

# Data Collection Planning with Curvature-Constrained Vehicles

Dubins Traveling Salesman Problem with Neighborhoods (DTSPN) and Dubins Orienteering Problem with Neighborhoods (DOPN)

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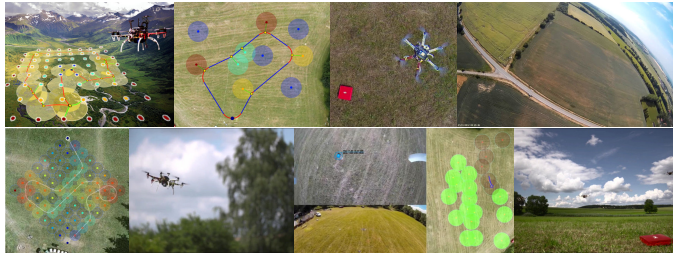
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Lecture 08

B4M36UIR – Artificial Intelligence in Robotics

## Motivation – Surveillance Missions with Aerial Vehicles

- Provide **curvature-constrained** path to collect the **most valuable measurements** with **shortest possible path/time** or under **limited travel budget**



- Formulated as routing problems with Dubins vehicle
  - Dubins Traveling Salesman Problem with Neighborhoods
  - Dubins Orienteering Problem with Neighborhoods

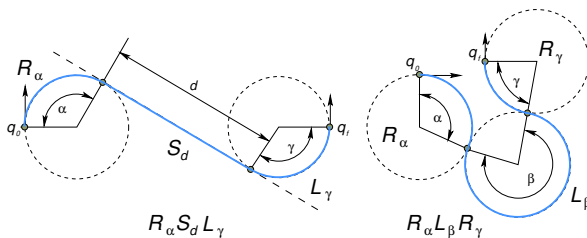
## Parametrization of Dubins Maneuvers

- Parametrization of each trajectory phase:

$$\{L_\alpha R_\beta L_\gamma, R_\alpha L_\beta R_\gamma, L_\alpha S_d L_\gamma, L_\alpha S_d R_\gamma, R_\alpha S_d L_\gamma, R_\alpha S_d R_\gamma\}$$

for  $\alpha \in [0, 2\pi)$ ,  $\beta \in (\pi, 2\pi)$ ,  $d \geq 0$

Notice the prescribed orientation at  $q_0$  and  $q_1$ .



## Overview of the Lecture

- Part 1 – Data Collection Planning – Aerial Surveillance Missions
  - Dubins Vehicle and Dubins Planning
  - Dubins Touring Problem (DTP)
  - Dubins Traveling Salesman Problem
  - Dubins Traveling Salesman Problem with Neighborhoods
  - Dubins Orienteering Problem
  - Dubins Orienteering Problem with Neighborhoods
  - Planning in 3D – Examples and Motivations
- Part 2 – Bonus HW03b – Data Collection Planning for Surveillance Missions
  - Task10 Bonus – Motivation and Assignment

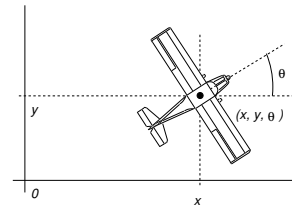
## Dubins Vehicle

- Non-holonomic vehicle such as car-like or aircraft can be modeled as the Dubins vehicle
  - Constant forward velocity
  - Limited minimal turning radius  $\rho$
  - Vehicle state is represented by a triplet  $q = (x, y, \theta)$ , where
  - Position is  $(x, y) \in \mathbb{R}^2$ , vehicle heading is  $\theta \in \mathbb{S}^1$ , and thus  $q \in SE(2)$

The vehicle motion can be described by the equation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{u}{\rho} \end{bmatrix}, \quad |u| \leq 1,$$

where  $u$  is the control input



## Planning with Dubins Vehicle – Summary

- The optimal path connecting two configurations can be found analytically  
*E.g., for UAVs that usually operates in environment without obstacles*
- The Dubins maneuvers can also be used in randomized-sampling based motion planners, such as RRT, in the control based sampling
- Dubins vehicle model can be considered in the multi-goal path planning
  - Surveillance, inspection or monitoring missions to periodically visits given target locations (areas)
- Dubins Touring Problem (DTP)
 

Given a sequence of locations, what is the shortest path visiting the locations, i.e., what are the headings of the vehicle at the locations
- Dubins Traveling Salesman Problem (DTSP)
 

Given a set of locations, what is the shortest Dubins path that visits each location exactly once and returns to the origin location
- Dubins Orienteering Problem (DOP)
 

Given a set of locations, each with associated reward, what is the Dubins path visiting the most rewarding locations and not exceeding the given travel budget

## Part I

# Part 1 – Data Collection Planning – Aerial Surveillance Missions

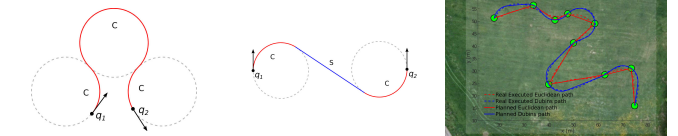
## Optimal Maneuvers for Dubins Vehicle

- For two states  $q_1 \in SE(2)$  and  $q_2 \in SE(2)$  in the environment **without obstacles**  $\mathcal{W} = \mathbb{R}^2$ , the optimal path connecting  $q_1$  with  $q_2$  can be characterized as one of two main types
  - CCC type: LRL, RLR;
  - CSC type: LSL, LSR, RSL, RSR;
 where S – straight line arc, C – circular arc oriented to left (L) or right (R)  
*L. E. Dubins (1957) – American Journal of Mathematics*
- The optimal paths are called **Dubins maneuvers**:
  - Constant velocity:  $v(t) = v$  and turning radius  $\rho$
  - Six types of trajectories connecting any configuration in  $SE(2)$  *without obstacles*
  - The control  $u$  is according to C and S type one of three possible values  $u \in \{-1, 0, 1\}$

## Dubins (Multi-Goal) Path

- Minimal turning radius  $\rho$
- Constant forward velocity  $v$
- State of the Dubins vehicle is  $q = (x, y, \theta)$ ,  $q \in SE(2)$ ,  $(x, y) \in \mathbb{R}^2$  and  $\theta \in \mathbb{S}^1$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{u}{\rho} \end{bmatrix}$$



Smooth Dubins path connecting a sequence of locations is also suitable for multi-rotor aerial vehicle

- Optimal path connecting  $q_1 \in SE(2)$  and  $q_2 \in SE(2)$  consists only of straight line arcs and arcs with the maximal curvature, i.e., two types of maneuvers CCC and CSC and the solution can be found analytically *(Dubins, 1957)*

