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Partition trees

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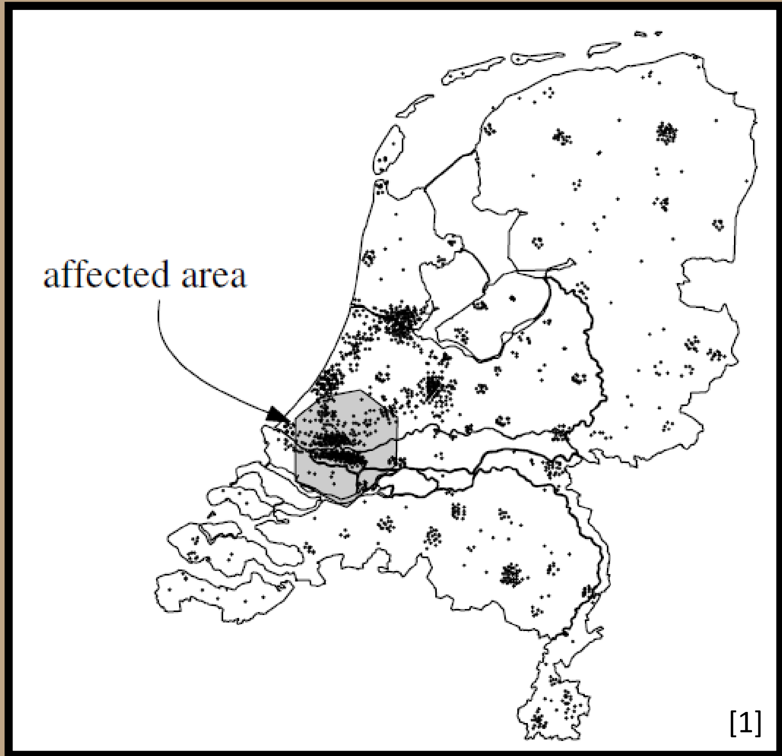
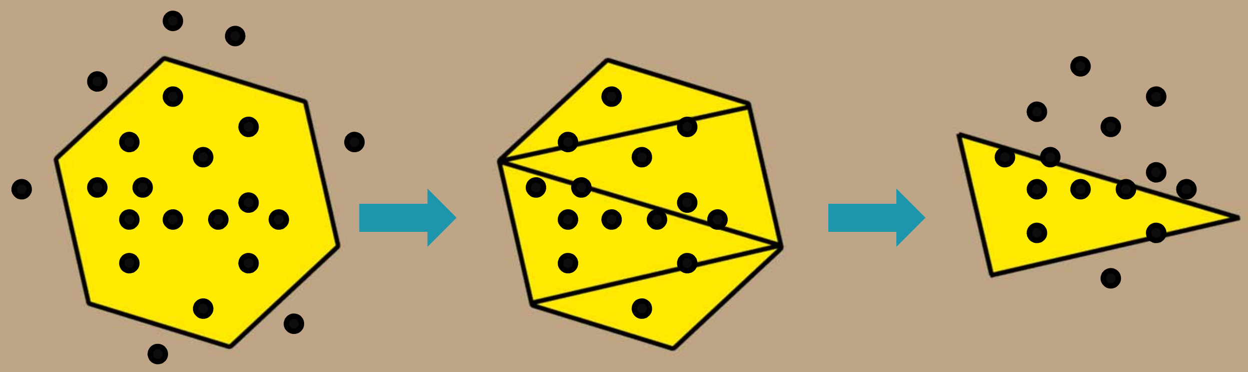
What is it about?

- set of points in the plane and we want to count the points lying inside a query region
- count number of cities in range



