



# **Parallel Evolutionary Algorithms. Coevolution.**

Petr Pošík



# Parallel Evolutionary Algorithms



# Motivation

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EAs applied on complex tasks need long run times to solve the problem:

- What is usually the most time-consuming task when solving real-world problems?

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  - Fitness evaluation!!!
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    - In GP, when evolving classifiers, functions, or programs, the fitness must be assessed by measuring the success when applying the classifier, function, or program on a set of training task instances

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⇒ PARALLELIZE!!!

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- Which of the above can be parallelized easily???

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# Agenda

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How can we parallelize?

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How can we parallelize?

1. Run several independent GAs in parallel.

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How can we parallelize?

1. Run several independent GAs in parallel.
2. Run a single GA, but distribute the time consuming things to parallel machines.  
**(Master-slave model.)**

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1. Run several independent GAs in parallel.
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3. Run several *almost independent* GAs in parallel; exchange a few individuals occasionally. (**Island model.**)

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4. Run a single GA with selection that takes only a few individuals into account. (**Spatially embedded model.**)

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6. Other, less standard possibilities. (**Injection model, heterogenous PGA.**)

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But first:

- The difference between parallel model and parallel implementation.

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# Parallel Implementation vs. Parallel Model

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## Sequential implementation:

- The algorithm is able to run on a single machine in a single process, often in a single thread only.

## Parallel implementation:

- The algorithm is able to take advantage of multiple CPU cores or multiple machines.

## The effect of parallelization:

- Reduction in the solution time by *increasing computational power*.
- The speed-up should be proportional to the number of parallel machines.

## Global model:

- The population is not divided in any way, the selection operator can consider all individuals.

## Parallel model:

- The population is somehow divided into subpopulations, which limits mainly the selection operator.

## The effect of parallelization:

- Changes the algorithm behavior substantially.



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## Possible combinations:

- Sequential implementation of the global model (usual case, simple GA)
- Parallel implementation of the global model (master-slave, brute-force speed-up)
- Sequential implementation of a parallel model (modified behavior)
- Parallel implementation of a parallel model (modified behavior, brute-force speed-up)

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## Parallelization of the Global Model



# Master-slave model

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## Master

- runs the evolutionary algorithm, and
- controls the slaves, distributes the work.

## Slaves

- take batches of individuals from the master,
- evaluate them, and
- send their fitness back to the master.

## Other possibilities:

- Sometimes we can parallelize also initialization, mutation, and (with a bit of care) crossover.
- The hardest parts to parallelize are selection and replacement.
- When does the parallelization actually pay off???

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Master-slave implementation does not change the behavior of the global model.

- Hints on implementation (locking, synchronizing) can be found in [Luk09, chap. 5].

[Luk09] Sean Luke. *Essentials of Metaheuristics*. 2009. available at <http://cs.gmu.edu/~sean/book/metaheuristics/>.

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## Island Model



# Island Model

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Also called *coarse-grained PGA* or *multi-deme GA*:

- By far the most often used model of PGA.

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- **Island Model**
- Migration
- Migration (cont.)

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The profit from island model:

- Demes are smaller:
  - converge faster,
  - can converge to different local optima, but
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DEMO: Island model of PGA applied on TSP

<http://labe.felk.cvut.cz/~posik/pga>

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**Migration topology:** Where should we take the migrants from and where should we put them?

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  - topologies with the same DOC exhibit similar behavior
  - in a comparison of fully-connected topology, 4D hypercube,  $4 \times 4$  toroidal net, and one-way and two-way rings, densely connected topologies were able to find the global optimum with lower number of evaluation

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  - diversity  $\rightarrow$  convergence; population convergence vs. convergence in time

## Migration (cont.)

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**Migration type:** Can the migration events occur individually or in batches?

- batch: all migration events occur in the same time, all demes send emigrants to their targets and take the immigrants from their sources
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Which individuals should be selected as emigrants? Which individuals should be replaced by immigrants?

- Best, worst
- Best, random
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- term *epoch* in the context of PGAs describes the part of evolution between 2 migration events



## Other Parallel Models



# Spatially Embedded Model

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Also called *fine-grained PGA*:

- Population has a structure (1D grid, 2D toroidal grid, 3D cube, etc.)
- Each individual has a position in this structure.

