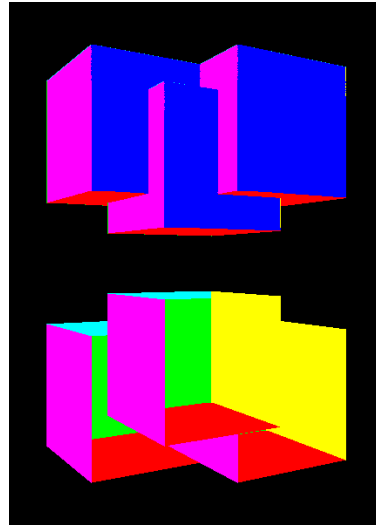
- Hidden surface removal
- Shadows
- Radiosity
- Ray Tracing
- Visibility culling
  
- Games / Multi-User Environments
- Streaming

# Hidden surface removal

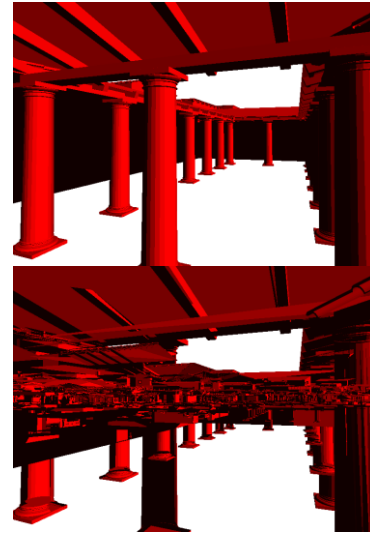
---

- Creating “correct” 2D image of 3D scene
  - Finding visible objects and their visible parts
  - Eliminating invisible objects and invisible parts

ON



OFF



# Visibility algorithms

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- Raster algorithms (image space)
  - Solve visibility for pixels
  - For each pixel
    - Find nearest object projected to pixel
    - Shade the pixel using object color
  - Algorithms: z-buffer, ray casting, painters alg.
- Vector algorithms (object space)
  - Vector based description of visibility
  - For each object
    - Find object parts not hidden by others
    - Draw visible/invisible parts
  - Algorithms: Naylor, Weiler-Atherton, Roberts
  - CAD systems, technical drawings, special applications

Complexity:  $O(P.N)$

P .. #pixels  
N .. #objects

Complexity:  $O(N^2)$

# Outline

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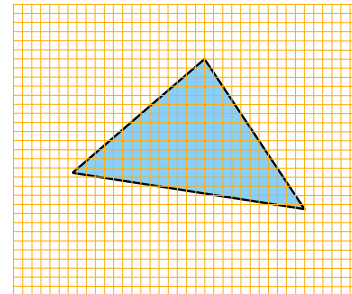
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# Depth buffer (Z-buffer)

- Ed Catmull – 1975
  - Co-founder and president of Pixar
- Wolfgang Strasser - 1975
- For each pixel depth of the nearest object
- Process objects in **arbitrary order**
  1. Rasterize to fragments
  2. Compare depth of each fragment with z-buffer content
  3. If closer overwrite z-buffer and pixel color



# Depth buffer – pseudocode

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- Two arrays: `z_buffer`, `color_buffer`

```
Clear color_buffer;  
Set z-buffer to “infinity”;  
for (each object) {  
  for (each object pixel P[x,y]) {  
    if (z-buffer[x,y] > P[x,y].depth) {  
      z_buffer[x,y] = P[x,y].depth;  
      color_buffer[x,y] = P[x,y].color;  
    }  
  }  
}
```

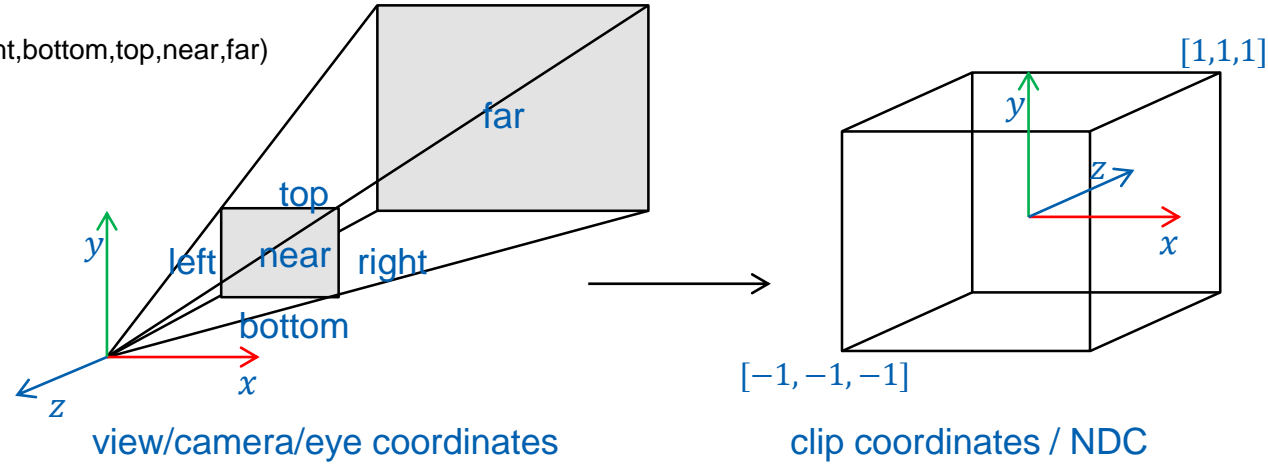
# Depth buffer - details

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- Computing pixel depth - interpolation
- Linear interpolation of  $z'' \sim 1/z$  ( $z''$  - device coordinates)
- For perspective projection depth resolution is non-uniform
  - Nearer objects have higher depth resolution
- **z-fighting** when rendering farther objects