Query region



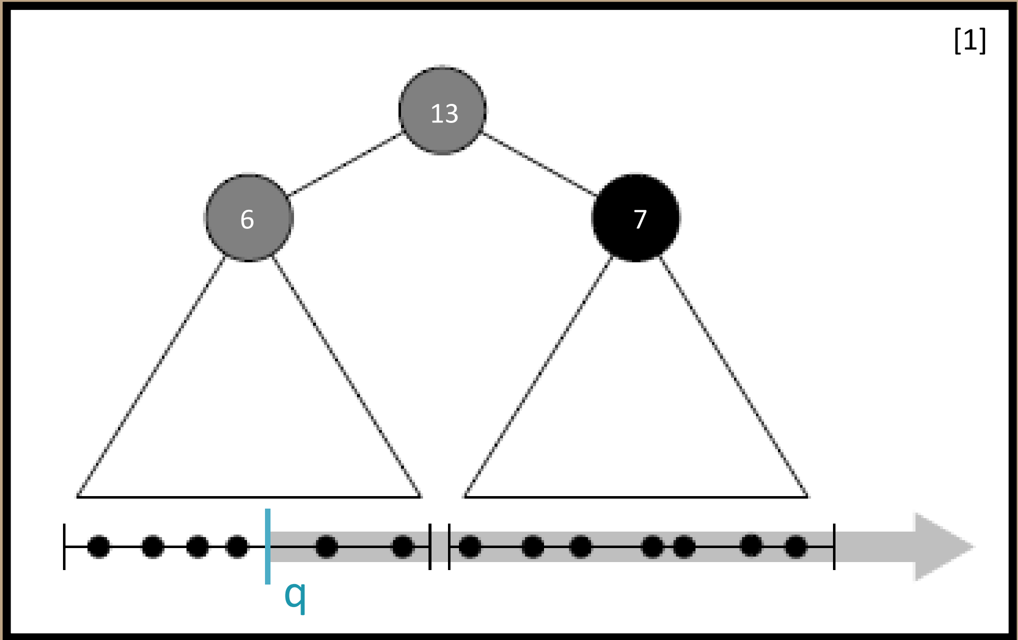
- preprocessing
- query region is a polygon (if no, we can approximate it)
- triangulate region
- query each of the resulting triangles
- return set of points in all triangles

… but first, let´s start with something easier

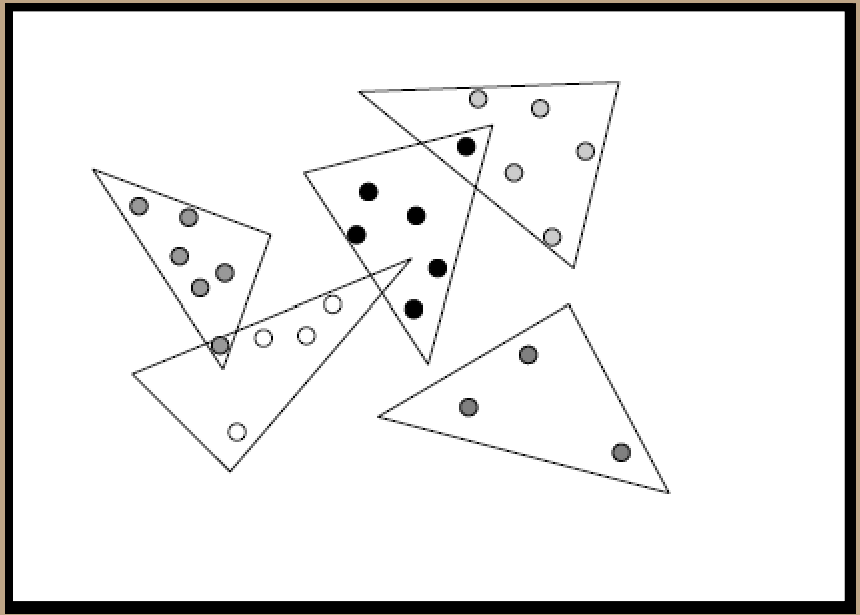
The 1D case

- how does it look like in 1D ?
- binary search tree
- **one region is completely contained in query line, one is disjoint**