PGA

Global model

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- Spatially Embedded Model
- Model Combinations
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- The best individuals do not spread in the population so fast. Diversity promotion.
- Easy parallelization via multithreading.
- Very efficient model for *vector processors*, often found on GPUs:
  - many identical operations can be performed in parallel in the same time

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Summary: PGAs

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Coevolution

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# Model Combinations

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## Hierarchical model:

- various combinations of the above mentioned models, e.g.
- island model where each deme uses master-slave fitness evaluation,
- island model where each deme uses spatially embedded model, etc.

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Other Parallel Models

- Spatially Embedded Model
- **Model Combinations**
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## Heterogenous model:

- Each deme uses a different optimizer
  - Different parameter settings
  - Different operators of selection, crossover, mutation and/or replacement
  - Completely different optimization algorithm (local search, differential evolution, ...)

PGA

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  - Completely different optimization algorithm (local search, differential evolution, ...)
  - Can each deme use a *different fitness function*???

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PGA

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Global model

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---

Other Parallel Models

- Spatially Embedded Model

- **Model Combinations**

- Injection Model

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Summary: PGAs

---

Coevolution

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Problems in coevolution

---

Summary: Coevolution



# Injection Model

---

Heterogenous island model where

- each deme uses a different fitness function!!!
- Usable when many quality criteria must be assessed; each deme
  - concentrates on one criterion and
  - submits partial solutions to other demes to be reworked using another criterion.
- Each deme preserves solutions of high quality when only its particular criterion is applied.

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## Summary: PGAs



# Summary

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- Parallelization can increase the speed the EA:
  - parallel implementations
  - parallel models

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Summary: PGAs

- [Summary](#)
- Learning outcomes

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# Summary

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- Parallelization can increase the speed the EA:
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- Parallel models change the behavior of the EA:
  - they can reduce the danger of premature convergence and speed-up the algorithm in the same time.

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# Summary

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- Parallelization can increase the speed the EA:
  - parallel implementations
  - parallel models
- Parallel models change the behavior of the EA:
  - they can reduce the danger of premature convergence and speed-up the algorithm in the same time.
- There are many possibilities on parallelization:
  - the optimal decision depends on the (parallel) computer architecture and on the task being solved
  - all possibilities introduce their own set of tunable parameters :-)

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---

Summary: PGAs

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- [Summary](#)
- Learning outcomes

Coevolution

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Problems in  
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# Learning outcomes

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After this lecture, a student shall be able to

- explain the main motivation for parallelization;
- explain the difference between parallel *implementation* and parallel *model* of EA;
- describe the features of individual combinations of sequential/parallel implementation and global/parallel model;
- know which parts of individual EAs (fitness evaluation, selection, replacement, mutation, crossover, model building, etc.) can be implemented in parallel easily, and explain why;
- explain the principle and features of the master-slave parallelization;
- implement island model and explain its features, describe the characteristics of migration operator (type, topology, degree of connectivity, trigger, selection/replacement strategy, count);
- describe spatially embedded model, heterogeneous model, injection model, and their use cases.

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---

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# Coevolution



# What is “coevolution”?

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Coevolution

- What?
- Types
- 1-pop comp.
- 2-pop comp.
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Problems in  
coevolution

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# What is “coevolution”?

---

Coevolution in EAs:

- The fitness of individuals in a population
  - is not given by the characteristics of the individual (only), but
  - is *affected by the presence of other individuals in the population.*
  
- It is closer to the biological evolution than ordinary EAs are.

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Coevolution can help in

- dealing with increasing difficulty of the problem,
- providing diversity in the system,
- producing not just high-quality, but also robust solutions,
- solving complex or high-dimensional problems by breaking them into nearly decomposable parts.

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Summary: PGAs

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Problems in  
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# Types of coevolution

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Summary:  
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# Types of coevolution

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By relation type:

- cooperative (synergic, compositional)
- competitive (antagonistic, test-based)

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Coevolution



# Types of coevolution

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By relation type:

- cooperative (synergic, compositional)
- competitive (antagonistic, test-based)

By the entities playing role in the relation:

- 1-population
  - intra-population
  - individuals from the same population cooperate or compete
- N-population
  - inter-population
  - individuals from distinct populations cooperate or compete

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Global model

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Summary: PGAs

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Problems in  
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# 1-population competitive coevolution

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**Example:** The goal is to evolve a game playing strategy

- successful against diverse opponents!!!