# Perspective projection - OpenGL

- glFrustum(left,right,bottom,top,near,far)



$$M = \begin{bmatrix} \frac{2near}{right - left} & 0 & \frac{right + left}{right - left} & 0 \\ 0 & \frac{2near}{top - bottom} & \frac{top + bottom}{top - bottom} & 0 \\ 0 & 0 & \frac{near + far}{near - far} & \frac{2 far near}{near - far} \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

# Perspective projection

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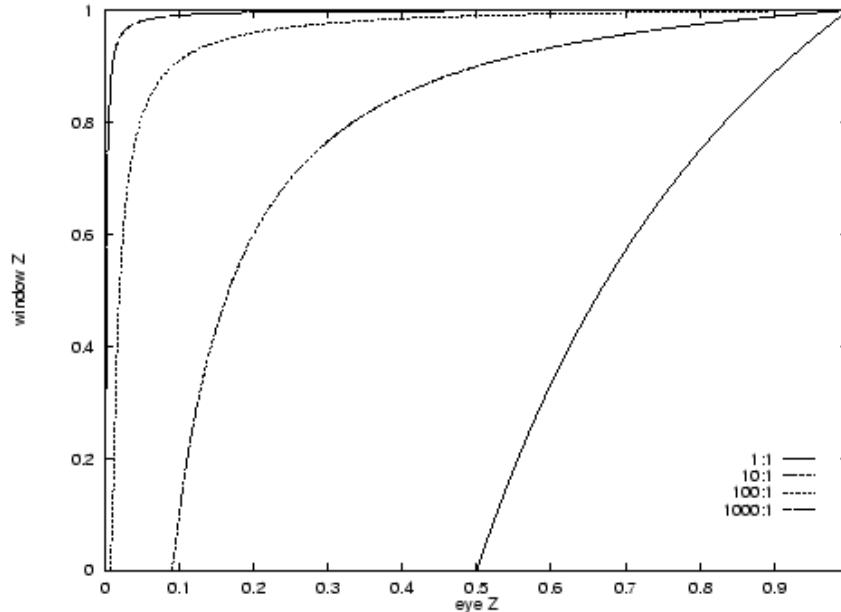
$$M = \begin{bmatrix} \frac{2near}{right - left} & 0 & \frac{right + left}{right - left} & 0 \\ 0 & \frac{2near}{top - bottom} & \frac{top + bottom}{top - bottom} & 0 \\ 0 & 0 & \frac{near + far}{near - far} & \frac{2 far near}{near - far} \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$x' = \frac{2 near}{left - right} \frac{x}{z} - \frac{right + left}{right - left}$$

$$z' = \frac{near + far}{far - near} + \frac{2 far near}{far - near} \frac{1}{z}$$

# Depth distributions in z-buffer

- Careful setting of near-far planes
  - near = 1 / far = 10 : 50% between 1.0 a 1.8
  - near = 0.01 / far = 10 : 90% between 0.01 – 0.1
  - Median =  $2 \cdot \text{near} \cdot \text{far} / (\text{near} + \text{far})$



# Resolving Z-fighting

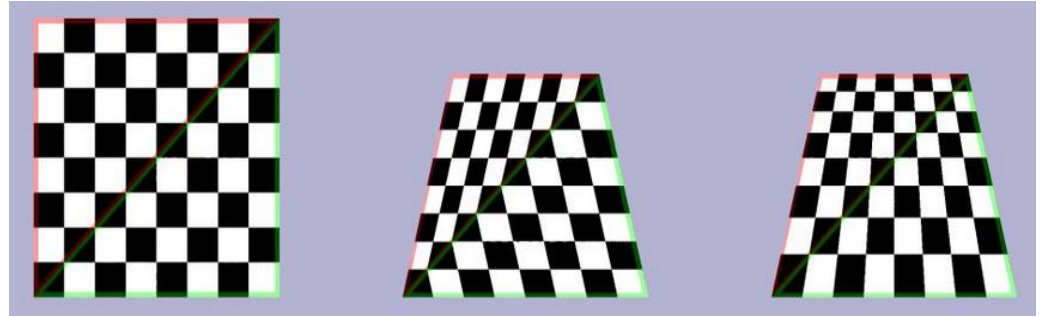
---

- Careful settings of near(!) and far planes
- Rendering close and far objects
  - Several passes, updating near/far
  - combine using stencil
- W-buffer
  - stores eye space  $z$ , linear depth distribution
  - reciprocal of  $z_i'$  for each pixel

# Perspectively correct interpolation

- LERP in screen space

- non linear in object space (hyperbola) !



- Solution for color

- Compute  $c' = c/z$  and  $z' = 1/z$
- LERP of  $c'$  and  $z'$
- For each pixel  $c_i = c'_i / z'_i$

- The same for texture coordinates  $u, v$  (!)