**The main difficulty is to determine the vehicle headings for a given sequence of waypoints**

## Dubins Touring Problem – DTP

- For a sequence of the  $n$  waypoint locations  $P = (p_1, \dots, p_n)$ ,  $p_i \in \mathbb{R}^2$ , the **Dubins Touring Problem (DTP)** stands to determine the **optimal headings**  $T = \{\theta_1, \dots, \theta_n\}$  at the waypoints  $q_i$  such that

$$\begin{aligned} \text{minimize } \tau \quad & \mathcal{L}(T, P) = \sum_{i=1}^{n-1} \mathcal{L}(q_i, q_{i+1}) + \mathcal{L}(q_n, q_1) \\ \text{subject to} \quad & q_i = (p_i, \theta_i), \theta_i \in [0, 2\pi), p_i \in P, \end{aligned}$$

where  $\mathcal{L}(q_i, q_j)$  is the length of the Dubins maneuver connecting  $q_i$  with  $q_j$

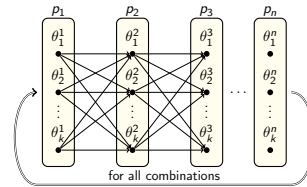
- The DTP is a **continuous optimization problem**
- The term  $\mathcal{L}(q_n, q_1)$  is for possibly closed tour that can be for example requested in the TSP with Dubins vehicle, a.k.a. DTSP  
*On the other, the DTP can also be utilized for open paths such as solutions of the OP with Dubins vehicle*
- In some cases, it may be suitable to relax the heading at the first/last locations in finding closed tours (i.e., solving DTSP)

## Sampling-based Solution of the DTP

- For a closed sequence of the waypoint locations

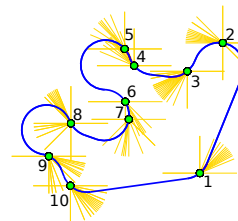
$$P = (p_1, \dots, p_n)$$

- We can sample possible heading values at each location  $i$  into a discrete set of  $k$  headings, i.e.,  $\Theta^i = \{\theta_1^i, \dots, \theta_k^i\}$  and create a graph of all possible Dubins maneuvers



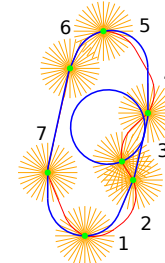
- For a set of heading samples, the optimal solution can be found by a forward search of the graph in  $O(nk^3)$   
*For open sequence we do not need to evaluate all possible initial headings, and the complexity is  $O(nk^2)$*

- The key is to **determined the most suitable heading samples per each waypoint**



## Example of Heading Sampling – Uniform vs. Informed

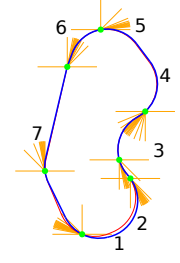
Uniform sampling



$$N = 224, T_{CPU} = 128 \text{ ms} \\ \mathcal{L} = 19.8, \mathcal{L}_U = 13.8,$$

- $N$  is the total number of samples, i.e., 32 samples per waypoint for uniform sampling
- $\mathcal{L}$  is the length of the tour (blue) and  $\mathcal{L}_U$  is the lower bound (red) determined as a solution of the **Dubins Interval Problem (DIP)**

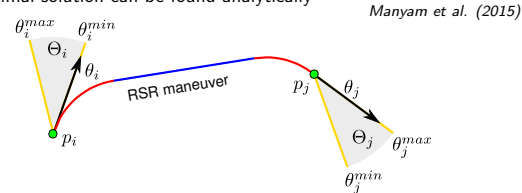
Informed sampling



$$N = 128, T_{CPU} = 76 \text{ ms} \\ \mathcal{L} = 14.4, \mathcal{L}_U = 14.2,$$

## Dubins Interval Problem (DIP)

- Dubins Interval Problem (DIP)** is a generalization of Dubins maneuvers to the shortest path connecting two points  $p_i$  and  $p_j$
- In the DIP, the leaving interval  $\Theta_i$  at  $p_i$  and the arrival interval  $\Theta_j$  at  $p_j$  are consider (not a single heading value)
- The optimal solution can be found analytically

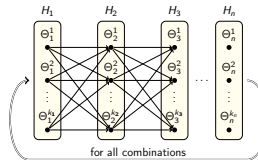


Manyam et al. (2015)

- Solution of the DIP is a tight lower bound for the DTP
- Solution of the DIP is not a feasible solution of the DTP  
*Notice, for  $\Theta_i = \Theta_j = (0, 2\pi)$  the optimal maneuver for DIP is a straight line segment*

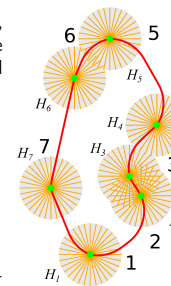
## Lower Bound of the DTP

- For a discrete set of heading intervals  $\mathcal{H} = \{H_1, \dots, H_n\}$ , where  $H_i = \{\Theta_i^1, \Theta_i^2, \dots, \Theta_i^{k_i}\}$ , a similar graph as for the DTP can be constructed with the edge cost determined by the solution of the associated DIP



- The forward search of the graph with dense samples provides a **tight lower bound of the DTP**

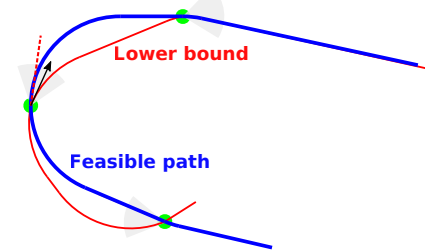
Manyam and Rathinam, 2015



## Lower Bound and Feasible Solution of the DTP

- The arrival and departure angles may not be the same

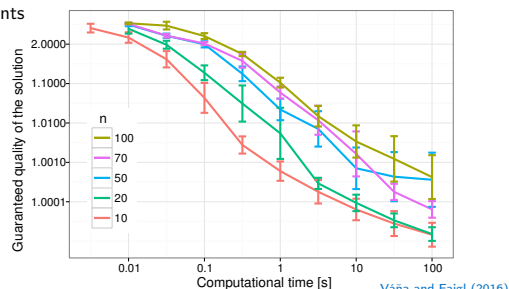
*The lower bound solution is not a feasible solution of the DTP*



- DTP solution** – use any particular heading of each interval in the lower bound solution

## The DIP-based Sampling of Headings in the DTP

- A similar forward search graph as for the DTP can be used for heading intervals instead of particular headings
- Using DIP for a sequence of waypoints is a **lower bound** of the DTP
- It can be used to inform how to splitting heading intervals
- The ratio between the lower bound and feasible solution of the DTP provides an estimation of the solution quality, e.g., for problems with  $n$  waypoints



Vána and Faigl (2016)

## Informed Sampling of Headings in Solution of the DTP

- Iterative refinement of the heading intervals  $\mathcal{H}$  up to the angular resolution  $\epsilon_{req}$

- The angular resolution is gradually decreased for the most promising intervals

- refinedDTP** – divide the intervals of the lower bound solution

- solvedTP** – solve DTP using the heading from the refined intervals

- It simultaneously provides **feasible** and **lower bound** solutions of the DTP