On each level is visited only 0 – 1 subtree recursively



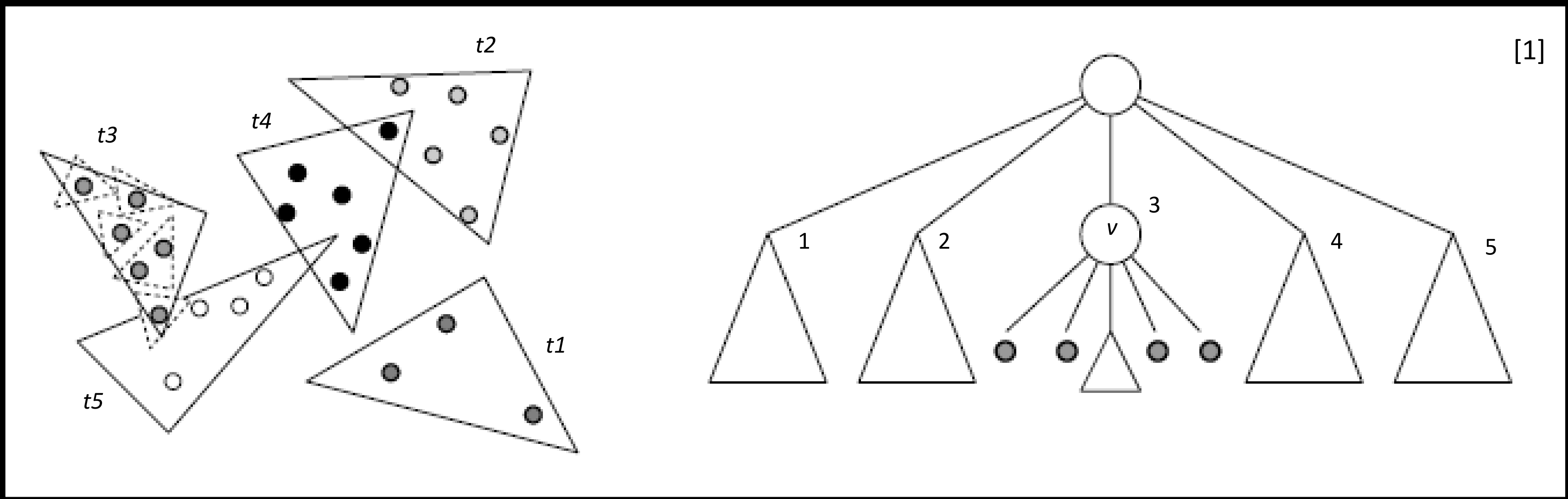
Can we use same approach in 2D?



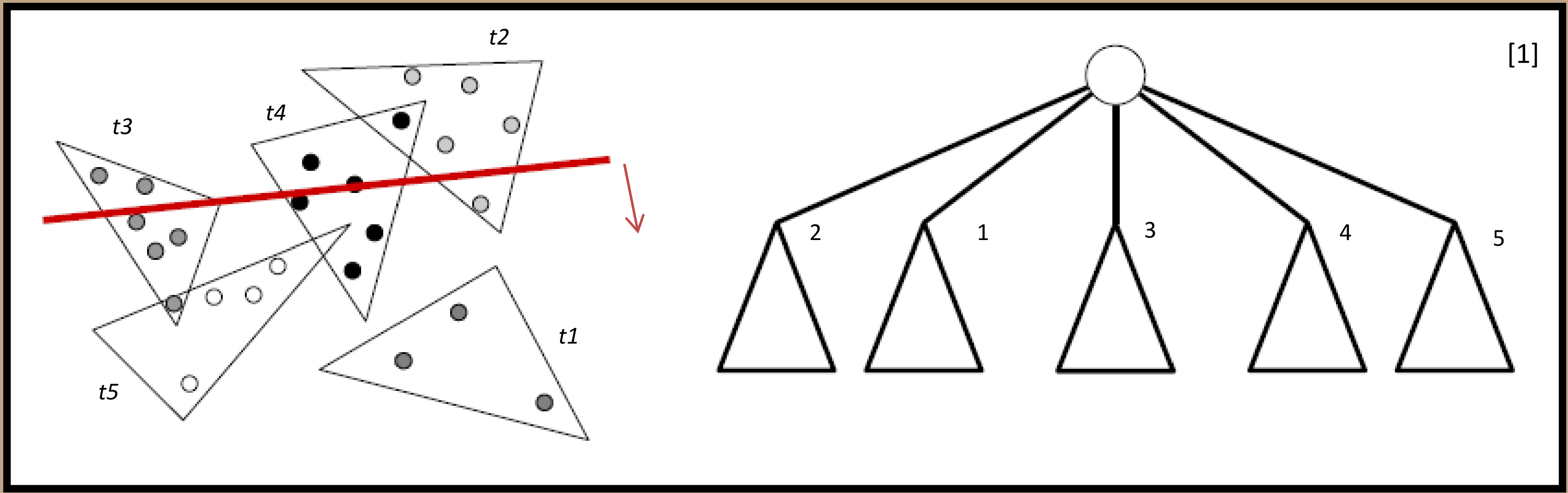
The 2D case – partition tree

- the structure is a tree *T* of branching degree *r*
- with each child *v* we store the triangle *t(v)*
- *crossing number* … maximum triangles crossed by any line
- *fine partition* … every group contains ≤ *2n/r* points