# Depth buffer - properties

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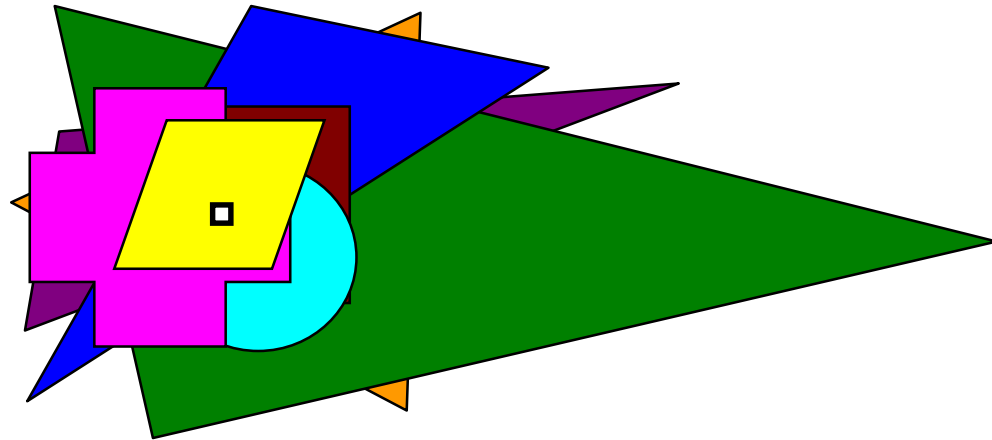
- **Benefits**
  - Simplicity
  - No preprocessing or sorting
  - Easy parallelization and HW implementation
- **Issues**
  - Pixel overdraw
  - Mapping depth to z-buffer bit range
  - Transparent objects
  - Alias

# Quiz – number of overdraws

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- 10 polygons project to pixel in random order
- What is the average number of overdraws?

- a) 3
- b) 5.5
- c) 7

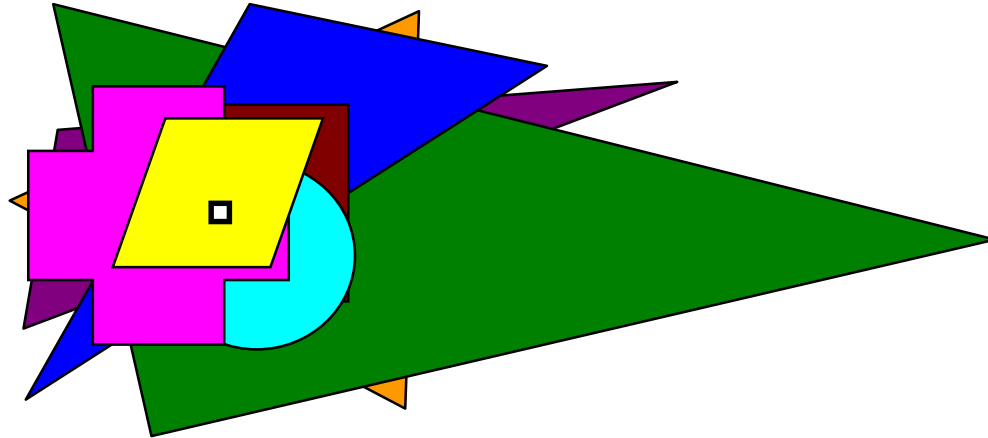


Source: Eric Haines - Subtle Tools

# Intuitive answer

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- Front-to-back 1x, back-to-front 10x
- So the average is 5.5 overdraws



# Correct answer

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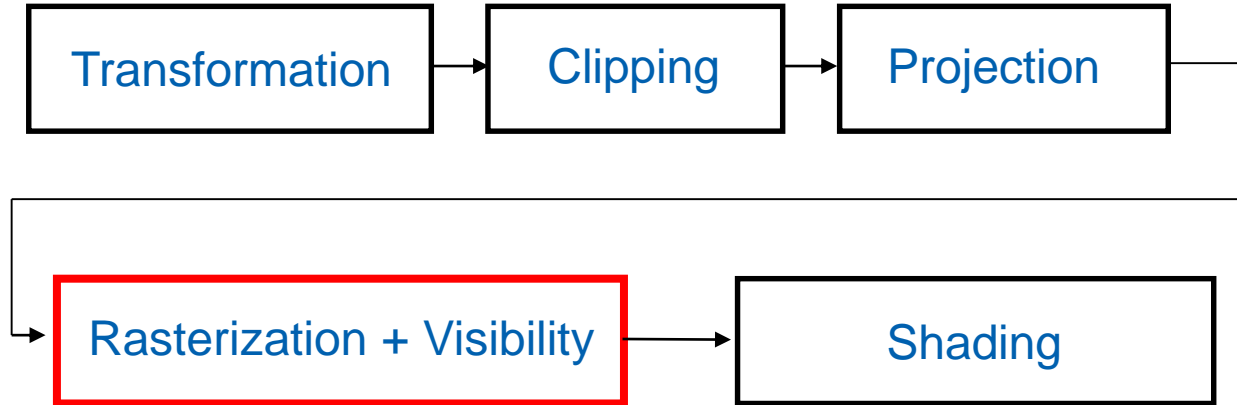
- The first polygon must cause overflow: 1
- The second is either back or front
  - Chance of overflow:  $\frac{1}{2}$
- Third polygon
  - $\frac{1}{3}$  chance that it is the closest and causes overflow
- Harmonic series:  $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{10} = 2.9289$

1 poly	1x
4 polys	2.08x
11 polys	3.02x
31 polys	4.03x
83 polys	5.00x
12,367 polys	10.00x