How would you proceed in an ordinary EA?

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  - A bit better... but beware (Blondie24)

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Summary:  
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  - Problem: No learning gradient! Needle in a haystack. All randomly generated players will almost surely lose against any advanced player.
- by playing several games against internet players?
  - A bit better... but beware (Blondie24)

**Solution:** Intra-population competitive coevolution

- by playing several games against other strategies in the population.
- All individuals of the same type.
- In the beginning, all are probably quite bad, but some of them are a bit better.
- The fitness (the number of games won) may not rise as expected since your opponents improve with you.

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Summary: PGAs

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Coevolution

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Problems in  
coevolution

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Summary:  
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## 2-population competitive coevolution

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**Example:** The goal is to evolve a sorting algorithm

- able to sort any sequence of numbers
- correctly and quickly.

How would you proceed in an ordinary EA?

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- Test all possible input sequences? Slow, intractable.
- Test only a fixed set of sequences? Which ones?

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Summary: PGAs

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Coevolution

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**Solution:** Inter-population competitive coevolution

- 2 populations, 2 species:
  - sorting algorithms
  - test cases (sequences to sort)

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Global model

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Other Parallel Models

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Summary: PGAs

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Coevolution

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Coevolution

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- Fitness evaluation:

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Coevolution

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- 2 populations, 2 species:
  - sorting algorithms
  - test cases (sequences to sort)
- Fitness evaluation:
  - Algorithm: by its ability to sort. How many sequences is it able to sort correctly? How quickly?
  - Test case: by its difficulty for the current sorting algorithms. How many algorithms did not sort it?
- Predator-prey relationship

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Summary: PGAs

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Problems in  
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Summary:  
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# N-population cooperative coevolution

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**Example:** The goal is to evolve a team consisting of

- a goalie, back, midfielder, and forward
- so that they form a good team together.

How would you proceed in an ordinary EA?

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How would you proceed in an ordinary EA?

Fitness evaluation:

- by simulating a number of games between teams

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- Represent all 4 strategies in 1 genome, evolve them all in 1 population.
- Theoretically possible, but the space is too large.
- May result in a team of players which wouldn't perform well if substituted to another team.

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- 4 separate populations
- Evolve players which would play well with any other team members

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**Cooperation:**

- symbiotic relationship
- good performance of the team  $\Rightarrow$  high contribution to fitness of all members

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Global model

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# 1-population cooperative coevolution

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# 1-population cooperative coevolution

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Example: Niching methods for

- diversity preservation
- maintaining several stable subpopulations in diverse parts of the search space

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Examples of niching methods:

- fitness sharing
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# 1-population cooperative coevolution

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Example: **Niching** methods for

- diversity preservation
- maintaining several stable subpopulations in diverse parts of the search space

Examples of niching methods:

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Principle:

- better individuals similar to others already in population are thrown away in favour of worse, but diverse individuals
- the selection process is affected by the presence of other individual in the neighborhood

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Global model

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## Problems in coevolution





# Fitness in coevolution

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Some important classifications of fitness

- by its time-dependence:
  - **static**: does not change with time
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# Fitness in coevolution

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  - **deterministic**: generates the same ordering of a set of individuals
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- by the role of other individuals in evaluation:
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- should be **static**, **deterministic** and **absolute**
- can easily be used as internal fitness

External fitness in coevolution:

- impossible (hard) to define
- often, it is **relative**, but measured with a carefully chosen, large enough set of other individuals (**static**) sufficiently many times (almost **deterministic**)

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Internal fitness in coevolution:

- **relative**: affected by other individuals
- **dynamic**: affected by evolving individuals (needs re-evaluation)
- **stochastic**: usually evaluated against a smaller number of individuals

# “Fitness” in sport

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Football league:



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- tournaments have different prize money to distribute to tournament winners
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- the player's level increases (decreases) if she recently won more (less) matches than expected

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None of these systems is static:

- Is Pete Sampras better than Roger Federer?
- Is Arnold Palmer better than Tiger Woods?
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The same holds for fitness assessment in coevolution!



