A0B17MTB – Matlab

Part #2

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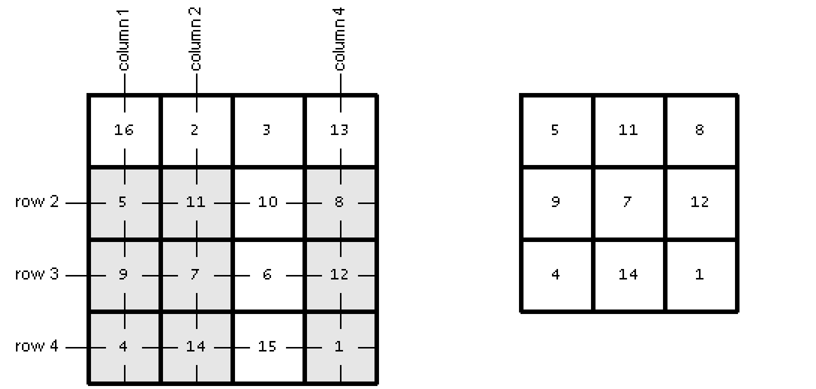
Learning how to …

Complex numbers

Matrix creation

Operations with matrices



`A`

`A([2 3 4], [1 2 4])`

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Complex numbers

- more entry options in Matlab
  - we want to avoid confusion
  - speed

```
>> C5 = sqrt(-1)
```

```
>> C1 = 1 + 1j
>> C2 = 1 + 5i % preferred
>> C3 = 1 + i
>> C4 = 1 + j5
```

- frequently used functions

`real, imag`	real and imaginary part of a complex number
`conj`	complex conjugate
`abs`	absolute value of a complex number
`angle`	angle in complex plane (in [rad])
`complex`	constructs complex number from real and imaginary components
`isreal`	checks if input is a complex number (more on that later)
`i, j`	complex unit
`cplxpair`	sorts complex numbers into complex conjugate pairs

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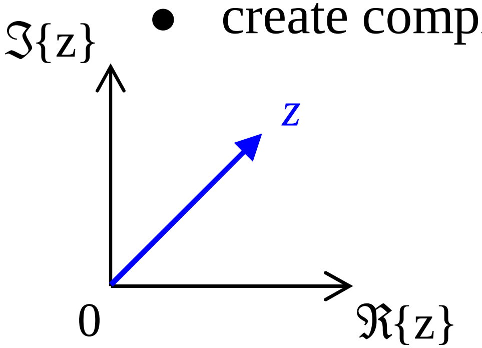
Complex numbers

300 s

- create complex number z and its complex conjugate



$$z = 1 + 1\mathrm{j}$$
$$s = z^*$$

- switch between Cartesian and polar form (find $|z|, \varphi$)

$$z = \Re\{z\} + \mathrm{j}\Im\{z\} = a + \mathrm{j}b$$
$$z = |z|\mathrm{e}^{\mathrm{j}\varphi},\ |z| = \sqrt{a^2 + b^2}$$
$$z = |z|\left(\cos(\varphi) + \mathrm{j}\sin(\varphi)\right)$$

- verify Moivre's theorem

$$z^n = \left(|z|\mathrm{e}^{\mathrm{j}\varphi}\right)^n$$
$$z^n = |z|^n\left(\cos(n\varphi) + \mathrm{j}\sin(n\varphi)\right)$$

$$Z = |z| = \sqrt{2} \approx 1.4142$$
$$\varphi = \arctan\left(\frac{\Im\{z\}}{\Re\{z\}}\right) = \arctan\left(\frac{1}{1}\right) \approx 0.7854\,\mathrm{rad}$$

Complex numbers

300 s

- find out magnitude of a complex vector (avoid indexing)
  - use `abs`, `sqrt`

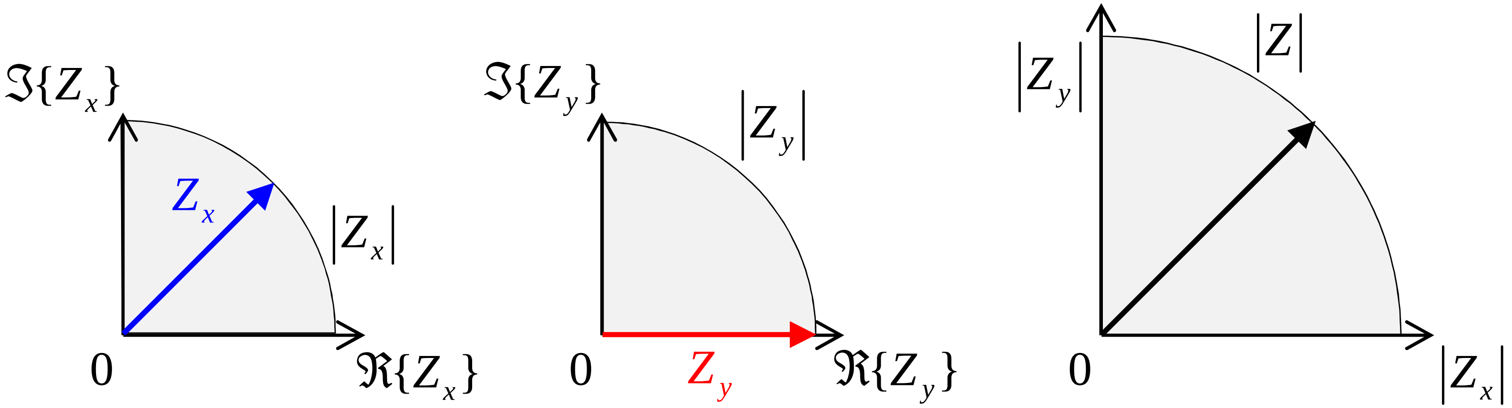
$$\mathbf{Z} = \begin{pmatrix} 1+1\mathrm{j} & \sqrt{2} \end{pmatrix}$$