Aproximation for big N  
 $\text{overflow}(N) = \ln(N) + 0.57721$

# Depth buffer in image pipeline

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# Depth buffer in OpenGL

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- `glutInitDisplayMode (... | GLUT_DEPTH | ... );`
- `glEnable(GL_DEPTH_TEST);`
- `glDepthFunc(GL_LESS);`
- `glClear(GL_DEPTH_BUFFER_BIT);`
- `glDepthMask(mask);`
  - `GL_TRUE` read/write
  - `GL_FALSE` read only

# Depth buffer and transparent objects

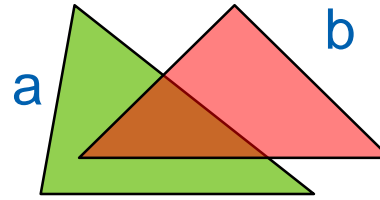
---

- Draw all non-transparent objects using z-buffer
- Sort all transparent objects back-to-front
- Render transparent objects with alpha-blending
  - OpenGL:
    - `glDepthMask(GL_FALSE);`
    - `glBlendFunc(gl.ONE, gl.ONE_MINUS_SRC_ALPHA);`
    - `glEnable(GL_BLEND);`

# Alpha blending – Over operator

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- $C = (r, g, b, \alpha)$
- $\alpha$  opacity
  - $\alpha = 0$  transparent
  - $\alpha = 1$  opaque



a over b

$$C = \alpha_a C_a + \alpha_b C_b (1 - \alpha_a)$$
$$\alpha = \alpha_a + \alpha_b (1 - \alpha_a)$$

$$c = c_a + c_b (1 - \alpha_a)$$
$$c_a = \alpha_a C_a$$
$$c_b = \alpha_b C_b$$

“pre-multiplied alpha”



# Depth buffer – Questions

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- Should we draw back to front or front to back? And should we care?
- How to increase depth resolution?
- When to perform the depth test?
- How to handle transparent objects?

# Outline

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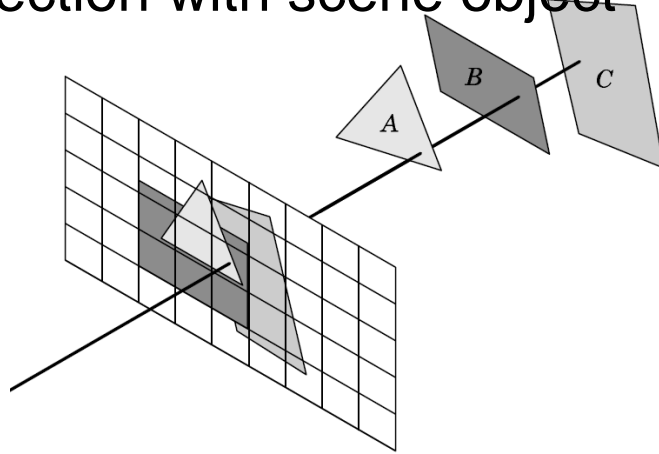
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# Ray casting

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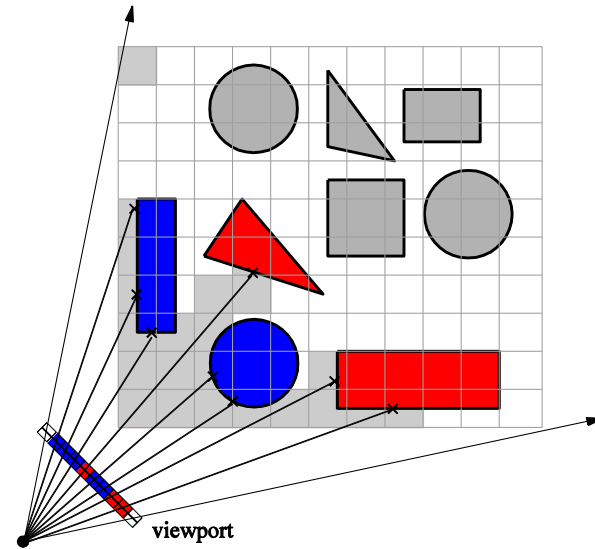
- Cast ray for each image pixel [Appel68]
- Find the nearest intersection with scene object



- Complexity
  - Naive:  $O(R \cdot N)$
  - With spatial data structure:  $O(R \cdot \log N)$

# Accelerated ray casting

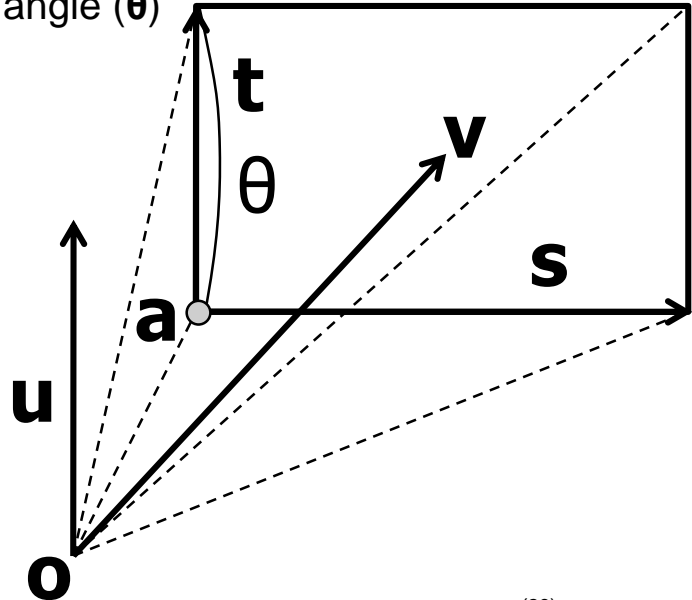
- Step 1: construct spatial DS
  - Preprocessing
  - BVH, kD-tree, octree, 3D grid
- Step 2: find the nearest intersection
  - Walk through cells intersected by the ray
  - Intersection found: terminate



