Mobile robot locomotion and kinematics principles

Robot trajectory control

Introduction & terms:

Robot actuators: Technical tools to physically influence environment (in general)

Specifically for mobile robots: Specialized systems allowing replacement of the robot in the environment

Technical solutions to mobility enabling systems:

- Wheeled traction (steered or skid-controlled)
- Tracked systems (always skid-control)
- Omnidirectional wheels (skid systems)
- Legged/walking systems (statically and dynamically stable)
- Aerial systems (hovers, gliders, multi-copters)
- Underwater vehicles

Featuring of locomotion systems

- Underwater vehicles (AUV)
  - Commonly constrained locomotion control and limitation on maneuvrability:

    Minimal forward speed assurance (gliders), braking and accelerations, curvature controls – for drifting and gliding systems
- Aerial systems (UAV)
  - Similar constrains on control and maneuvrability as underwater systems
  - Problem of static stabilization of the system (i.e. hovering multi-copter appears in principle unstable) brings control engineering problems
  - Power/fuel consumption also at wait-states (hovering, gliding)
- Walking/legged systems
  - High terrain throughput (advantage)
  - High complexity of locomotion control, many joints, drives and DOFs
  - Problem of the general robot body stabilization (to allow efficient sensing)
  - Problem of static/dynamic stabilization of the robot body (i.e. mono-peds, bi-peds, etc.).
  - Walking/legged systems bring up very complex and hard control problems!

- Wheeled and tracked systems
  - Purely wheeled systems (with a heading-steering system)
  - Tracked systems and systems with hard-coupled wheels (the both are skid-controlled)
  - Omnidirectional wheels (Mecanum wheels)

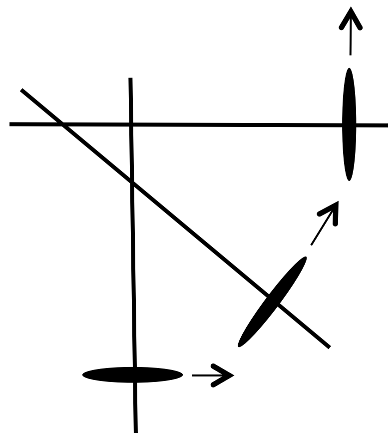
Kinematic construction of robot basic locomotion systems – types:

- Diferential drive
- Ackermann control
- Car-like
- Synchronous drive
- Omnidirectional wheels

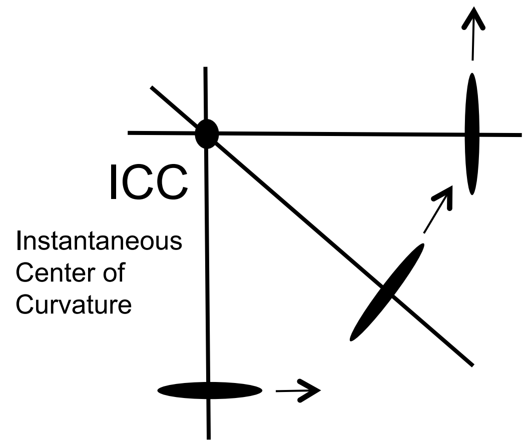
Purely wheeled systems and the slip

Modes of pure rotational motion vs. wheel slip:

Lemma: To assure pure wheel rotation, an ***instantaneous center of curvature (ICC)*** should exist, i.e. all the wheels on the robots' undercarriage in rotation should drive along the same curvature.