$$\|\mathbf{Z}\| = ?,\ \ \mathbf{Z} \in \mathbb{C}^2$$

(1) $|Z_x|,\ |Z_y|$

(2) $|\mathbf{Z}| = \sqrt{|Z_x|^2 + |Z_y|^2} = \sqrt{Z_x Z_x^* + Z_y Z_y^*}$
$= \sqrt{\mathbf{Z} \cdot \mathbf{Z}^*} = \sqrt{|\mathbf{Z}|^2}$

  - alternatively, use following functions:
    - `norm`
    - `dot` (*dot product*)
    - `hypot` (*hypotenuse*)



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Transpose and matrix conjugate

- Pay attention to situations where the matrix is complex, $\mathbf{A} \in \mathbb{C}^{M \times N}$
- two distinct operations:

transpose	$\mathbf{A}^{\mathrm{T}} = \left[A_{ij}\right]^{\mathrm{T}} = \left[A_{ji}\right]$	`transpose(A) % <- don't use`	`A.'`
transpose + conjugate	$\mathbf{A}^{\mathrm{H}} = \mathbf{A}_{ij}^{\mathrm{H}} = \left[\mathbf{A}^{*}\right]^{\mathrm{T}}$	`conj(A) % <- don't use`	`A'`

```
>> A = magic(2) + 1j*magic(2)'

A =

   1.0000 + 1.0000i   3.0000 + 4.0000i
   4.0000 + 3.0000i   2.0000 + 2.0000i
```

```
>> A.'

ans =

   1.0000 + 1.0000i   4.0000 + 3.0000i
   3.0000 + 4.0000i   2.0000 + 2.0000i
```

```
>> A'

ans =

   1.0000 - 1.0000i   4.0000 - 3.0000i
   3.0000 - 4.0000i   2.0000 - 2.0000i
```

Entering matrices – „ : “

- large vectors and matrices with regularly increasing elements can be typed in using colon operator
  - `a` is the smallest element („from“) , `incr` is increment, `b` is the largest element („to“)

```
>> A = a:incr:b
```

```
>> A = 1:4:17

A =

     1     5     9    13    17
```

- `b` doesn’t have to be element of the series in question
  - last element $N \cdot incr$ then follows the inequality:

$$\left| a + N \cdot incr \right| \leq \left| b \right|$$

```
>> A = 1:4:7

A =

     1     5
```

- if `incr` is ommited, the increment is set equal to 1

```
>> A = a:b
```

```
>> A = 0:10

A =

     0     1     2     3     4     5     6     7     8     9    10
```


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Entering matrices

300 s

- Using the colon operator „:“ create
  - following vectors

$$\mathbf{u} = \begin{pmatrix} 1 & 3 & \dots & 99 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} 25 & 20 & \dots & -5 \end{pmatrix}^{\mathrm{T}}$$

  - matrix
    - caution, the third column cant be created using colon operator ":" only

$$\mathbf{T} = \begin{pmatrix} -4 & 1 & \dfrac{\pi}{2} \\ -5 & 2 & \dfrac{\pi}{4} \\ -6 & 3 & \dfrac{\pi}{6} \end{pmatrix}$$

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Entering matrices – `linspace`, `logspace`

- colon operator defines vector with evenly spaced points
- In the case fixed number of elements of a vector is required, use `linspace`:

```
>> A = linspace(a, b, N);
```

```
>> A = linspace(0,2,5)

A =

         0    0.5000    1.0000    1.5000    2.0000
```

  - When the `N` parameter is left out, 100 elements of the vector are generated:

```
>> A = linspace(a, b);
```

- the function `logspace` works analogically, except that logarithmic scale is used

```
>> A = logspace(a, b, N);
```


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Entering matrices

200 s

- create a vector of 100 evenly spaced points in the interval <-1.15,75.4>

- create a vector of 201 evenly spaced points in the interval <100,-100>

- create a vector with spacing of -10 in the interval <100,-100>
  - try both options using `linspace` and colon ":"

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Entering matrices using functions

- special types of matrices of given size are needed quite often
  - Matlab offers number of functions to serve this purpose
- example: matrix filled with zeros
  - will be used quite often

```
zeros(m)                  % matrix B of size m×m
zeros(m, n)               % matrix B of size m×n
zeros(m, n, p,...)        % matrix B of size m×n×p×...
zeros([m n])              % matrix B of size m×n

B = zeros(m, 'single') % matrix B of size m×m, of type ‘single’)

% see Help for other options
```