# Ray Casting – Generating (Primary) Rays

- Implicit camera parameters
  - MVP matrix inversion
- Explicit knowledge of camera parameters
  - position ( $\mathbf{o}$ ), view direction ( $\mathbf{v}$ ), up vector ( $\mathbf{u}$ ), view angle ( $\theta$ )

1. Compute view coordinate system:  $\mathbf{a}$ ,  $\mathbf{s}$ ,  $\mathbf{t}$
2. Ray through pixel  $x$ ,  $y$  (image size width  $\times$  height):  
ray\_origin =  $\mathbf{o}$ ;  
ray\_dir =  $\text{Normalize}(\mathbf{a} + x/\text{width} * \mathbf{s} + y/\text{height} * \mathbf{t} - \mathbf{o})$ ;



# Ray casting - properties

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- Benefits

- Flexibility (adaptive raster, ray tracing)
- Efficient culling of occluded objects

- Drawbacks

- Lower use of coherence
- Requires spatial DS
  - Issue for dynamic scenes and HW implementation

# Z-buffer vs. Ray Casting

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	Scan-line coherence	Requires preprocessing	Efficient handling of occluded objects
<b>Z-buffer</b>	yes +	no +	no -
<b>Ray casting</b>	no -	yes -	yes +

Z-buffer better for dynamic scenes with low occlusion

Ray casting better for complex highly occluded scenes

# Z-buffer GPU optimizations

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- Z-cull
  - $z_{\min}, z_{\max}$  for 8x8 pixel blocks
  - If  $\text{tri}_{z_{\min}} > \text{tile}_{z_{\max}}$  discard
- Early-z test (for each pixel)
  - Apply z-test before shader execution
  - On newer GPUs used by default
  - Switched off when modifying “z” in shader
- HW occlusion queries, conditional rendering



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# Painter's algorithm

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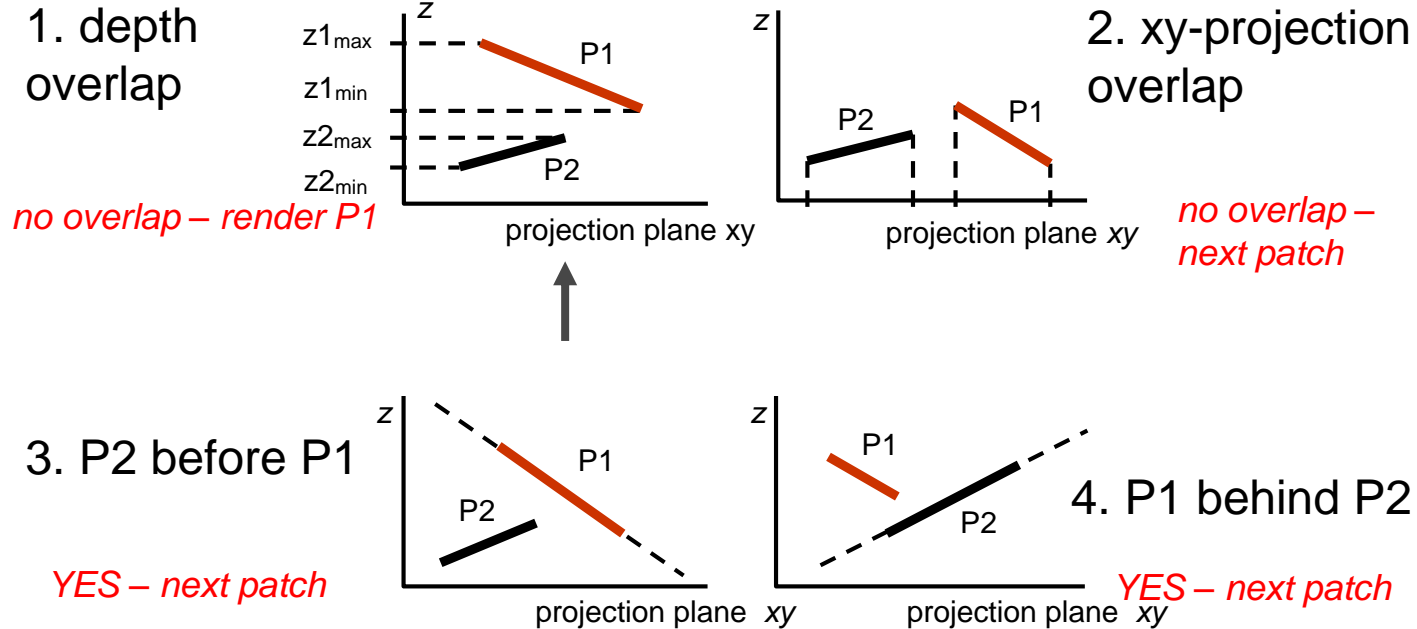
- Rendering back to front
- Farther patches overwritten by closer ones
- Used in 2D drawing tools (layers)
  
- In 3D without explicit ordering more complicated
- Depth sort algorithm [Newell72]

# Depth Sort Painter's algorithm

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- Sort patches using  $z_{\max}$  of each patch
- Farthest patch = candidate for rendering (P1)
- Series of tests to confirm the candidate using remaining patches