Rotation and slip



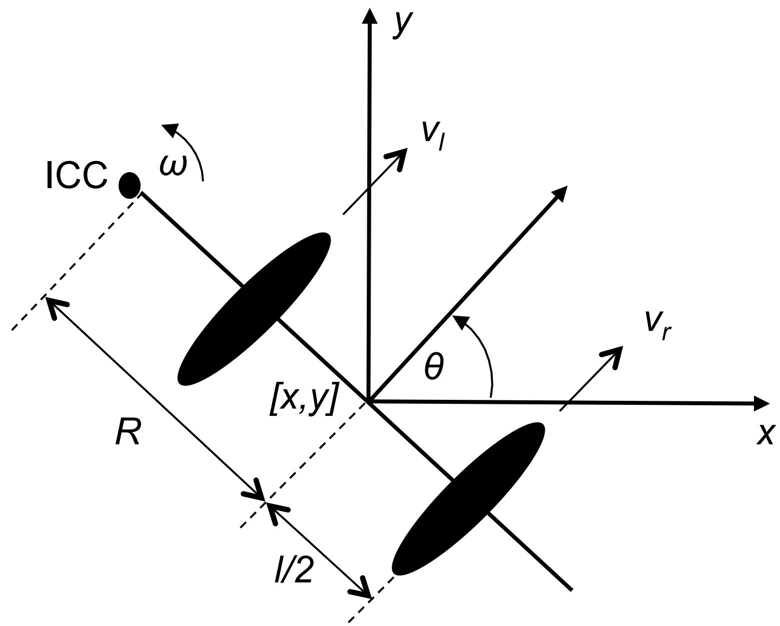
Rotation only

Remark 1: Holds only for ideally thin wheels, width -> 0

Remark 2: Other cases exhibiting skid are hard to be described by analytic tools

Diferential drive

Principal setup



$$ICC = \left[x - R\sin\Theta, y + R\cos\Theta\right]$$ Coordinates of Instantaneous Center of Curvature

$$R = \frac{l\left(v_l + v_r\right)}{2\left(v_l - v_r\right)}$$ Radius of the curvature

$$\omega = \frac{v_r - v_l}{l}$$ Angular speed along the curvature

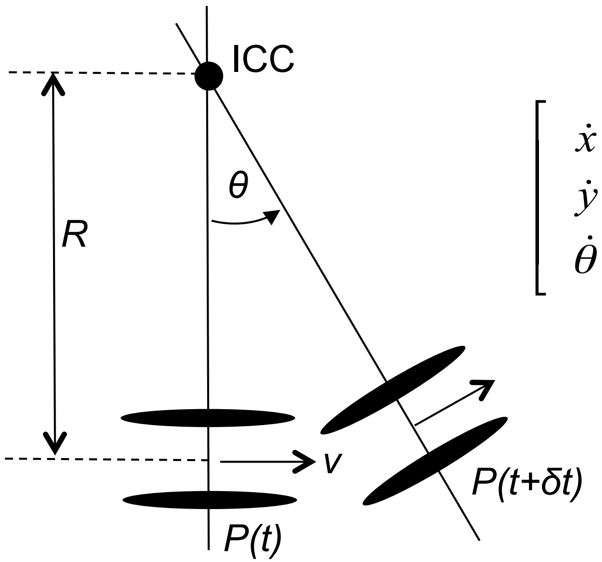
$$v_l = \omega\left(R - l/2\right)$$

$$v_r = \omega\left(R + l/2\right)$$

Forward velocities for each of the wheels

Generalized kinematics equation - holds for any type of drive

Principal setup



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega\delta t \end{bmatrix}$$

Differentiates the driving angle as $\omega\delta t \approx \delta\theta$

Where *v* and *θ* stand for the forward velocity*,* resp. the curvature angle

By integrating the afore we obtain:

$$x(t) = \int_0^t v(t).cos(\theta(t))dt \qquad y(t) = \int_0^t v(t).\sin(\theta(t))dt \qquad \theta(t) = \int_0^t \omega(t)dt$$

What implies:

Substituting the basic kinematic equation, also holds:

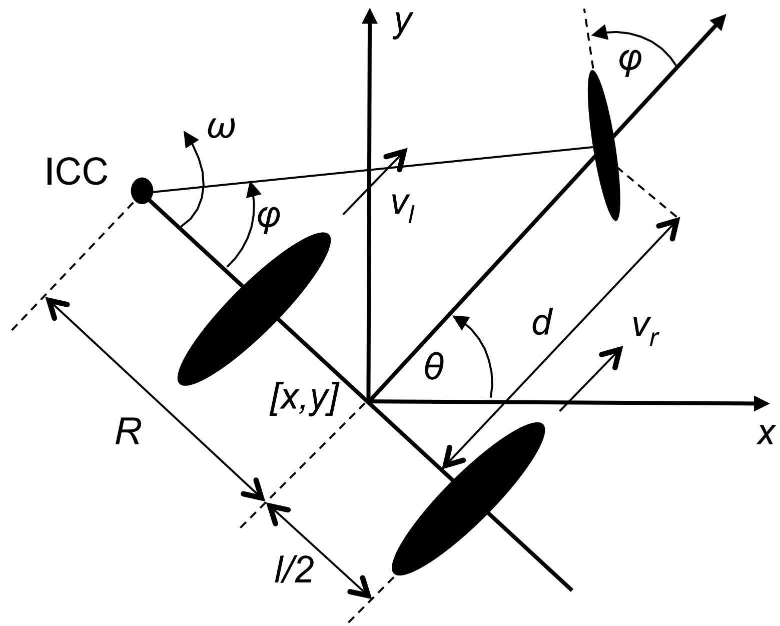
$$x(t) = \frac{1}{2}\int_0^t \left(v_r(t) + v_l(t)\right).\cos\left(\theta(t)\right)dt$$