Entering matrices using functions

- following useful functions analogical to the `zeros` function are available

`ones`	matrix filled with ones
`eye`	identity matrix
`NaN, Inf`	matrix filled with NaN, matrix filled with Inf
`magic`	matrix suitable for Matlab experiments, notice its interesting properties
`rand, randn, randi`	matrix filled with random numbers coming from uniform and normal distribution, matrix filled with uniformly distributed random integers
`randperm`	returns a vector containing a random permutation of numbers
`diag`	creates diagonal matrix or returns diagonal of a matrix
`blkdiag`	constructs block diagonal matrix from input arguments
`cat`	groups several matrices into one (depending on dimension)
`true, false`	creates a matrix of logical ones and zeros
`pascal, hankel`	Pascal matrix, Hankel matrix

- for further functions see Matlab → Mathematics → Elementary Math → Constants and Test Matrices

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Entering matrices using functions

360 s

- create following matrices
  - use Matlab functions
  - begin with matrices you find easy to cope with

$$\mathbf{M}_1 = \begin{pmatrix} \mathrm{NaN} & \mathrm{NaN} \\ \mathrm{NaN} & \mathrm{NaN} \end{pmatrix}$$

$$\mathbf{M}_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathbf{M}_3 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{pmatrix}$$

$$\mathbf{M}_4 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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Entering matrices using functions

360 s

- try to create empty 3-dimensional array of type `double`

- can you find another option?

Entering matrices

- quite often there are several options how to create given matrix
  - it is possible to use <u>output of one function as an input of another</u> function in Matlab:
- consider
  - clarity
  - simplicity
  - speed
  - convention

```
>> plot(diag(randn(10, 1), 1))
```

- e.g. band matrix with '1' on main diagonal and with '2' and '3' on secondary diagonals

```
>> N = 10;
>> diag(ones(N,1)) + diag(2*ones(N-1,1),1) + diag(3*ones(N-1,1),-1)
```

  - can be sorted out using `for` cycle as well (see next slides), might be faster ...
  - some other idea?

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Dealing with sparse matrices

- Matlab provides support for working with sparse matrices
  - most of the elements of sparse matrices are zeros and it pays off to store them in a more efficient manner
- to create sparse matrix S out of a matrix A:

```
S = sparse(A),
```

- conversion of a sparse matrix to a full matrix :

```
B = full(S),
```

  - in the case of need see Help for other functions

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Matrix operations #1

- there are other useful functions apart from transpose (`transpose`) and matrix diagonal (`diag`) :

```
P =

    0.3404    0.2551    0.9593    0.2575
    0.5853    0.5060    0.5472    0.8407
    0.2238    0.6991    0.1386    0.2543
    0.7513    0.8909    0.1493    0.8143
```

  - upper triangular matrix

```
>> U = triu(P),
```

```
>> U = triu(P)

U =

    0.3404    0.2551    0.9593    0.2575
         0    0.5060    0.5472    0.8407
         0         0    0.1386    0.2543
         0         0         0    0.8143
```

  - lower triangular matrix

```
>> L = tril(P),
```

```
>> L = tril(P)

L =

    0.3404         0         0         0
    0.5853    0.5060         0         0
    0.2238    0.6991    0.1386         0
    0.7513    0.8909    0.1493    0.8143
```

  - a matrix can be modified taking into account secondary diagonals as well

```
>> L = triu(P, -1),
```

```
>> U2 = triu(P,-1)

U2 =

    0.3404    0.2551    0.9593    0.2575
    0.5853    0.5060    0.5472    0.8407
         0    0.6991    0.1386    0.2543
         0         0    0.1493    0.8143
```


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Matrix operations #2

- function `repmat` is used to copy (part of) a matrix

```
>> B = repmat(A, m, n),
```

  - e.g.

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \end{pmatrix}$$

```
>> B = repmat(A, 1, 2),
>> B = repmat(A, [2 1]),
```

$$\boxed{\begin{matrix} A_{11} & A_{12} & A_{13} & A_{11} & A_{12} & A_{13} \end{matrix}}$$

$$\boxed{\begin{matrix} A_{11} & A_{12} & A_{13} \\ A_{11} & A_{12} & A_{13} \end{matrix}}$$

- `repmat` is a very fast function
  - comparison of execution time of creating a 1e4 × 1e4 matrix filled with zeros :

```
>> X = zeros(1e4, 1e4);      % computed in 0.18s
>> Y = repmat(0, 1e4, 1e4); % computed in 0.0004s, BUT... don't use it
```

  - it is for you to consider the way of matrix allocation …

Matrix operations #3

- function `reshape` is used to reshuffle a matrix

```
>> B = reshape(A, m, n),
```

- e.g.