# Depth Sort Painter's algorithm – cont.

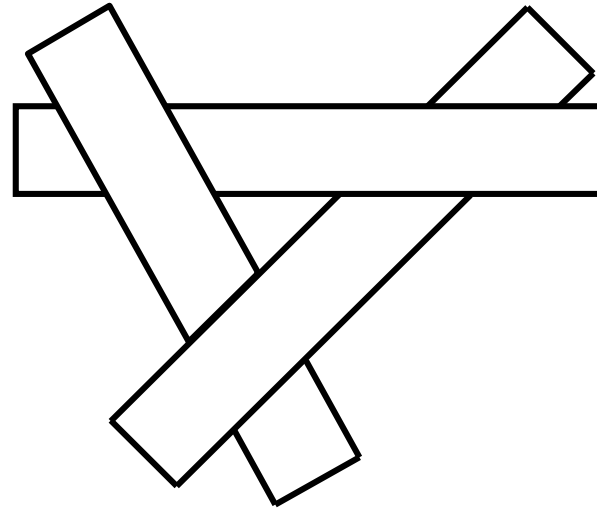
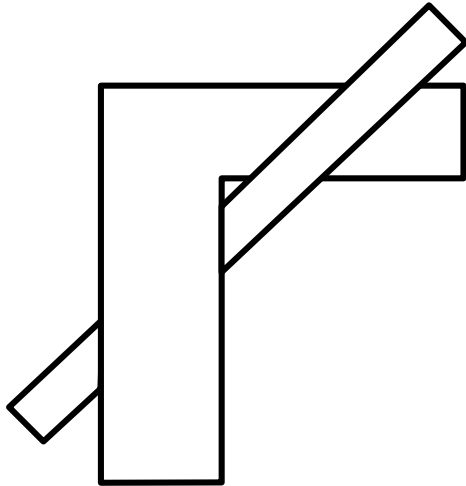


Tests failed: swap (P2 = new candidate)

# Cycle of candidates

---

- Can be detected using counter for candidate
- Solved by cutting the patch



# Painter's algorithm - properties

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- **Benefits**
  - No depth buffer needed
  - Simplified version: easy implementation
- **Issues**
  - Overdraw
  - Correct depth order
  - Self intersections of patches not allowed

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# Binary Space Partitioning (*BSP*)

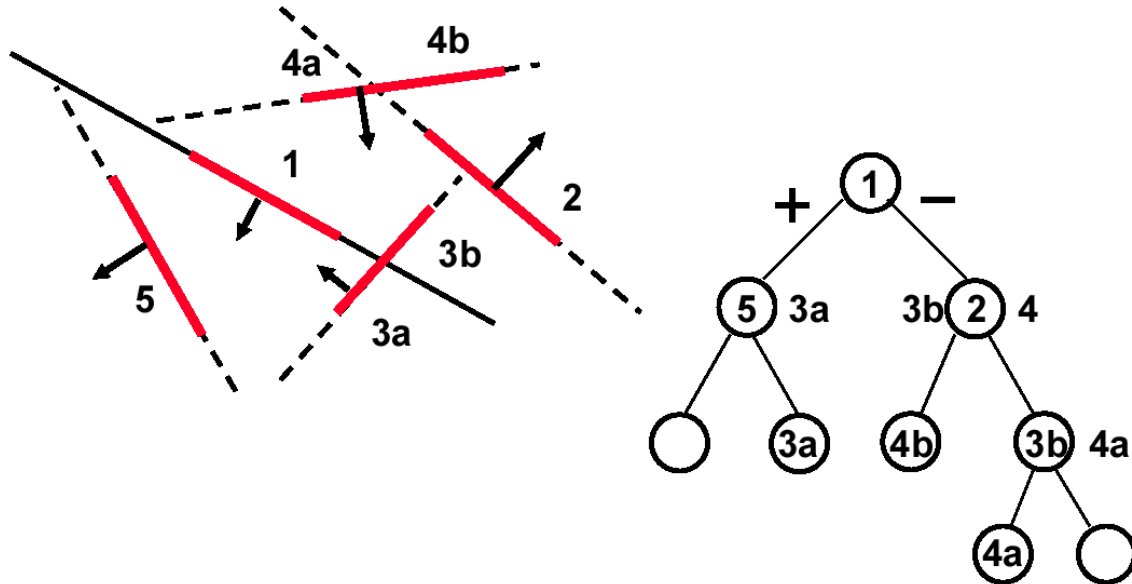
---

- View independent sorting of the scene [Fuchs80]
- Two phases
  - BSP tree construction (1x)
  - Tree traversal and rendering (as painter's alg.)



# BSP Tree Construction

- Recursive splitting by planes
- Planes typically defined using scene polygons



# Rendering with BSP tree

---

```
void RenderBSP (Node S)
if (camera in front of S.plane) {
    RenderBSP (S.back);
    Render(S.polygons);
    RenderBSP (S.front);
}
else {
    RenderBSP (S.front);
    Render(S.polygons);
    RenderBSP (S.back);
}
}
```

# BSP tree and Z-buffer

---

- Reduce number of overdraws
- Traverse front-to-back (reverse order compared to painter's alg.)
- Alternatives to BSP tree
  - kD tree, octree, BVH