$$y(t) = \frac{1}{2}\int_0^t \left(v_r(t) + v_l(t)\right).\sin\left(\theta(t)\right)dt$$

$$\theta(t) = \frac{1}{l}\int_0^t v_r(t) - v_l(t)dt$$

Ackermann drive (tricycle structure)

Principal setup



$ICC = \left[x - R\sin\Theta, y + R\cos\Theta\right]$ where $R = {d}/{\tan\varphi}$ for the center of curvature

$R = \dfrac{l\left(v_l + v_r\right)}{2\left(v_r - v_l\right)}$ Radius of the curvature

$\omega = \dfrac{v_r - v_l}{l}$ Angular speed

$v_l = \omega\left(R - l/2\right)$

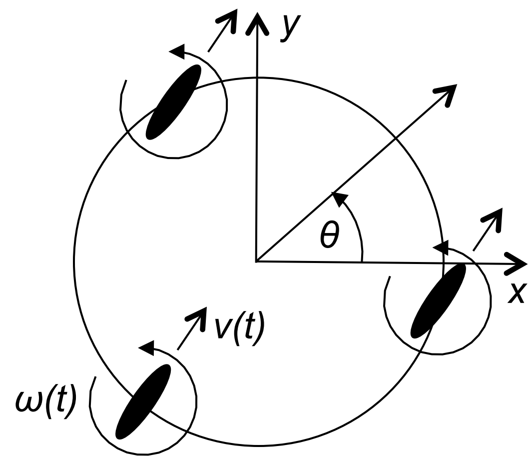
$v_r = \omega\left(R + l/2\right)$

Forward velocities of each of the wheels

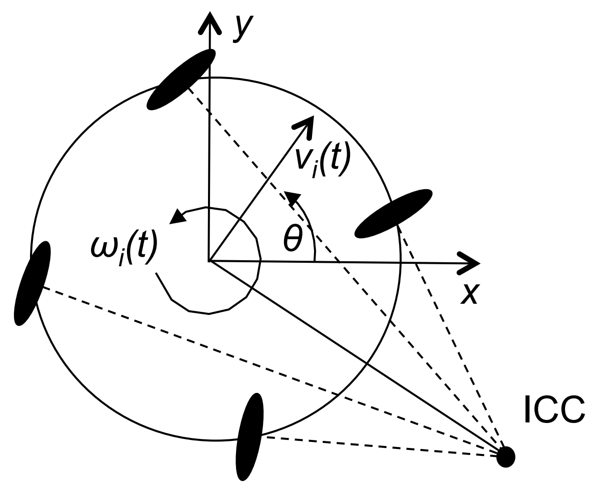
…and can also be substituted into kinematic equations as afore…

Special construction drives I

Synchronous drive



XR400 drive



Where for both the drives stands:

$$x(t) = \int_0^t v(t).cos(\theta(t))dt$$

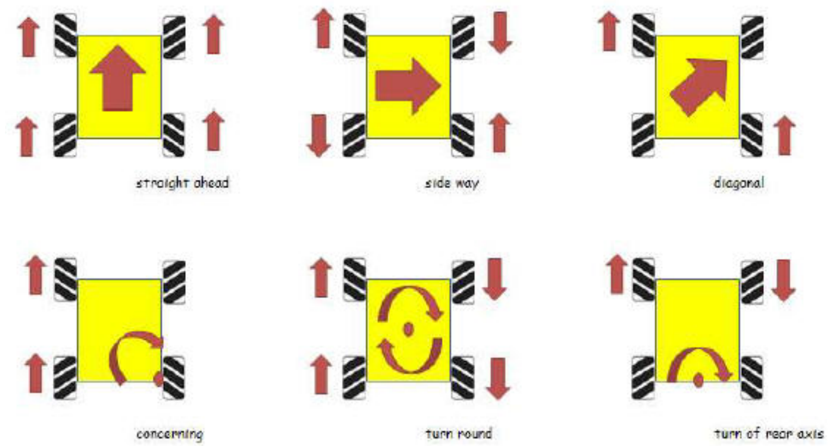
$$y(t) = \int_0^t v(t).\sin(\theta(t))dt$$