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

```
>> B = reshape(A, [4 1]),
>> B = reshape(A, 1, 4),
```

$$\boxed{\begin{matrix} A_{11} \\ A_{21} \\ A_{12} \\ A_{22} \end{matrix}}$$

$$\boxed{\begin{matrix} A_{11} & A_{21} & A_{12} & A_{22} \end{matrix}}$$

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Matrix operations #4

- following functions are used to swap the order of
  - columns: `fliplr`

```
>> B = fliplr(A),
```

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} A_{13} & A_{12} & A_{11} \\ A_{23} & A_{22} & A_{21} \end{pmatrix}$$

  - rows: `fliud`

```
>> B = flipud(A),
```

$$\mathbf{A} = \begin{pmatrix} A_{21} & A_{22} & A_{23} \\ A_{11} & A_{12} & A_{13} \end{pmatrix}$$

  - row-wise or column-wise: `flipdim`

```
>> B = flipdim(A, 1),
>> B = flipdim(A, 2),
```

- the same result is obtained using indexing (see next slides)

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Matrix operations #5

- circular shift is also available
  - can be carried out in chosen dimension (row-wise/ column-wise)
  - can be carried out in both directions (back / forth)

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

```
>> B = circshift(A, -2),
```

$$\mathbf{A} = \begin{pmatrix} A_{31} & A_{32} & A_{33} \\ A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{pmatrix}$$

```
>> B = circshift(A, [-2 1]),
```

$$\mathbf{A} = \begin{pmatrix} A_{33} & A_{31} & A_{32} \\ A_{13} & A_{11} & A_{12} \\ A_{23} & A_{21} & A_{22} \end{pmatrix}$$

- Consider the difference between `flipdim` a `circshift`

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Matrix operations #1

450 s

- convert the matrix $\mathbf{A} = \begin{pmatrix} 1 & \pi \\ \mathrm{e} & -\mathrm{i} \end{pmatrix}$ to have the form of matrices $\mathbf{A}_1$ to $\mathbf{A}_4$
  - use `repmat`, `reshape`, `triu`, `tril` and `conj`

$$\mathbf{A}_1 = \begin{pmatrix} 1 & \pi & 1 & \pi & 1 & \pi \\ \mathrm{e} & -\mathrm{i} & \mathrm{e} & -\mathrm{i} & \mathrm{e} & -\mathrm{i} \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} 1 & \pi & \mathrm{e} & -\mathrm{i} \end{pmatrix}$$

$$\mathbf{A}_3 = \begin{pmatrix} 1 & \pi \\ \mathrm{e} & +\mathrm{i} \\ 1 & \pi \\ \mathrm{e} & +\mathrm{i} \\ 1 & \pi \\ \mathrm{e} & +\mathrm{i} \end{pmatrix}$$

$$\mathbf{A}_4 = \begin{pmatrix} 1 & \pi & 0 & 0 & 0 & 0 \\ \mathrm{e} & -\mathrm{i} & \mathrm{e} & 0 & 0 & 0 \\ 0 & \pi & 1 & \pi & 0 & 0 \\ 0 & 0 & \mathrm{e} & -\mathrm{i} & \mathrm{e} & 0 \\ 0 & 0 & 0 & \pi & 1 & \pi \\ 0 & 0 & 0 & 0 & \mathrm{e} & -\mathrm{i} \end{pmatrix}$$

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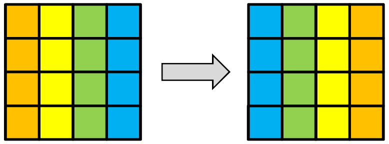
Matrix operations #2

450 s ↑

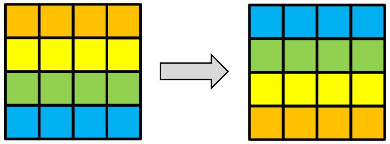
- create following matrix (use advanced techniques)

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 1 & 2 & 3 \\ 0 & 2 & 4 & 0 & 2 & 4 \\ 0 & 0 & 5 & 0 & 0 & 5 \end{pmatrix}$$

  - save the matrix in file named ‘`matrix.mat`‘
  - create matrix **B** by swapping columns in matrix **A**



  - create matrix **C** by swapping rows in matrix **B**



  - add matrices **B** and **C** in the file ‘`matrix.mat` ‘

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Matrix operations #3

150 s

- compare and interpret following commands:

```
>> x = (1:5)';                  % entering vector

>> X = repmat(x, [1 10]),   % 1. option
>> X = x(:, ones(10, 1)),   % 2. option
```

```
>> x = (1:5)'

x =

     1
     2
     3
     4
     5
```

- `repmat` is powerful, but not always the most time-efficient function

```
>> X = repmat(x,[1 10])

X =

     1     1     1     1     1     1     1     1     1     1
     2     2     2     2     2     2     2     2     2     2
     3     3     3     3     3     3     3     3     3     3
     4     4     4     4     4     4     4     4     4     4
     5     5     5     5     5     5     5     5     5     5

>> X = x(:,ones(10,1))

X =

     1     1     1     1     1     1     1     1     1     1
     2     2     2     2     2     2     2     2     2     2
     3     3     3     3     3     3     3     3     3     3
     4     4     4     4     4     4     4     4     4     4
     5     5     5     5     5     5     5     5     5     5
```

Vector and matrix operations

- remember that matrix multiplication is not commutative, i.e. $\mathbf{AB} \neq \mathbf{BA}$

- remember that vector × vector product results in

$$\mathbf{v}_{M,1}\mathbf{u}_{1,N} = \mathbf{w}_{M,N} \qquad\qquad \mathbf{v}_{1,M}\mathbf{u}_{M,1} = w_{1,1}$$

$$\begin{array}{c|cc} & u_{11} & u_{12} \\ \hline v_{11} & w_{11} & w_{12} \\ v_{21} & w_{21} & w_{22} \\ v_{31} & w_{31} & w_{32} \end{array} \qquad\qquad \begin{array}{ccc|c} & & & u_{11} \\ & & & u_{21} \\ & & & u_{31} \\ \hline v_{11} & v_{12} & v_{13} & w_{11} \end{array}$$

- … pay attention to the dimensions of matrices!