# Outline

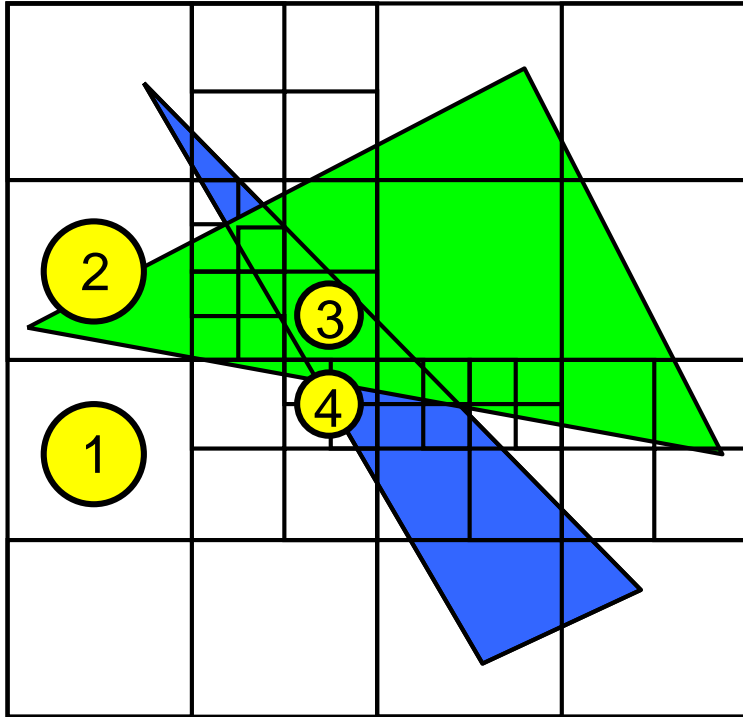
---

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# Image Subdivision – Warnock's alg.

- Recursive fast rectangle clipping tests
- Recursion terminates in pixel /subpixel



Divide and Conquer [Warnock69]

1. No object: *background color*
2. One object: *render*
3. More objects, one closest: *render closest*
4. *Rozděl rekurzivně*

# Scan-line Algorithms

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- Sort primitives by scan lines (Y)
- Compute spans: intersections of primitives and scan lines
- Elementary spans: intersection of spans
- Sort elementary spans (X)
- Find the closest object for each elementary span (Z)
- [Watkins70]
  - Bubble sort for X and Y
  - $O(\log n)$  search for Z

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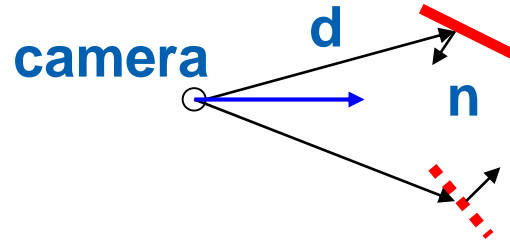
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# Back-face Culling

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- Eliminates ~ 50% polygons
- If  $d \cdot n > 0$  : cull
- In NDC: just check for sign of  $n'_z$ 
  - Computed from transformed vertices (not shading normal)
- OpenGL:

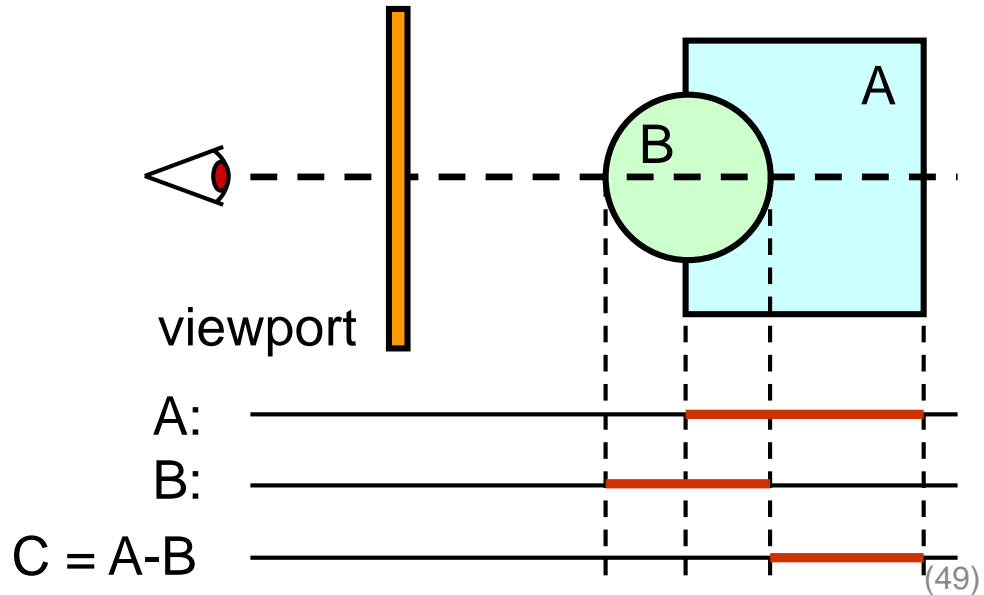
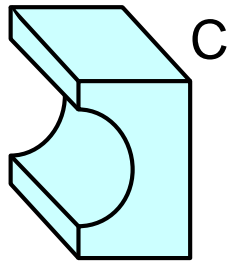
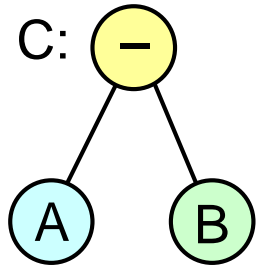
```
glFrontFace(GL_CCW);  
glCullFace(GL_FRONT);  
glEnable(GL_CULL_FACE);
```





