Parallel programming

HW1 assignment

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LU decomposition

- A decomposition of matrix **A** to lower and upper triangular matrices **L** and **U**, respectively.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$$A = LU$$

- **Application:**
  - Useful to quickly resolve a linear system of equation if only the right-hand side is changed (Ax = b).

LU decomposition

- LU decomposition is basically Gaussian elimination (GE)

$$E_3 E_2 E_1 A = U$$

E_i represent one step of GE, e.g., subtraction of a row multiple from another row

- Example of E_i matrix and its inverse (subtract 2 times the first row from the third row)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

E_i $\qquad\qquad$ E_i^{-1}

- Inverting E_i matrices we get L

$$A = \underbrace{E_1^{-1} E_2^{-1} E_3^{-1}}_{L} U$$

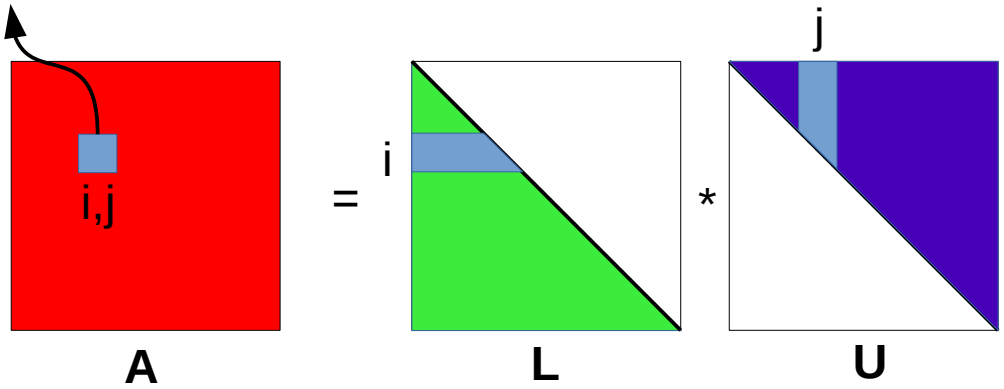
LU decomposition

- How does L look like?

$$L = E_1^{-1} E_2^{-1} E_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 4 & 1 \end{pmatrix}$$

LU decomposition - derivation

$$a_{ij} = \sum_{r=0}^{n-1} l_{ir} u_{rj} \qquad \forall i, j \in \{0, \ldots, n-1\}$$



$$l_{ir} = \begin{cases} 0 \text{ if } r > i \\ 1 \text{ if } r = i \\ l_{ir} \text{ otherwise} \end{cases}$$

$$u_{rj} = \begin{cases} 0 \text{ if } r > j \\ u_{rj} \text{ otherwise} \end{cases}$$

$$a_{ij} = \sum_{r=0}^{n-1} l_{ir} u_{rj} = \sum_{r=0}^{i} l_{ir} u_{rj} = \sum_{r=0}^{i-1} l_{ir} u_{rj} + l_{ii} u_{ij} = \sum_{r=0}^{i-1} l_{ir} u_{rj} + 1\, u_{ij}$$

$$\Rightarrow \quad u_{ij} = a_{ij} - \sum_{r=0}^{i-1} l_{ir} u_{rj}$$

$$a_{ij} = \sum_{r=0}^{n-1} l_{ir} u_{rj} = \sum_{r=0}^{j} l_{ir} u_{rj} = \sum_{r=0}^{j-1} l_{ir} u_{rj} + l_{ij} u_{jj}$$

$$\Rightarrow \quad l_{ij} = \frac{1}{u_{jj}} \left(a_{ij} - \sum_{r=0}^{j-1} l_{ir} u_{rj} \right)$$