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Element-by-element vector product

- it is possible to multiply arrays of the same size in the element-by-element manner in Matlab
  - result of the operation is an array
  - size of all arrays are the same, e.g. in the case of 1×3 vectors:

$$\mathbf{a} = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}$$

```
>> a*b
```

$\boxed{a_1 \quad a_2 \quad a_3}, \boxed{b_1 \quad b_2 \quad b_3} \rightarrow$

```
Error using _*_
(Inner matrix dimensions must agree.)
```

```
>> a.*b
```

$\boxed{a_1 \quad a_2 \quad a_3}, \boxed{b_1 \quad b_2 \quad b_3} \rightarrow \boxed{a_1 b_1 \quad a_2 b_2 \quad a_3 b_3} = \left[a_i b_i\right]$

Element-by-element matrix product

- if element-by-element multiplication of two matrices of the same size is needed, use the '`.*`' operator
  - i.e. two cases of multiplication are distinguished

```
>> A*B
```

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \rightarrow \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

```
>> A.*B
```

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \rightarrow \begin{bmatrix} A_{11}B_{11} & A_{12}B_{12} \\ A_{21}B_{21} & A_{22}B_{22} \end{bmatrix}$$

  - It is so called *Hadamard product* / *element-wise product* / *Schur product*: $\mathbf{A} \circ \mathbf{B}$

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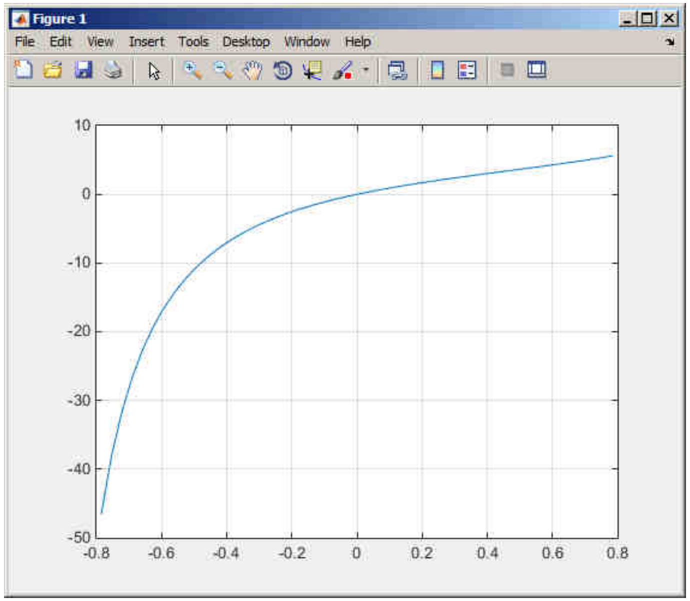
Element-wise operations #1

- element-wise operations can be applied to vectors as well in Matlab. Element-wise operations can be usefully combined with vector functions
- it is possible, quite often, to eliminate 1 or even 2 for-loops!!!
- these operations are exceptionally efficient
  $\longrightarrow$ allow the use of so called vectorization (see later)

- e.g.: $f(x) = \dfrac{10}{(x+1)}\tan(x),$

$$x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

```
>> x = -pi/4:pi/100:pi/4;
>> fx = 10./(1+x).*tan(x);
>> plot(x, fx);
>> grid on;
```



Element-wise operations #1

300 s

- evaluate functions $f_1(x) = \sin(x)$ of the variable $x \in [0, 2\pi]$

$$f_2(x) = \cos^2(x)$$

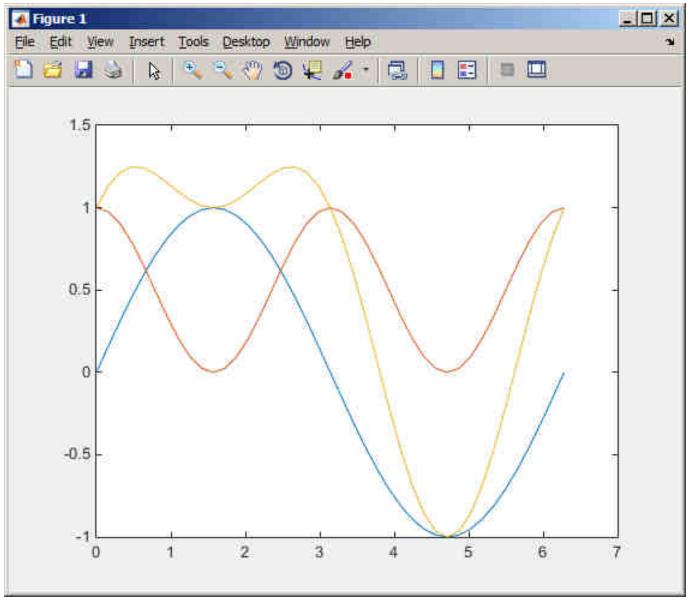
$$f_3(x) = f_1(x) + f_2(x)$$

  - evaluate the functions in evenly spaced points of the interval, the spacing is $\Delta x = \pi/20$