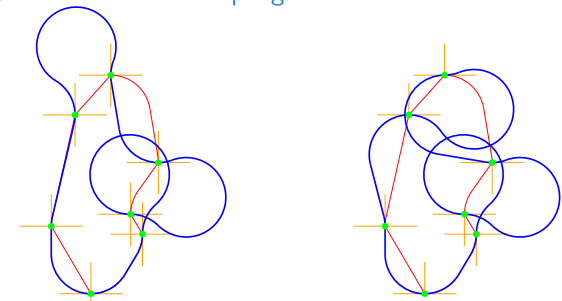
*The lower bound provides a tight estimation of the solution quality*

- The first solution is provided very quickly – **any-time algorithm**

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Algorithm 1: Iterative Informed Sampling-based DTP Algorithm
Vistup: P – Target locations to be visited
Vistup:  $\epsilon_{req}$  – Requested angular resolution
Vistup:  $\epsilon_{max}$  – Requested quality of the solution
Vistup: T – A tour visiting the targets
 $\epsilon = 2\pi$  // initial angular resolution:
 $\mathcal{H} = \text{createIntervals}(P, \epsilon)$  // initial intervals:
 $\mathcal{L}_L = 0$  // init lower bound:
 $\mathcal{L}_U = \infty$  // init upper bound:
while  $\epsilon > \epsilon_{req}$  and  $\mathcal{L}_U / \mathcal{L}_L > \epsilon_{max}$  do
   $\epsilon = \epsilon / 2$ ;
   $(\mathcal{H}, \mathcal{L}_L) = \text{refinedDTP}(P, \epsilon, \mathcal{H})$ ;
   $(T, \mathcal{L}_U) = \text{solvedTP}(P, \mathcal{H})$ ;
end
return T;
```

Faigl, Vána et al. (2017)

## Uniform vs Informed Sampling

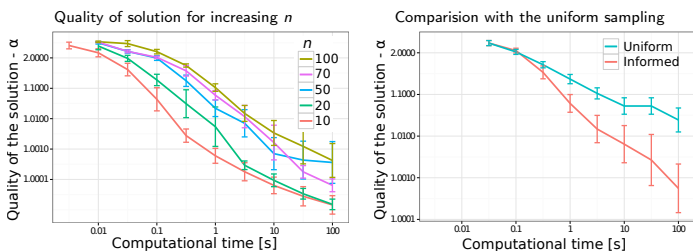


$$\epsilon = 2\pi/4, N = 28, T_{CPU} = 8 \text{ ms} \\ \mathcal{L} = 27.9, \mathcal{L}_U = 13.2$$

$$\epsilon = 2\pi/4, N = 21, T_{CPU} = 8 \text{ ms} \\ \mathcal{L} = 29.9, \mathcal{L}_U = 13.2$$

## Results and Comparison with Uniform Sampling

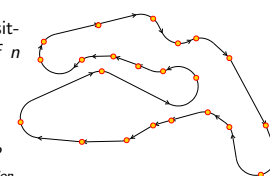
- Random instances of the DTSP with a sequence of visits to the targets determined as a solution of the Euclidean TSP
- The waypoints placed in a squared bounding box with the side  $s = (\rho\sqrt{n})/d$  for the  $\rho = 1$  and density  $d = 0.5$  *It matters on the Density of targets!*



- The Informed sampling-based approach provides solutions up to 0.01% from the optima
- A solution of the DTP is a fundamental building block for **routing problems with Dubins vehicle**

## Dubins Traveling Salesman Problem (DTSP)

- Determine a closed shortest Dubins path visiting each location  $p_i \in P$  of the given set of  $n$  locations  $P = \{p_1, \dots, p_n\}$ ,  $p_i \in \mathbb{R}^2$
- Permutation  $\Sigma = (\sigma_1, \dots, \sigma_n)$  of visits *Sequencing part of the problem*
- Headings  $\Theta = \{\theta_{\sigma_1}, \theta_{\sigma_2}, \dots, \theta_{\sigma_n}\}$  for  $p_{\sigma_i} \in P$  *Continuous optimization*



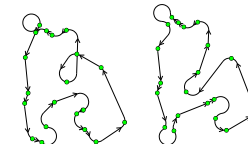
- DTSP is an optimization problem over all possible **permutations  $\Sigma$**  and **headings  $\Theta$**  in the states  $(q_{\sigma_1}, q_{\sigma_2}, \dots, q_{\sigma_n})$  such that  $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$

$$\begin{aligned} \text{minimize}_{\Sigma, \Theta} \quad & \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \quad (1) \\ \text{subject to} \quad & q_i = (p_i, \theta_i) \quad i = 1, \dots, n, \quad (2) \end{aligned}$$

where  $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$  is the length of Dubins path between  $q_{\sigma_i}$  and  $q_{\sigma_j}$ .

## Challenges of the Dubins Traveling Salesman Problem

- The key difficulty of the DTSP is that the path length mutually depends on
    - Order of the visits to the locations
    - Headings at the target locations
- We need the sequence to determine headings, but headings may influence the sequence*



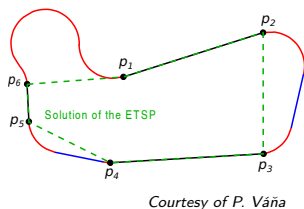
Two fundamental approaches can be found in literature

- Decoupled** approach based on a given sequence of the locations *E.g., found by a solution of the Euclidean TSP*
  - Sampling-based** approach with sampling of the headings at the locations into discrete sets of values and considering the problem as the variant of the **Generalized TSP**
- Besides, further approaches are
- Genetic and memetic techniques (evolutionary algorithms)
  - Unsupervised learning based approaches

## Decoupled Solution of the DTSP – Alternating Algorithm

**Alternating Algorithm (AA)** provides a solution of the DTSP for an **even** number of targets  $n$  *Savla et al. (2005)*

- Solve the related Euclidean TSP *Relaxed motion constraints*
- Establish headings for even edges using straight line segments
- Determine optimal maneuvers for odd edges using the analytical form for Dubins maneuvers *Headings are known*



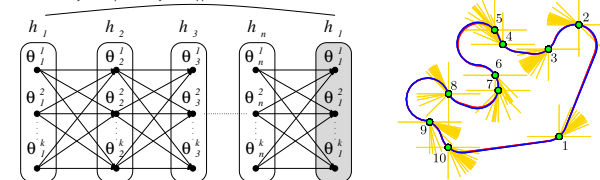
## DTSP with the Given Sequence of the Visits to the Targets

- If the sequence of the visits  $\Sigma$  to the target locations is given
- the problem is to determine the optimal heading at each location
- and the problem becomes the **Dubins Touring Problem (DTP)** *Váňa and Faigl (2016)*

- Let for each location  $g_i \in G$  sample possible heading to  $k$  values, i.e., for each  $g_i$  the set of headings be  $h_i = \{\theta_1^i, \dots, \theta_k^i\}$ .
- Since  $\Sigma$  is given, we can construct a graph connecting two consecutive locations in the sequence by all possible headings
- For such a graph and particular headings  $\{h_1, \dots, h_n\}$ , we can find an optimal headings and thus, **the optimal solution of the DTP**.