# Problems with fitness assessment: 1-pop. competitive coevolution

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## Cycles, etc.

- What if A beats B, B beats C, but C beats A?

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  - A player that beats the most other “good” players?
  - A player that wins by the most total points on average?
- Often, additional matches are executed.
- But, do you want to spend your fitness budget
  - on evaluating current individuals more precisely, or
  - on searching further?

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## 2 competitive populations (illustration)

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Lotka-Volterra model (Predator-prey population dynamics):

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = -\gamma y + \delta xy$$

where  $x$  is the number of prey (rabbits)  
and  $y$  is the number of predators (wolves).

Assumptions:

1. The prey population has always food enough.
2. The predators eat only the prey.
3. The rate of change of population is proportional to its size.
4. The environment is static.

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Meaning:

- The change of the prey population ( $dx/dt$ ) is composed of
  - increase due to the newly born individuals (proportional to the population size,  $\alpha x$ ) and
  - decrease caused by the predation (which is proportional to the rate of predator-prey meetings,  $\beta xy$ ).
- The change of the predator population ( $dy/dt$ ) is composed of
  - decrease due to natural death (proportional to the population size,  $\gamma y$ ) and
  - increase allowed by the food supply (proportional to the rate of predator-prey meetings,  $\delta xy$ ).

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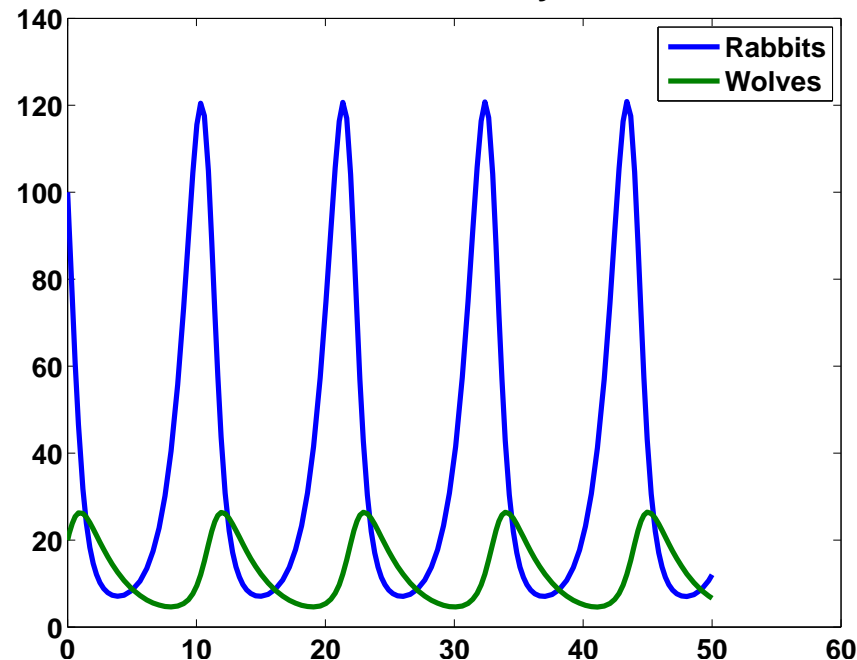
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Time history





# Problems with fitness assessment: 2-pop. competitive coevolution

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## Arms races

- one population learns a trick and forces the second population to learn a new trick to beat the first one. . .
- one population may evolve faster than the other:

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  - external fitness in both populations drops until the gradient re-emerges
- not exactly what was shown by Lotka-Volterra, but similar
- Solution:
  - detect such situation (but how?)
  - delay the evolution of the better population until the worse one catches up

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# Problems with fitness assessment: N-pop. cooperative coevolution

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**Hijacking** (in team of goalie, back, midfield, and forward):

- a really good forward takes over one population, any team will play well thanks to him
- members of all other populations have almost the same fitness  $\Rightarrow$  uniform random selection
- Solution: apply some form of *credit assignment*