  - for verification:

```
>> plot(x,f1, x,f2, x,f3),
```

  - Matlab also enables symbolic solution (see later)



Element-wise operations #2

240 s

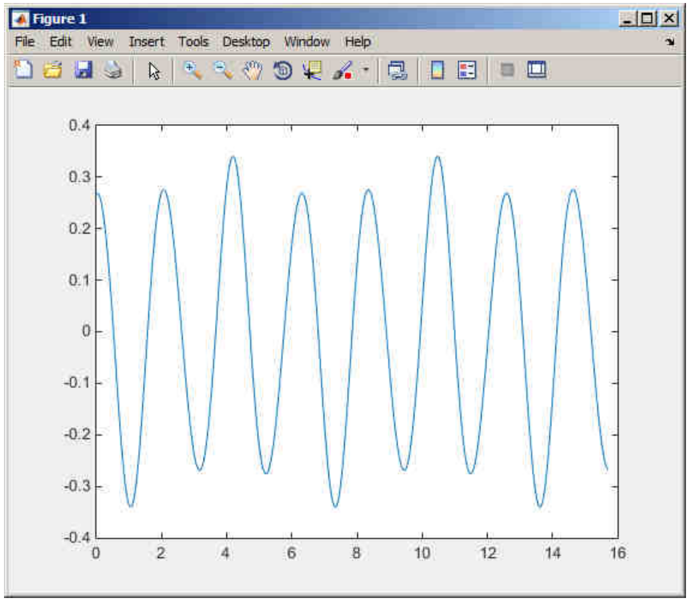
- depict graphically following functional dependence in the interval

$$x \in [0, 5\pi]$$

$$f_4(x) = \frac{-\cos(3x)}{\cos(x)\sin\left(x - \frac{\pi}{5}\right) - \pi}$$

  - plot the result using following function

```
>> plot(x, f4);
```



- explain the difference in the way of

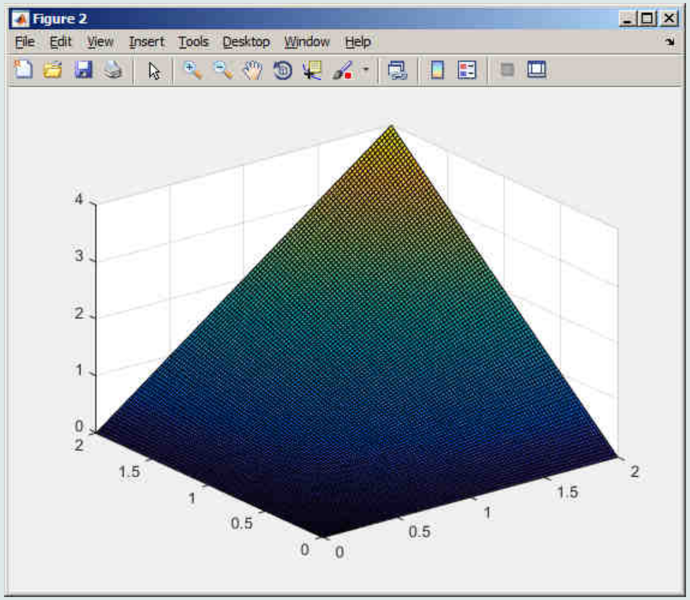
```
>> A*B,
```
```
>> A.*B,
```
```
>> A'.*B,
```

multiplication of matrices of the same size

Element-wise operations #3

360 s

- evaluate the function $f(x,y)=xy,\quad x,y\in[0,\ 2]$, use 101 evenly spaced points in both *x* and *y*
- the evaluation can be carried out either using vectors, matrix element-wise vectorization or using two for loops
  - plot the result using `surf(x,y,f)`
  - when ready, try also $f(x,y)=x^{0.5}y^2$ on the same interval



Matrix operations

- construct block diagonal matrix: `blkdiag`

$$\boxed{A_{11}} \qquad \boxed{\begin{matrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{matrix}}$$

```
>> A = 1; B = [2 3; -4 -5];
>> C = blkdiag(B, A);
```

$$\begin{matrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & A_{11} \end{matrix}$$

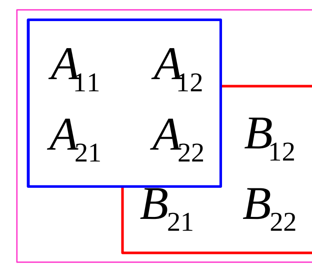
- arranging two matrices of the same size: `cat`

$$\boxed{\begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{matrix}} \qquad \boxed{\begin{matrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{matrix}}$$

```
C = cat(2, A, B)
C = cat(1, A, B)
C = cat(3, A, B)
```

$$\begin{matrix} A_{11} & A_{12} & B_{11} & B_{12} \\ A_{21} & A_{22} & B_{21} & B_{22} \end{matrix}$$

$$\begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ B_{11} & B_{12} \\ B_{21} & B_{22} \end{matrix}$$