## DTSP as a Solution of the DTP

*The first layer is duplicated layer to support the forward search method*



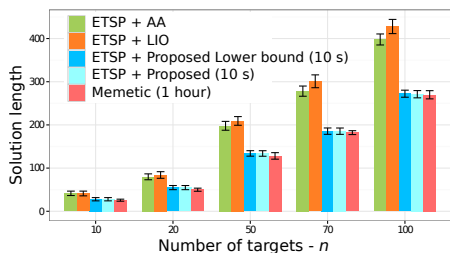
- The edge cost corresponds to the length of Dubins maneuver
- Better solution of the DTP can be found for more samples, which will also improve the DTSP but only for the given sequence

*Two questions arise for a practical solution of the DTP*

- How to sample the headings? Since more samples makes finding solution more demanding** *We need to sample the headings in a "smart" way, i.e., guided sampling using lower bound of the DTP*
- What is the solution quality? Is there a tight lower bound?** *Yes, the lower bound can be computed as a solution of Dubins Interval Problem (DIP)*

## DTP Solver in Solution of the DTSP

- The solution of the DTP can be used to solve DTSP for the given sequence of the waypoints *E.g., determined as a solution of the Euclidean TSP as in the Alternating Algorithm*
- Comparison with the Alternating Algorithm (AA), Local Iterative Optimization (LIO), and Memetic algorithm *AA – Savla et al., 2005, LIO – Váňa & Faigl, 2015, Memetic – Zhang et al. 2014*



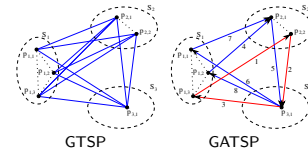
## DTSP – Sampling-based Approach

- Sampled heading values can be directly utilized to find the sequence as a solution of the **Generalized Traveling Salesman Problem (GTSP)** *Notice For Dubins vehicle, it is the Generalized Asymmetric TSP (GATSP)*

The problem is to determine a shortest tour in a graph that visits all specified subsets of the graph's vertices

*The TSP is a special case of the GTSP when each subset to be visited consists just a single vertex.*

- GATSP  $\rightarrow$  ATSP *Noon and Bean (1991)*
- ATSP can be solved by LKH
- ATSP  $\rightarrow$  TSP, which can be solved optimally, e.g., by Concorde



## Dubins Traveling Salesman Problem with Neighborhoods

- In surveillance planning, it may be required to visit a set of target regions  $G = \{R_1, \dots, R_n\}$  by the Dubins vehicle
- Then, for each target region  $R_i$ , we have to determine a particular point of the visit  $p_i \in R_i$  and DTSP becomes the **Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)** *In addition to  $\Sigma$  and headings  $\Theta$ , waypoint locations  $P$  have to be determined*
- DTSPN is an optimization problem over all permutations  $\Sigma$ , headings  $\Theta = \{\theta_{\sigma_1}, \dots, \theta_{\sigma_n}\}$  and points  $P = (p_{\sigma_1}, \dots, p_{\sigma_n})$  for the states  $(q_{\sigma_1}, \dots, q_{\sigma_n})$  such that  $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$  and  $p_{\sigma_i} \in R_{\sigma_i}$ :

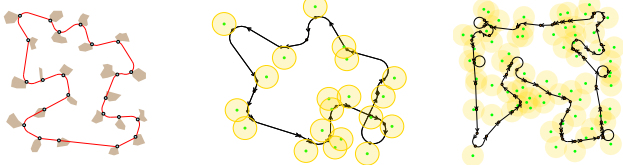
$$\begin{aligned} \text{minimize}_{\Sigma, \Theta, P} \quad & \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \quad (3) \\ \text{subject to} \quad & q_i = (p_i, \theta_i), p_i \in R_i \quad i = 1, \dots, n \quad (4) \end{aligned}$$

- $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$  is the length of the shortest possible Dubins maneuver connecting the states  $q_{\sigma_i}$  and  $q_{\sigma_j}$



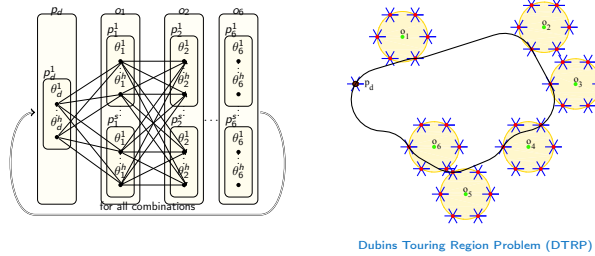
## DTSPN – Approches and Examples of Solution

- Similarly to the DTSP, also DTSPN can be addressed by
  - Decoupled approaches** for which a sequence of visits to the regions can be found as a solution of the ETSP(N)
  - Sampling-based approaches** and formulation as the GATSP
    - Clusters of sampled waypoint locations each with sampled possible heading values
  - Soft-computing** techniques such as memetic algorithms
  - Unsupervised learning** techniques
- Similarly to the lower bound of the DTSP based on the **Dubins Interval Problem (DIP)** a lower bound for the DTSPN can be computed using **Generalized DIP (GDIP)**



## DTSPN – Decoupled Approach

- Determine a sequence of visits to the  $n$  target regions as the solution of the ETSP
- Sample possible waypoint locations and for each such a location sample possible heading values, e.g.,  $s$  locations per each region and  $h$  heading per each location
- Construct a search graph and determine a solution in  $O(n(sh)^3)$
- An example of the search graph for  $n = 6, s = 6,$  and  $h = 6$



## DTSPN – Local Iterative Optimization (LIO)

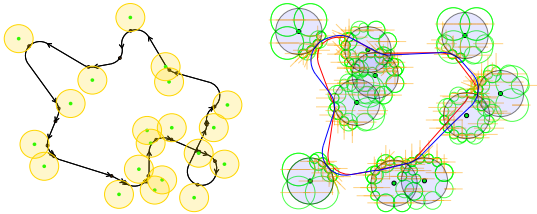
- Instead of sampling into a discrete set of waypoint locations each with sampled possible headings, we can perform local optimization, e.g., hill-climbing technique
- At each waypoint location  $p_i$ , the heading can be  $\theta_i \in [0, 2\pi)$
- A waypoint location  $p_i$  can be parametrized as a point on the boundary of the respective region  $R_i$ , i.e., as a parameter  $\alpha \in [0, 1)$  measuring a normalized distance on the boundary of  $R_i$
- The multi-variable optimization is treated independently for each particular variable  $\theta_i$  and  $\alpha_i$  iteratively