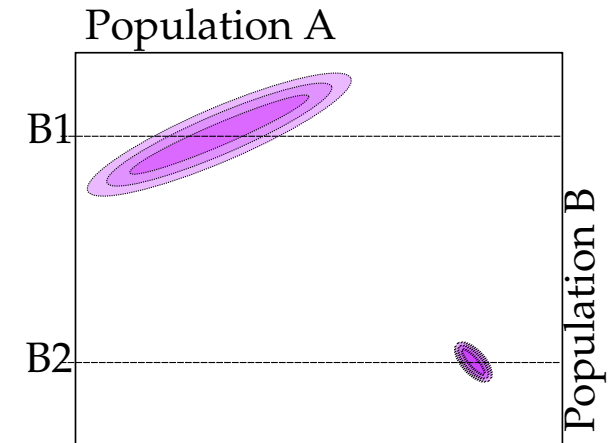
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## Relative overgeneralization

- when evaluated by average score, worse (but more robust) individual B1 will have higher score than better (but volatile) B2
- use maximum score (more tests needed)
- but again, the choice depends on what we want — a player able to get the highest score, or a player that would form a good team with the most other team members?



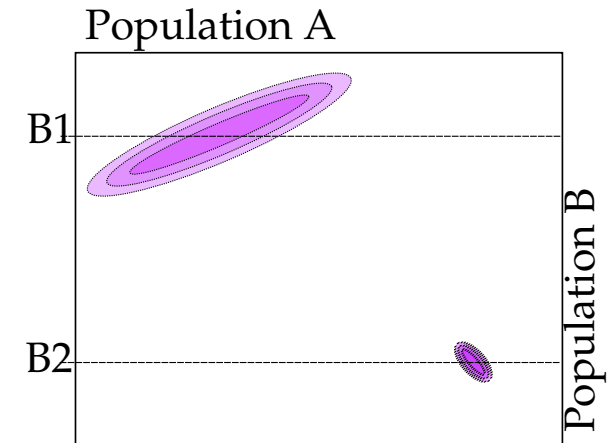
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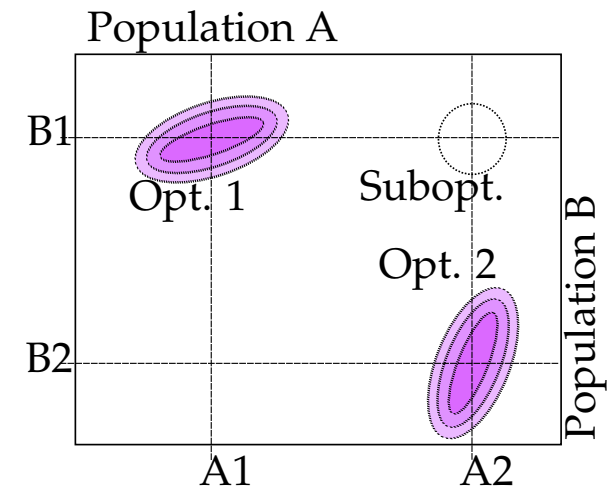
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## Miscoordination

- when the team components are not independent
- Pop. A evolved A2 (but not A1), pop. B evolved B1 (but not B2)
- Neither A2 nor B1 survives





## Summary: Coevolution



# Summary

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## Coevolution

- can be cooperative or competitive (or both)
- can take place in 1 population or in more populations
- fitness is not fixed during evolution
- introduces new unexpected dynamics to the system (new issues to be solved)

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# Summary

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## Coevolution

- can be cooperative or competitive (or both)
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- fitness is not fixed during evolution
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## Appropriate when

- no explicit fitness function can be formed
- there are too many fitness cases
- the problem is modularizable (divide and conquer)

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# Learning outcomes

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After this lecture, a student shall be able to

- define “coevolution”, explain what makes it different from ordinary evolution in the context of optimization algorithms;
- explain differences between *cooperative* and *competitive* coevolution;
- explain differences between *intra-* and *inter-*population coevolution;
- define features of fitness measure (static/dynamic, deterministic/stochastic, absolute/relative, internal/external);
- describe the features of an ideal external (and internal) fitness, and describe the features of internal fitness in coevolution;
- exemplify individual types of cooperative/competitive intra-/inter-population coevolution; and
- exemplify various types of issues that can be observed in them (how to order individuals based on inconsistent matches, loss of diversity when one population evolves faster than the other, hijacking, relative overgeneralization, miscoordination).

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