Size of matrices and other structures

- it is often needed to know size of matrices and arrays in Matlab
- function `size` returns a vector giving the size of the matrix / array

```
>> A = randn(3,5);
>> d = size(A) % d = [3 5]
```

- function `length` returns largest dimension of an array
  - i.e. `length(A) = max(size(A))`

```
>> A = randn(3,5,8);
>> e = length(A) % e = 8
```

- function `ndims` returns number of dimensions of a matrix / array
  - i.e. `ndims(A) = length(size(A))`

```
>> m = ndims(A) % m = 3
```

- function `numel` returns number of elements of a matrix / array

```
>> n = numel(A) % n = 120
```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Size of matrices and other structures

250 s

- create an arbitrary 3D array
  - you can make use of the following commands :

```
>> A = rand(2+randi(10), 3+randi(5));
>> A(:,:,2) = flipud(fliplr(A)),
```

- and now:
  - find out the size of A
  - find out the number of elements of A
  - find out the number of elements of A in the 'longest' dimension
  - find out the number of dimensions of A

Data types in Matlab

- can be postponed for later …

```
>> whos
  Name                     Size            Bytes  Class      Attributes

  D                       50x1               400  double
  DD                       1x20              160  double
  DWx                     20x20             3200  double
  DWy                     20x20             3200  double
  Eps                      1x1                 8  double
  KA                      20x20             3200  double
  L                        1x1                 8  double
  Lcheck                  20x20             3200  double
  N                        1x1                 8  double
  Nth                      1x1                 8  double
  OK                       1x1                 1  logical
  PR                      20x20             3200  double
  Pr                       1x1                 8  double
  SOL                     20x20              400  logical
  Tcross                   1x1                 4  single
  lam                      1x1                 8  double
  omWA                    20x20             3200  double
  psi                      1x1                 8  double
  type_of_connection       1x6                12  char
```

```
>> class(type_of_connection)

ans =

char
```

Bonus: function `gallery`

- function enabling to create a vast set of matrices that can be used for Matlab code testing

- most of the matrices are special-purpose
  - function gallery offers significant coding time reduction for advanced Matlab users

- see `help gallery / doc gallery`

- try for instance:

```
>> gallery('pei', 5, 4)
>> gallery('leslie', 10)
>> gallery('clement', 8)
```


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Function why

- it is a must to try that one! :)
  - try `help why`
  - try to find out how many answers exist

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Discussed functions

`real, imag, cong, angle, complex`	complex numbers related functions	
`norm, cumsum`	norm (of a matrix / vector), cummulative sum	●
`hypot`	square root of sum of squares (real / complex numbers)	
`linspace, logspace`	vector generation - evenly spaced, linear / logarithmic scale	
`zeros, ones, eye, NaN, magic`	create matrix	
`rand, randn, randi`	matrix of random numbers with uniform or normal distribution, matrix of random integers	
`randperm`	vector containing a random permutation of numbers	
`true, false`	create matrix (logical)	
`pascal, hankel, gallery`	special purpose matrices	●
`blkdiag, cat`	block diagonal matrix, groups several matrices into one	
`diag, triu, tril,`	diagonal matrix, upper and lower triangular matrix	●
`fliplr, flipud, circshift`	element swapping, circular shift	
`repmat, reshape`	matrix operation (replication, reshaping)	●
`length, size, ndims, numel`	length of a vector, size of a matrix, number of dim. and elements	
`sparse, full`	sparse and full matrix operations	
`grid on, grid off`	Turns grid of a graph on / off	
`figure, surf`	opens new figure, 3D graph `surf`	●

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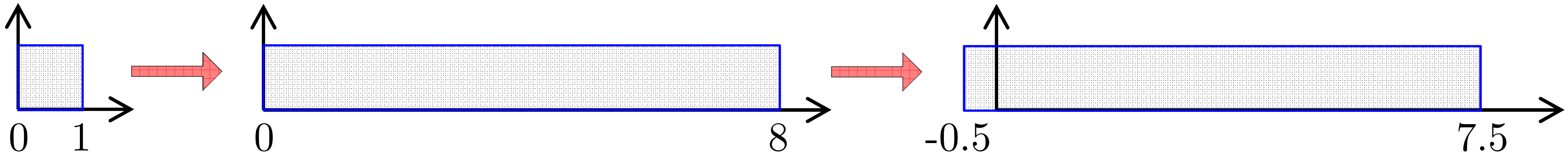
Exercise #1

360 s

- create matrix **M** of size `size(M) = [3 4 2]` containing random numbers coming from uniform distribution on the interval [-0.5,7.5]



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Exercise #2

200 s

- Consider the operation `a1^a2`, is this operation is applicable to following cases?