```

Algorithm 2: Local Iterative Optimization (LIO) for the DTSPN
Data: Input sequence of the goal regions  $G = (R_{\sigma_1}, \dots, R_{\sigma_n})$ , for the permutation  $\Sigma$ 
Result: Waypoints  $(q_{\sigma_1}, \dots, q_{\sigma_n})$ ,  $q_i = (p_i, \theta_i)$ ,  $p_i \in \delta R_i$ ;
initialization() // random assignment of  $q_i \in \delta R_i$ ;
while global solution is improving do
  for every  $R_i \in G$  do
     $\theta_i := \text{optimizeHeadingLocally}(\theta_i)$ ;
     $\alpha_i := \text{optimizePositionLocally}(\alpha_i)$ ;
     $q_i := \text{checkLocalMinima}(\alpha_i, \theta_i)$ ;
  end
end
    
```

## Lower Bound for the DTSP with Neighborhoods Generalized Dubins Interval Problem

- In the DTSPN, we need to determined not only the **headings**, but the waypoint locations themselves
- Dubins Interval Problem is not sufficient to provide tight lower-bound



- Generalized Dubins Interval Problem (GDIP)** can be utilized for the DTSPN similarly as the DIP for the DTSP

■ Váňa and Faigl: *Optimal Solution of the Generalized Dubins Interval Problem*, RSS 2018, **best student paper finalist**.

## Generalized Dubins Interval Problem (GDIP) and its Optimal Solution

- Determine the shortest Dubins maneuver connecting  $P_i$  and  $P_j$  given the angle intervals  $\theta_i \in [\theta_i^{min}, \theta_i^{max}]$  and  $\theta_j \in [\theta_j^{min}, \theta_j^{max}]$

**Full problem (GDIP)**      **One-side version (OS-GDIP)**

- Optimal solution** – Closed-form expressions for (1–6) and convex optimization (7)

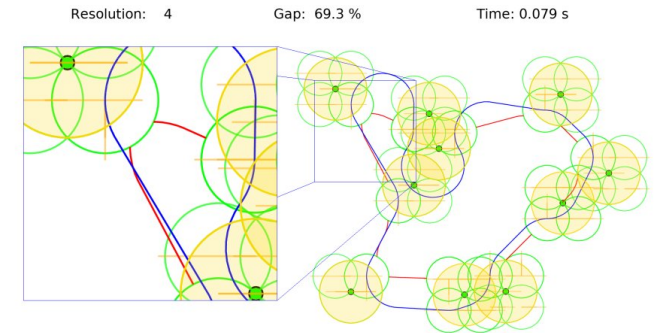
Problem	Average computational time [μs]	Ratio
1) S type	0.58	1.00
2) CS type	2.86	4.93
3) C <sub>ψ</sub> type	12.63	21.78
4) CSC type		
5) CSC <sub>ψ</sub> type		
6) CC <sub>ψ</sub> C type		
7) CC <sub>ψ</sub> type		

<https://github.com/comrob/gdip>

■ Váňa and Faigl: *Optimal Solution of the Generalized Dubins Interval Problem*, RSS 2018, **best student paper finalist**.

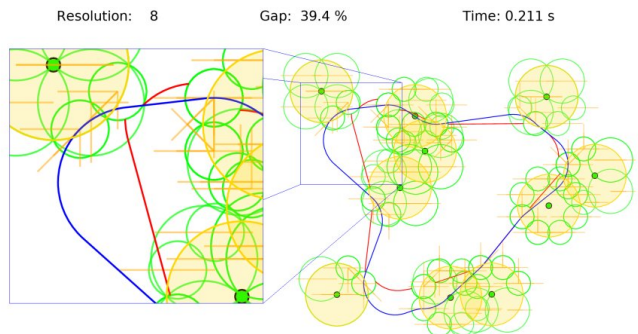
## GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples



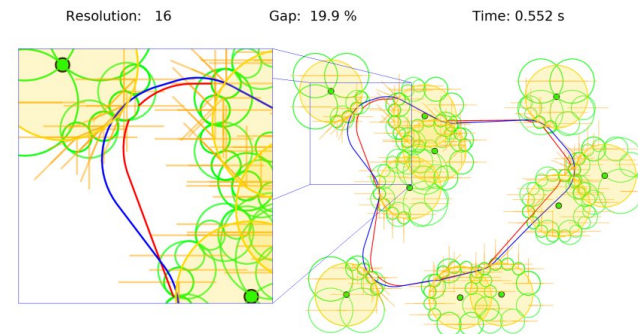
## GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples



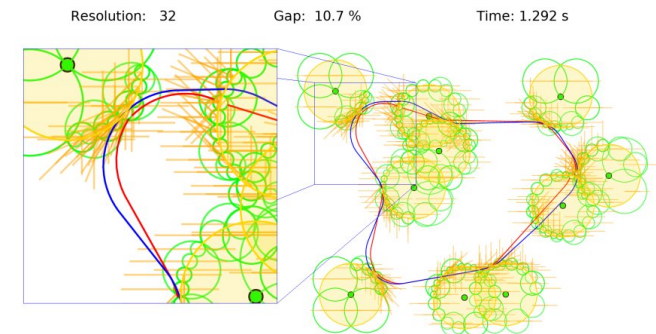
## GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples



## GDIP-based Informed Sampling for the DTSPN

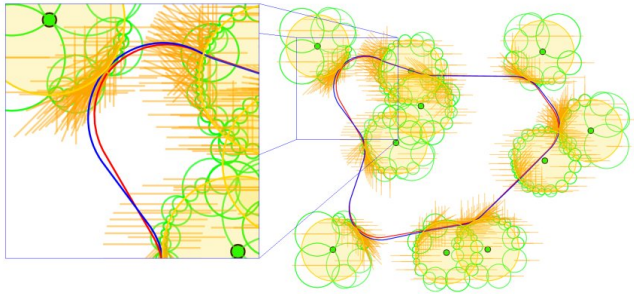
- Iterative refinement of the neighborhood samples and heading samples



## GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples

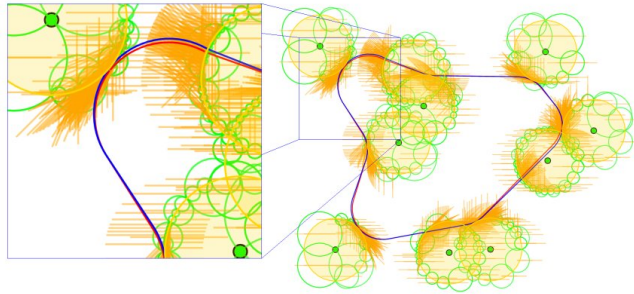
Resolution: 64 Gap: 5.3 % Time: 3.183 s



## GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples

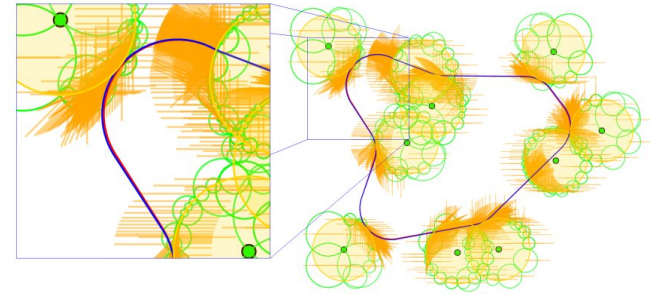
Resolution: 128 Gap: 2.6 % Time: 8.994 s



## GDIP-based Informed Sampling for the DTSPN

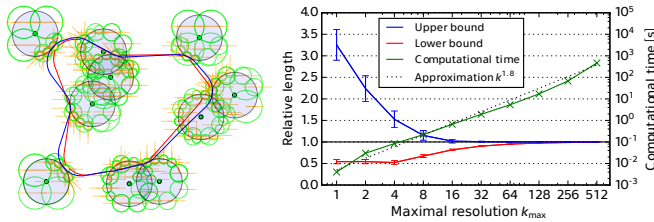
- Iterative refinement of the neighborhood samples and heading samples

Resolution: 256 Gap: 1.3 % Time: 33.474 s



## DTSPN – Convergence to the Optimal Solution

- For a given sequence of visits to the target regions (locations)



- The algorithm scales linearly with the number of locations
- Complexity of the algorithm is approximately  $\mathcal{O}(nk^{1.8})$

<https://github.com/comrob/gdip>

Váňa and Faigl: *Optimal Solution of the Generalized Dubins Interval Problem*, RSS 2018, best student paper finalist.

## Variable Neighborhood Search (VNS)

- Variable Neighborhood Search (VNS)** is a general metaheuristic for combinatorial optimization (routing problems)

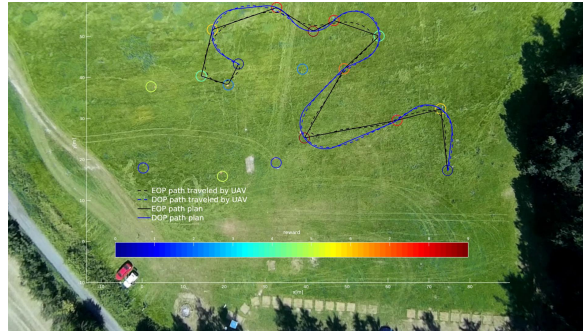
Hansen, P. and Mladenović, N. (2001): *Variable neighborhood search: Principles and applications*. European Journal of Operational Research.

- The VNS is based on **shake** and **local search** procedures
  - Shake** procedure aims to escape from local optima by changing the solution within the neighborhoods  $N_{1, \dots, k_{max}}$
  - The neighborhoods are particular operators*
  - Local search** procedure searches fully specific neighborhoods of the solution using  $l_{max}$  predefined operators