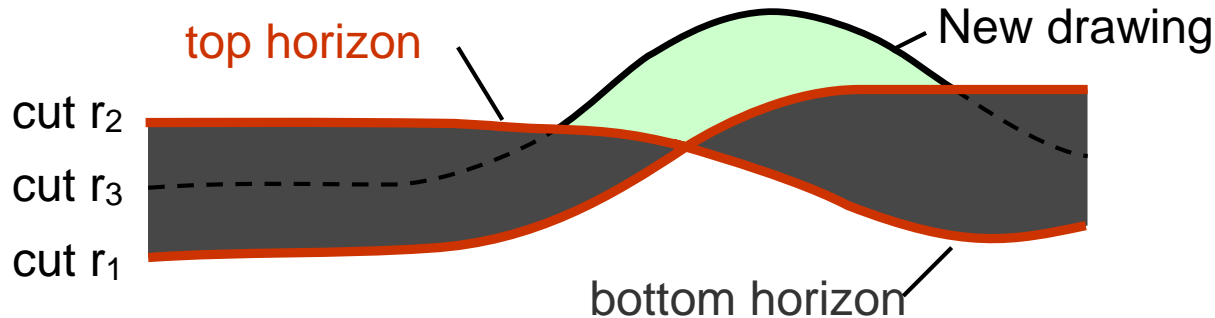
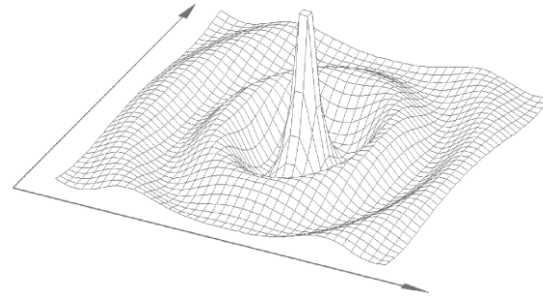
# Direct rendering of CSG models

- Specialized ray casting
- Intervals of ray/object intersections
- Solving set operations = set operations on intervals



# Floating horizon algorithm

- Graphs of functions  $z = f(x,y)$
- Terrains (height field )
- Algorithm outline
  - Render front-to-back
  - Keep bottom and top horizon



# A-Buffer

---

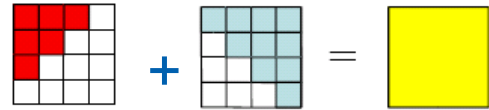
- Antialiasing, correct transparency
  - [Carpenter84], Lucasfilm: “The Road To Point Reyes”
  - Later used in RenderMan (Pixar)
- Ordered list of primitives for each pixel
- Storing not just depth
  - transparency, coverage, object ID, normal,...
- Polygon rasterization
  - Non-transparent polygon covers the whole pixel – add to list and **remove** farther ones
  - Transparent polygon or partial pixel coverage – insert to list, **do not remove** farther ones

# A-Buffer

---

- Rendering pass

- For each pixel process the list
- Composition (subpixel rasterization, coverage mask 4x4)
- Similar to MSAA



- Benefits

- More general than z-buffer
- Used in production rendering
- Handles transparency

# Other buffers...

---

A-buffer - Carpenter, 1984  
G-buffer - Saito & Takahashi, 1991  
M-buffer - Schneider & Rossignac, 1995  
P-buffer - Yuan & Sun, 1997  
T-buffer - Hsiung, Thibadeau & Wu, 1990  
W-buffer - 3dfx, 1996?  
Z-buffer - Catmull, 1973 (?)  
ZZ-buffer - Salesin & Stolfi, 1989

Accumulation Buffer - Haerberli & Akeley, 1990  
Area Sampling Buffer - Sung, 1992  
Back Buffer - Baum, Cohen, Wallace & Greenberg, 1986  
Close Objects Buffer - Telea & van Overveld, 1997  
Color Buffer  
Compositing Buffer - Lau & Wiseman, 1994  
Cross Scan Buffer - Tanaka & Takahashi, 1994  
Delta Z Buffer - Yamamoto, 1991  
Depth Buffer - 1984  
Depth-Interval Buffer - Rossignac & Wu, 1989  
Double Buffer - 1993

Escape Buffer - Hepting & Hart, 1995  
Frame Buffer - Kajjiya, Sutherland & Cheadle, 1975  
Hierarchical Z-Buffer - Greene, 1993  
Item Buffer - Weghorst, Hooper & Greenberg, 1984  
Light Buffer - Haines & Greenberg, 1986  
Mesh Buffer - Deering, 1995  
Normal Buffer - Curington, 1985  
Picture Buffer - Ollis & Borgwardt, 1988  
Pixel Buffer - Peachey, 1987  
Ray Distribution Buffer - Shinya, 1994  
Ray-Z-Buffer - Lamparter, Muller & Winckler, 1990  
Refreshing Buffer - Basil, 1977  
Sample Buffer - Ke & Change, 1993  
Shadow Buffer - GIMP, 1999  
Sheet Buffer - Mueller & Crawfis, 1998  
Stencil Buffer - 1997?  
Super Buffer - Gharachorloo & Pottle, 1985  
Super-Plane Buffer - Zhou & Peng, 1992  
Triple Buffer  
Video Buffer - Scherson & Punte, 1987  
Volume Buffer - Sramek & Kaufman, 1999

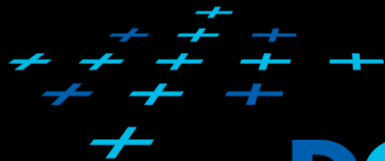
Source: Eric Haines - Is the Hardware Z-Buffer Doomed?

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**Questions?**