    the subsets are fairly equally distributed



2D example

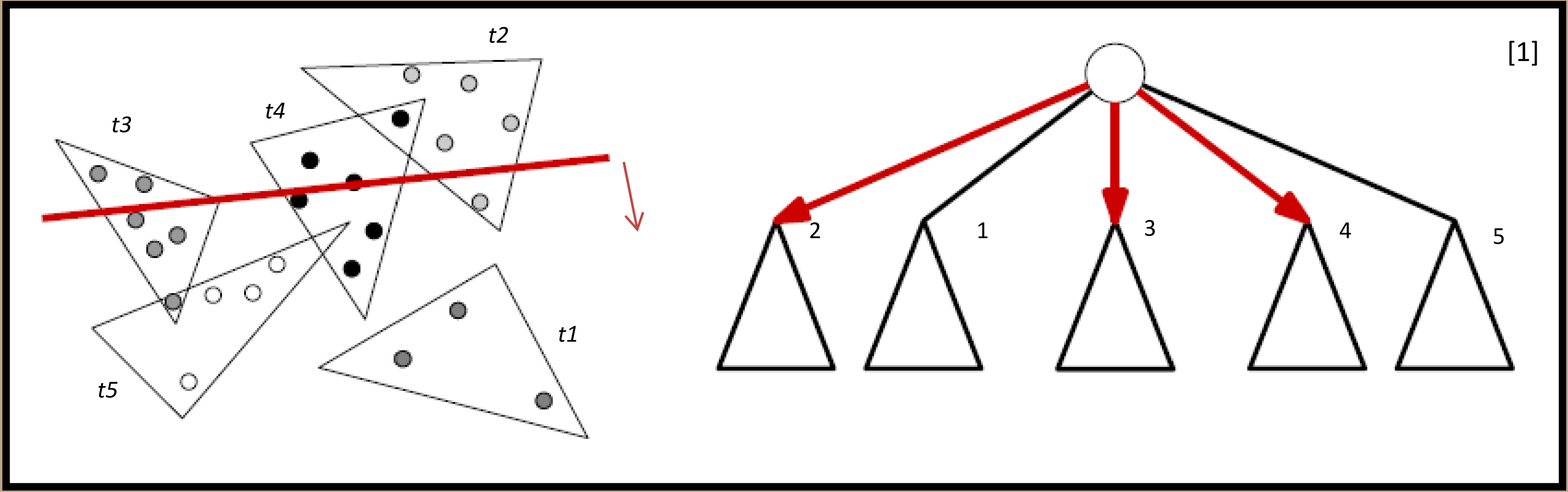


Pseudo-code

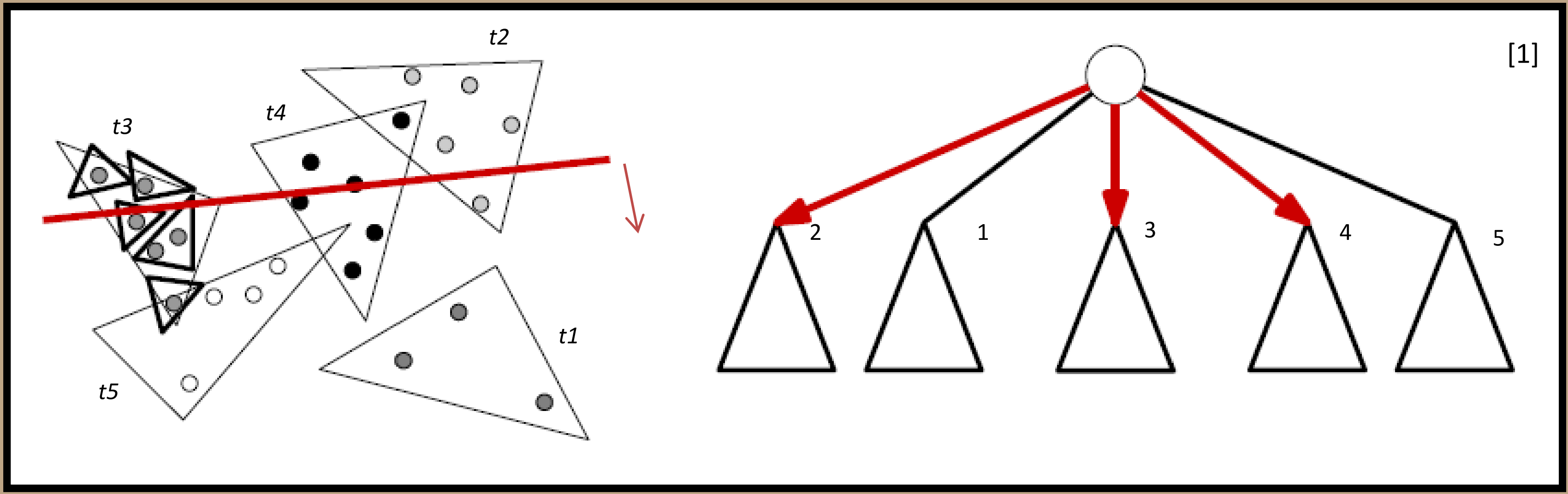
```
SelectInHalfplane(half-plane h, partition tree 𝒯)
N ← ∅    { set of selected nodes }

if 𝒯 = {μ} then
    if point stored at μ lies in h then