## Data Collection / Surveillance Planning with Travel Budget

- Visit the most important targets because of limited travel budget
- The problem can be formulated as the **Orienteering Problem** with Dubins vehicle, a.k.a. **Dubins Orienteering Problem (DOP)**

Robert Pěnička, Jan Faigl, Petr Váňa and Martin Saska, RA-L 2017



## Variable Neighborhood Search (VNS) for the DOP

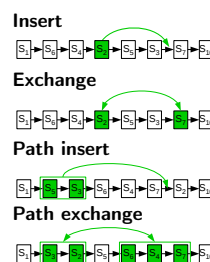
- The solution is the first  $k$  locations of the sequence of all target locations satisfying  $T_{max}$
- It is an improving heuristics, i.e., an initial solution has to be provided
- A set of predefined neighborhoods are explored to find a better solution

VNS for the OP – Sevklı, Z. et al. (2006)

- Shake** – explores the configuration space and escape from a local minima using
  - Insert** – moves one random element
  - Exchange** – exchanges two random elements

- Local Search** – optimizes the solution
  - Path insert** – moves a random sub-sequence
  - Path exchange** – exchanges two random sub-sequences

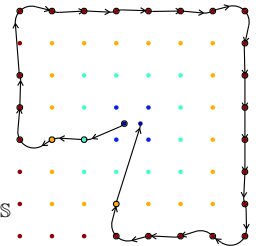
- Randomized VNS** – examines only  $n^2$  changes in the *Local Search* procedure in each iteration



## Dubins Orienteering Problem

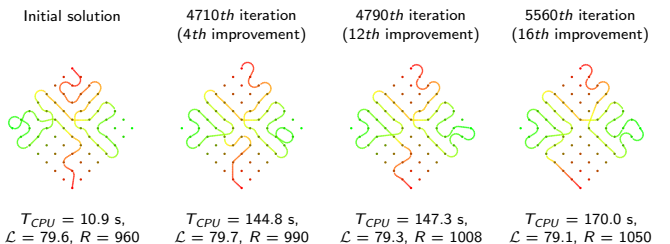
- Curvature-constrained data collection path respecting Dubins vehicle model with the minimal turning radius  $\rho$  and constant forward velocity  $v$
- The path is a sequence of waypoints  $q_i \in SE(2)$ ,  $q = (s, \theta)$ ,  $\theta \in \mathbb{S}^1$ .
- In addition to  $S_k, k, \Sigma$  (OP) determine headings  $\Theta = (\theta_{\sigma_1}, \dots, \theta_{\sigma_k})$  such that

$$\begin{aligned} & \underset{k, S_k, \Sigma}{\text{maximize}} && R = \sum_{i=1}^k r_{\sigma_i} \\ & \text{subject to} && \sum_{i=2}^k \mathcal{L}(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{max}, \\ & && q_{\sigma_i} = (s_{\sigma_i}, \theta_{\sigma_i}), s_{\sigma_i} \in S, \theta_{\sigma_i} \in \mathbb{S}^1 \\ & && s_{\sigma_1} = s_1, s_{\sigma_k} = s_n \end{aligned}$$



The problem combines discrete combinatorial optimization (OP) with the continuous optimization for determining the vehicle headings

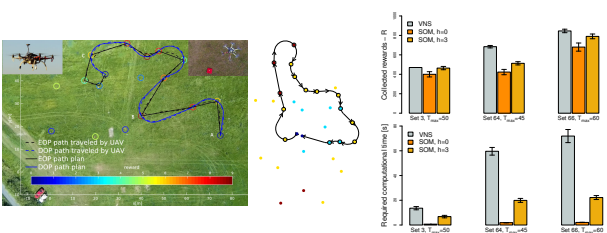
## Evolution of the VNS Solution to the DOP





## Possible Solutions of the Dubins Orienteering Problem

- Solve the Euclidean OP (EOP) and then determine Dubins path  
*The final path may exceed the budget and the vehicle can miss the locations because of motion control*
- Directly solve the **Dubins Orienteering Problem (DOP)**, e.g.,
  - Sample possible heading values and use Variable Neighborhood Search (VNS) *Pěnička, Faigl, Vaňa, Saska (RA-L 2017)*
  - Unsupervised learning based on Self-Organizing Maps (SOM) *Faigl, (WSOM+ 2017)*

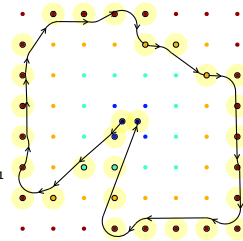


VNS-based approach provides better solutions than SOM, but it tends to be more computationally demanding

## Dubins Orienteering Problem with Neighborhoods

- Curvature-constrained path respecting Dubins vehicle model
- Each waypoint consists of location  $p \in \mathbb{R}^2$  and the heading  $\theta \in \mathbb{S}^1$
- In addition to  $S_k, k, \Sigma$  determine locations  $P_k = (p_{\sigma_1}, \dots, p_{\sigma_k})$  and headings  $\Theta = (\theta_{\sigma_1}, \dots, \theta_{\sigma_k})$  such that

$$\begin{aligned} & \text{maximize}_{k, S_k, \Sigma} && R = \sum_{i=1}^k r_{\sigma_i} \\ & \text{subject to} && \sum_{i=2}^k \mathcal{L}(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{max}, \\ & && q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i}), p_{\sigma_i} \in \mathbb{R}^2, \theta_{\sigma_i} \in \mathbb{S}^1 \\ & && \|p_{\sigma_i}, s_{\sigma_i}\| \leq \delta, s_{\sigma_i} \in S_k \\ & && p_{\sigma_1} = s_1, p_{\sigma_k} = s_n \end{aligned}$$



We need to solve the continuous optimization for determining the vehicle heading at each waypoint and the waypoint locations  $P_k = \{p_{\sigma_1}, \dots, p_{\sigma_k}\}, p_{\sigma_i} \in \mathbb{R}^2$

## Variable Neighborhoods Search (VNS) for the DOPN

Algorithm 3: VNS based method for the DOPN