  - `a1` – matrix, `a2` – scalar
  - `a1` – matrix, `a2` – matrix
  - `a1` – matrix, `a2` – vector
  - `a1` – scalar, `a2` – scalar
  - `a1` – scalar, `a2` – matrix
  - `a1`, `a2` – matrix, `a1.^a2`

  you can always create the matrices `a1`, `a2` and make a test …

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Exercise #3

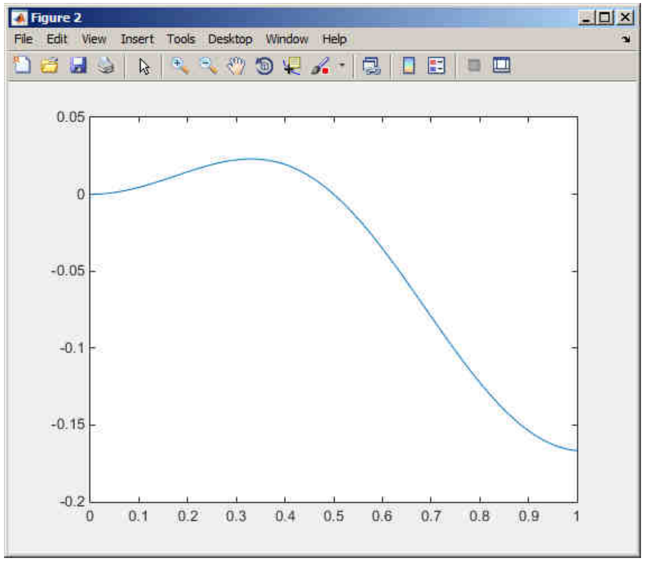
420 s

- make corrections to the following piece of code to get values of the function *f*(*x*) for 200 points on the interval [0, 1] :

$$f(x) = \frac{x^2 \cos(\pi x)}{(x^3+1)(x+2)}$$

```
>> % erroneous code
>> x = linspace(0, 1);
>> clear;
>> g = x^3+1; H = x+2;
>> y = cos xpi; z = x.^2;
>> f = y*z/gh
```

  - find out the value of the function for $x = 1$ by direct accessing the vector
  - what is the value of the function for $x = 2$?
  - to check, plot the graph of the function $f(x)$



Exercise #4

200 s

- think over how many ways there are to calculate the length of the hypotenuse when two legs of a triangle are given

  - make use of various Matlab operators and functions
  - consider also the case where the legs are complex numbers

Exercise #5

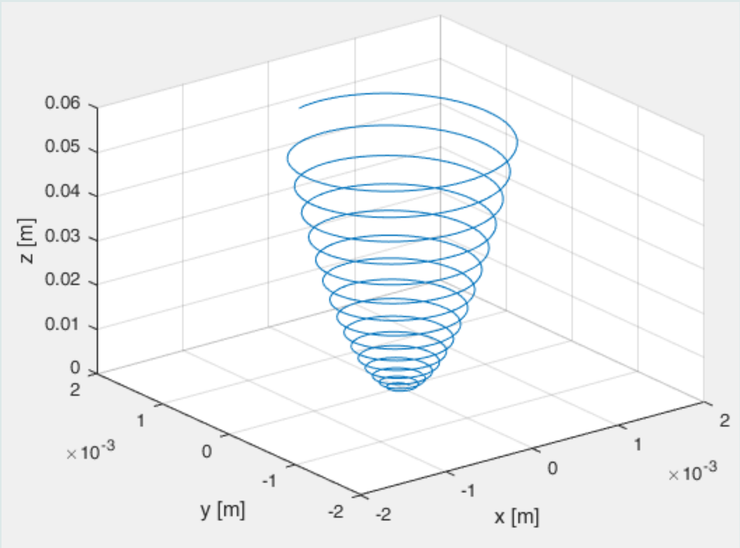
- A proton, carrying a charge of $Q = 1.602 \cdot 10^{-19}\ \mathrm{C}$ and of a mass of $m = 1.673 \cdot 10^{-31}\ \mathrm{kg}$ enters a homogeneous magnetic and electric field in the direction of the z axis in the way that the proton follows a helical path; the initial velocity of the proton is $v_0 = 1 \cdot 10^{7}\ \mathrm{m/s}$. The intensity of the magnetic field is $B = 0.1\mathrm{T}$, the intensity of the electric field is $E = 1 \cdot 10^{5}\ \mathrm{V/m}$

  - velocity of the proton along the z axis is $v = \dfrac{QE}{m} t + v_0$
  - where t is time, travelled distance along the z axis is $z = \dfrac{1}{2}\dfrac{QE}{m} t^2 + v_0 t$
  - radius of the helix is $r = \dfrac{vm}{BQ}$
  - frequency of orbiting the helix is $f = \dfrac{v}{2\pi r}$
  - the x and y coordinates of the proton are $x = r\cos(2\pi f\, t), \quad y = r\sin(2\pi f\, t)$

Exercise #6

500 s

- plot the path of the proton in space in the time interval from 0 ns to 1 ns in 1001 points using function `comet3(x,y,z)`




```
>> clear; close all; clc;
```

```
>> % put your code here
>> % ...
>> % ...
>> % ...
>> % ...
>> % ...
>> % ...
>> % ...
>> % ...
>> % ...
>> % ...
>> % ...
```

```
>> comet3(x, y, z)
```

Thank you!

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