$$\theta(t) = \int_0^t \omega(t)dt$$

Motion in resulting direction of forces imposed by *v(t)* and *ω(t)*

Special construction drives II

Omnidirectional (mecanum) wheels



$$v_y = \frac{1}{4}\left(v_0 + v_1 + v_2 + v_3\right)$$

$$v_x = \frac{1}{4}\left(v_0 - v_1 + v_2 - v_3\right)$$

$$v_\theta = \frac{1}{4}\left(v_0 + v_1 - v_2 - v_3\right)$$

$$v_{error} = \frac{1}{4}\left(v_0 - v_1 - v_2 + v_3\right)$$

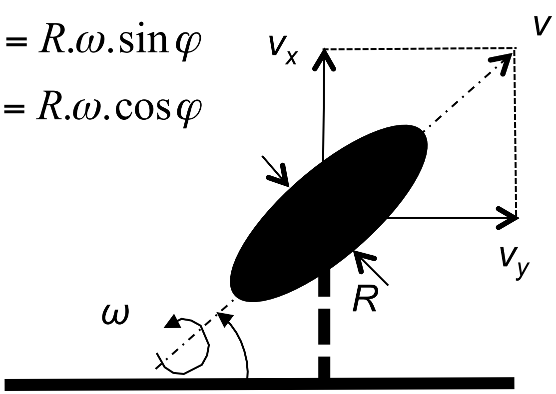
Where:

v_y – forward velocity
v_x – forward velocity
v_θ – forward velocity

Example of performance at:
https://www.youtube.com/watch?v=TXTo16KKm8Q

$$v_x = R.\omega.\sin\varphi$$

$$v_y = R.\omega.\cos\varphi$$



Robot undercarriage

Each of the sub-wheels performs distribution of forces (velocities) as in the figure.

Holonomic vs. non-holonomic systems, constrains

- Lets' have a system description in generalized coordinate system $\vec{q} \in Q$, where Q stands for robot state space (configuration space, *C*-space)
- Given the robot trajectory *q(t),* the vector of the system velocity stands $\dot{\vec{q}}(t) \in Tq(Q)$
- The system in state $\vec{q}$ under controls $\vec{u} \in U$ from the control space holds the fuction: $F:[Q,U] \rightarrow Tq(Q)$ denoting the space of generalized coordinates, so that the fundamental kinematic equation can be rewritten as: $\vec{q} = F(\vec{q},\vec{u})$

An example: A plotter without dynamics, input controls are u_x, u_y

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$

□

- Realistic systems always exhibit some kinematic constrains of the generalized coordinate system (i.e. do not allow attaining certain values of these coordinates); these constrains can be defined for *i=1..k*, for all *k<n*, where *n* stands for the system dimension as:

$$a_i(\vec{q},\dot{\vec{q}}) = 0$$

- The kinematic constrains may also be rewritten into form: $a_i.\vec{q}.\dot{\vec{q}} = 0$, for all *i=1..k, k<n*
  - If the kinematic constrain can be integrated; the system is holonomic.
  - If all the kinematic constraining conditions are holonomic, the whole system stands also holonomic.

An example:

Yet simple constraint for the previous plotter system can be: $2x\dot{x} + 2y\dot{y} = 0$; integrating this constraint (along *dxdy*) delivers: $x^2 + y^2 = c^2$, which denotes circular trajectories only. □

Consequeces of holonomy

- Holonomic constrains limit the state-space of the robot; i.e. the robot can not execute all types of locomotions, but only such, which comply with the given constraints (this is sometimes also entitled as “geometric constrains”)
- Major advantage of holonomic systems is, that control synthesis can be obtained by making use of general approaches (i.e. deriving unambiguous controls for execution of trajectory A->B)
- Non-holonomic systems are hardly controllable; no unified approach for control synthesis (can not be made asymptotically stable) − the task is mainly resolved via ”strategy switching”