```

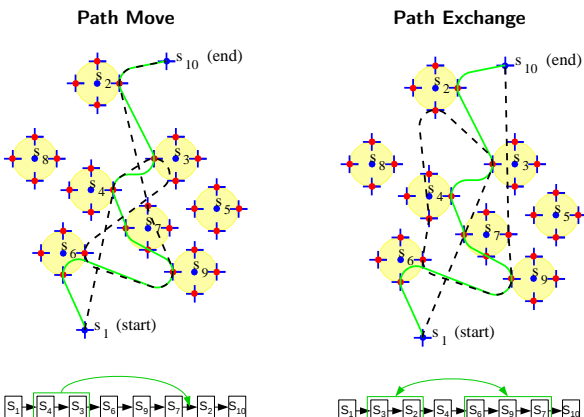
Input : S – Set of the target locations
Input : T_max – Maximal allowed budget
Input : o – Initial number of position waypoints for each target
Input : m – Initial number of heading values for each waypoints
Input : r_l – Local waypoint improvement ratio
Input : l_max – Maximal neighborhood number
Output : P – Found data collecting path
S_r ← getReachableLocations(S, T_max)
P ← createInitialPath(S_r, T_max) // greedy
while Stopping condition is not met do
  l ← 1
  while l ≤ l_max do
    P' ← shake(P, l)
    P'' ← localSearch(P', l, r_l)
    if C_d(P'') ≤ T_max and
       [(R(P'') > R(P)) or (R(P'') == R(P) and
         C_d(P'') < C_d(P))] then
      P ← P''
      l ← l + 1
    else
      l ← l + 1
  end
end
end
    
```

The particular  $l$  for the individual operators of the **shake** procedure are:

- Waypoint Shake** ( $l = 1$ )
  - Path Move** ( $l = 2$ )
  - Path Exchange** ( $l = 3$ )
- The **local search** procedure consists of three operators and the particular  $l$  for the individual operators of the **local search** procedure are:
- Waypoint Improvement** ( $l = 1$ )
  - One Point Move** ( $l = 2$ )
  - One Point Exchange** ( $l = 3$ )

Pěnička, R., Faigl, J., Saska, M., Vaňa, P. (2017)

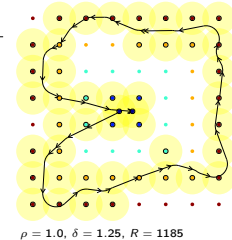
## VNS for DOPN – Example of the Shake Operators



## Comparison of the DOPN Solvers

- VNS-based DOPN solver with  $s = 16$  sampled waypoint locations per sensor and  $h = 16$  heading samples per waypoint location *Pěnička, Faigl, et al. (ICUAS 2017)*
- SOM-based DOPN solver with  $h = 3$  *Faigl, Pěnička (IROS 2017)*
- Aggregate results using average relative percentage error (ARPE) and relative percentage error (RPE) to the reference (best found) solution

Problem set	VNS-based		SOM-based ( $h = 3$ )		
	ARPE	T <sub>cpu</sub> * [s]	RPE	ARPE	T <sub>cpu</sub> [s]
Set 3, $\delta = 0.0$	1.0	1,178.9	3.6	7.4	7.0
Set 3, $\delta = 0.5$	0.9	13,273.3	6.6	10.6	7.9
Set 3, $\delta = 1.0$	0.5	13,304.4	5.5	9.2	8.3
Set 64, $\delta = 0.0$	1.9	5,272.2	17.4	23.8	17.9
Set 64, $\delta = 0.5$	2.8	13,595.6	18.7	24.2	20.2
Set 64, $\delta = 1.0$	1.3	13,792.3	9.9	15.2	22.2
Set 66, $\delta = 0.0$	1.5	6,546.6	3.6	9.1	22.9
Set 66, $\delta = 0.5$	1.4	13,650.1	6.7	11.8	25.5
Set 66, $\delta = 1.0$	3.2	13,824.5	16.1	21.3	26.7

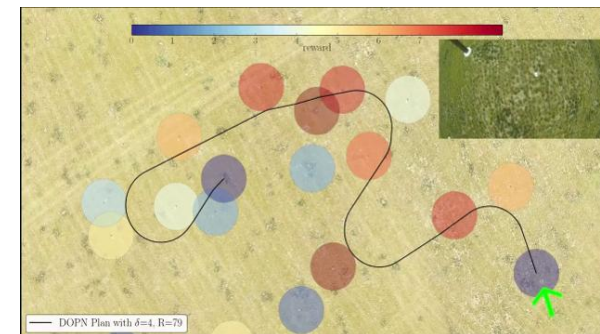


\*The results have been obtained with a grid Xeon CPUs running at 2.2 GHz to 3.4 GHz due to computational requirements

## DOPN – Example of Solution and Practical Deployment

- VNS-based solution of the DOPN

Robert Pěnička, Jan Faigl, Martin Saska and Petr Vaňa, ICUAS 2017



<http://mrs.felk.cvut.cz/jint17dopn>

## 3D Data Collection Planning with Dubins Airplane Model

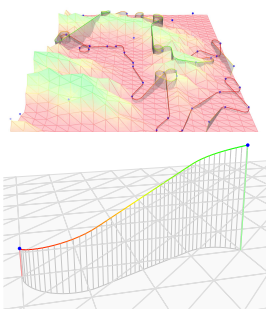
- Dubins Airplane model describes the vehicle state  $q = (p, \theta, \psi)$ ,  $p \in \mathbb{R}^3$  and  $\theta, \psi \in \mathbb{S}^1$  as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \cdot \cos \psi \\ \sin \theta \cdot \cos \psi \\ \sin \psi \\ u_{\theta} \cdot \rho^{-1} \end{bmatrix} \quad (5)$$

Chitsaz, H., LaValle, S.M. (2017)

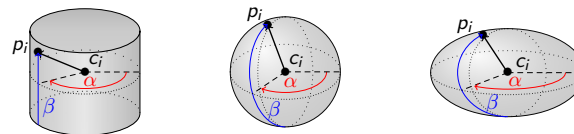
- Constant forward velocity  $v$ , the minimal turning radius  $\rho$ , and limited pitch angle, i.e.,  $\psi \in [\psi_{min}, \psi_{max}]$

- $u_{\theta}$  controls the vehicle heading,  $|u_{\theta}| \leq 1$ , and  $v$  is the forward velocity
- Generation of the 3D trajectory is based on the 2D Dubins maneuver
- If altitude changes are too high, additional helix segments are inserted



## DTSPN in 3D

- Using the same principles as for the DTSPN in 2D, we can generalize the approaches for 3D planning using the Dubins Airplane model instead of simple Dubins vehicle
- The regions can be generalized to 3D and the problem can be addressed by decoupled or sampling-based approaches, i.e., using GATSP formulation
- In the case of LIO, we need a parametrization of the possible waypoint location, e.g.,



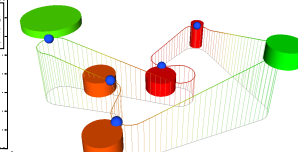
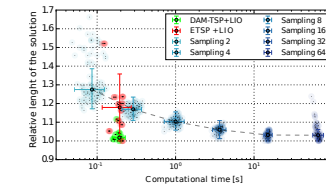
## Solutions of the 3D-DTSPN

Algorithm 4: LIO-based Solver for 3D-DTSPN