LU decomposition - derivation

$$a_{ij}^{(k)} = a_{ij}^{(k-1)} - \sum_{r=0}^{k-1} l_{ir} u_{rj} \qquad \forall k \in \{0, \ldots, n-1\}$$

iteration

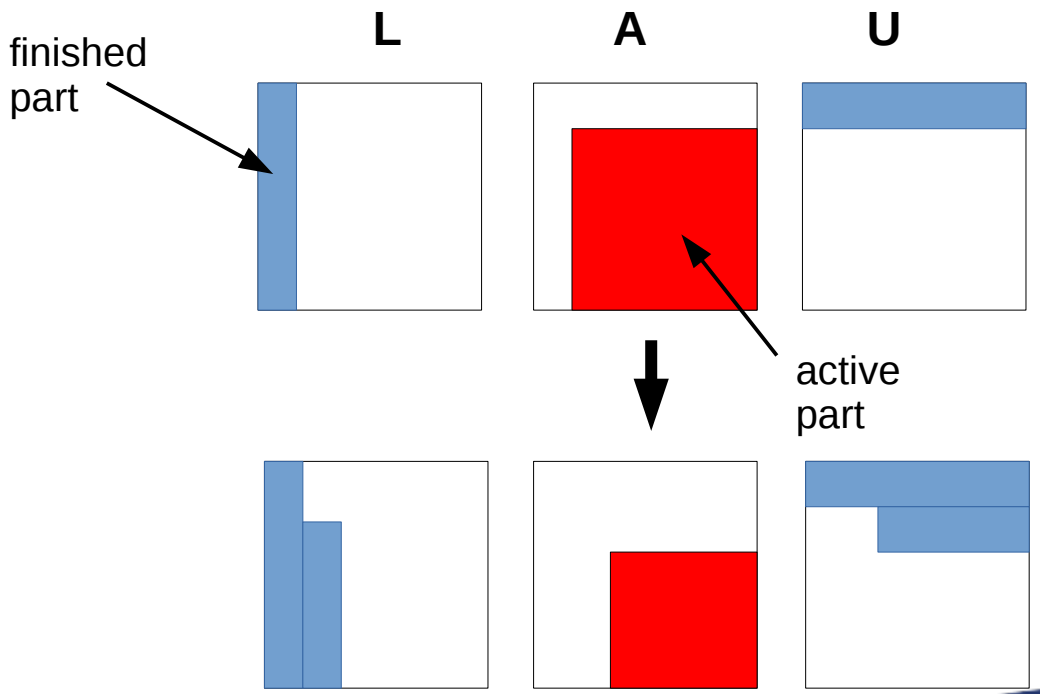
$$u_{ij} = a_{ij}^{(i)} \qquad l_{ij} = \frac{a_{ij}^{(j)}}{u_{jj}}$$

$$l_{ij} = \frac{1}{u_{jj}} \left(a_{ij} - \sum_{r=0}^{j-1} l_{ir} u_{rj} \right)$$

$$u_{ij} = a_{ij} - \sum_{r=0}^{i-1} l_{ir} u_{rj}$$

Pseudocode:

for k = 0 **to** n - 1 **do**
  for j = k **to** n - 1 **do**
    $u_{kj} = a_{kj}^{(k)}$
  $l_{kk} = 1$
  for i = k + 1 **to** n - 1 **do**
    $l_{ik} = a_{ik}^{(k)} / u_{kk}$
  for i = k + 1 **to** n - 1 **do**
    for j = k + 1 **to** n - 1 **do**
      $a_{ij}^{(k+1)} = a_{ij}^{(k)} - l_{ik} u_{kj}$



HW1 assignment

- Use the provided **code skeleton**
  - ➔ reads test problems, measures runtime, prints the matrices **L**, **U**, and **A**.
- **Assignment:**
  - ➔ implement LU decomposition
  - ➔ parallelize the code using C++11 threads
  - ➔ upload your solution to UploadSystem
    - ➔ What should I upload?
- **Flags for g++ (used by UploadSystem):**
  - `-pthread -Ofast -std=c++14 -march=native`
- **Tricky issues:**
  - For large problems the performance is memory bound. You can mitigate the issue by implementing a cache-friendly version (cache blocking technique).
  - In practice, a partial pivoting is used to have numerically stable results (avoid possible division by zero).