```

Data: Regions R
Result: Solution represented by Q and Σ
Σ ← getInitialSequence(R);
Q ← getInitialSolution(R, Σ);
while terminal condition do
  Q ← optimizeHeadings(Q, R, Σ);
  Q ← optimizeAlpha(Q, R, Σ);
  Q ← optimizeBeta(Q, R, Σ);
end
return Q, Σ;
    
```

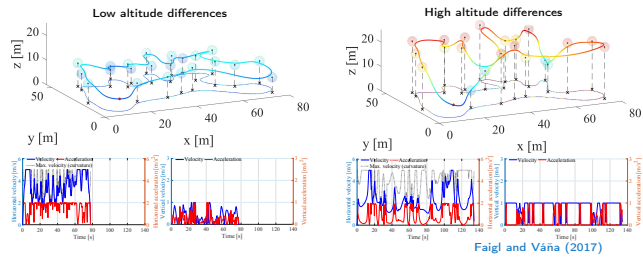
- Solutions based on LIO (ETSP+LIO), TSP with the travel cost according to Dubins Airplane Model (DAM-TSP+LIO), and sampling-based approach with transformation of the GTSP to the ATSP solved by LKH



Vaňa and Faigl (2017)

## 3D Surveillance Planning

- Parametrization of smooth 3D multi-goal trajectory as a sequence of Bézier curves
- Unsupervised learning for the TSPN can be generalized for such trajectories
- During the solution of the sequencing part of the problem, we can determine a velocity profile along the curve and compute the so-called *Travel Time Estimation* (TTE)
- Bézier curves better fit the limits of the multi-rotor UAVs that are limited by the maximal accelerations and velocities rather than minimal turning radius as for Dubins vehicle



- Low altitude differences saturate horizontal velocity while high altitudes changes saturate vertical velocity

## Assignment – Task10 Bonus

## Topic: Data Collection Planning for Surveillance Missions

**Goal:** Solve data collection planning problem formulated as the DTSP (DTSPN) and deploy the planned path to the model of UAVs and eventually experimentally verify the paths using real UAV

**Assignment:** <https://cw.fel.cvut.cz/wiki/courses/b4m36uir/hw/task10bonus>

Up to additional **5 points** can be gained for the implementation of the DTS and/or DTSPN, and execution of the trajectories in the MRS simulation framework

- Implement a solution of the DTSP(N), e.g., one of the following methods
  - (1 points) for simple ETSP and Alternating Algorithm (AA), a.k.a ETSP+AA;
  - (4 points) become familiar with the MRS simulation framework and deploy the planned trajectories within the simulator
- **Additional** implementation of the DTSPN sampling-based solver
  - (5 points) Employ a solution of the Generalized Dubins Interval Problem (GDIP) in sampling-based solution of the DTSPN and determining the lower bound of the DTSPN for a particular sequence of visits (e.g., using solution of the ETSP)

## Part II

## Part 2 – Data Collection Planning for Surveillance Missions

## Summary of the Lecture

## Motivation

- There is a framework for testing and evaluation of UAVs control strategies developed and maintained by the winners of the Mohamed Bin Zayed International Robotics Challenge (MBZIRC) 2017

## Multi-robot Systems (MRS) group

<http://mrs.felk.cvut.cz>



<https://www.youtube.com/watch?v=jU3YbCtXpEw>

- The framework allows a direct evaluation of the planned trajectories, i.e., Dubins trajectories, in the same way for the simulator and also for real vehicles
- It provides an unique opportunity to become more familiar with multi-rotor unmanned aerial vehicles and gain experience with practical deployment of the planned trajectories to UAVs
- Full support of the evaluation environment is provided together with setuped computers at the dedicated computer lab of the MRS (KNE-118)
- A practical deployment on real UAVs would be possible during the first campaigns in spring 2018

## Topics Discussed

- Dubins vehicles and planning – Dubins maneuvers
- Dubins Interval Problem (DIP)
- Dubins Touring Problem (DTP)
- Dubins Traveling Salesman Problem (DTSP) and Dubins Traveling Salesman with Neighborhoods (DTSPN)
  - Decoupled approaches – Alternating Algorithm
  - Sampling-based approaches – GATSP
- Dubins Orienteering Problem (OP) and Dubins Orienteering Problem with Neighborhoods (DOPN)
- Data collection and surveillance planning in 3D
- **Next: Multirobot Path Planning (MPP)**