        N ← {μ}
else
    for each child ν of the root of 𝒯 do
        if t(ν) ⊂ h then
            N ← N ∪ {ν}
        else
            if t(ν) ∩ h ≠ ∅ then
                N ← N ∪ SelectInHalfplane(h, 𝒯_ν)

return N
```

[1]

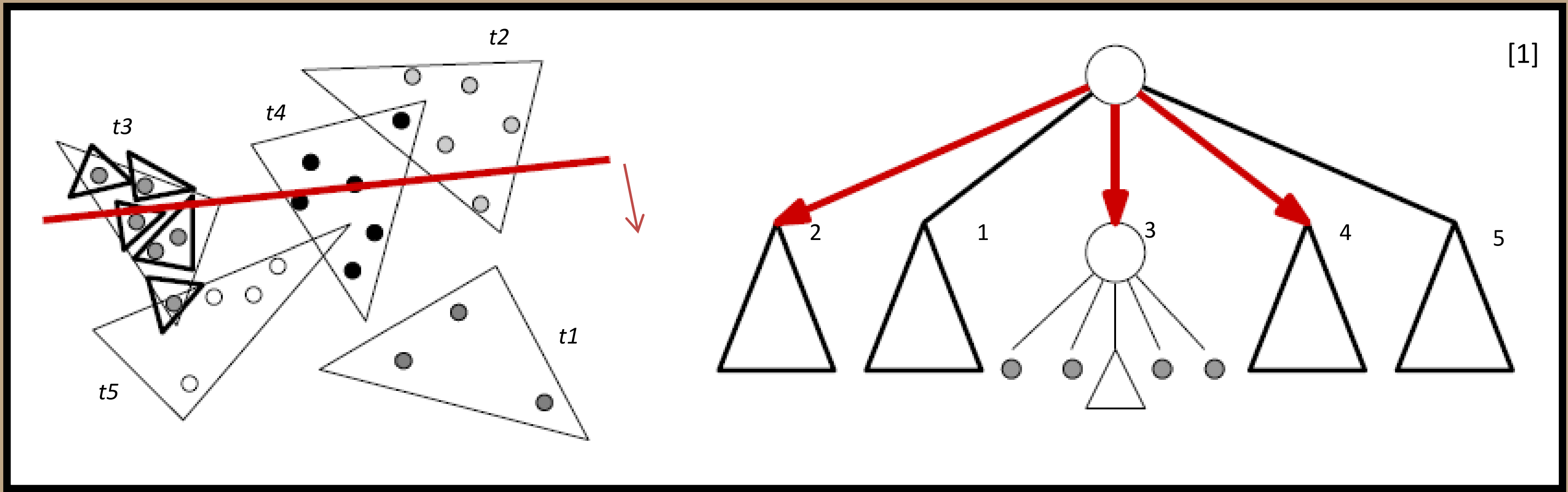
2D example



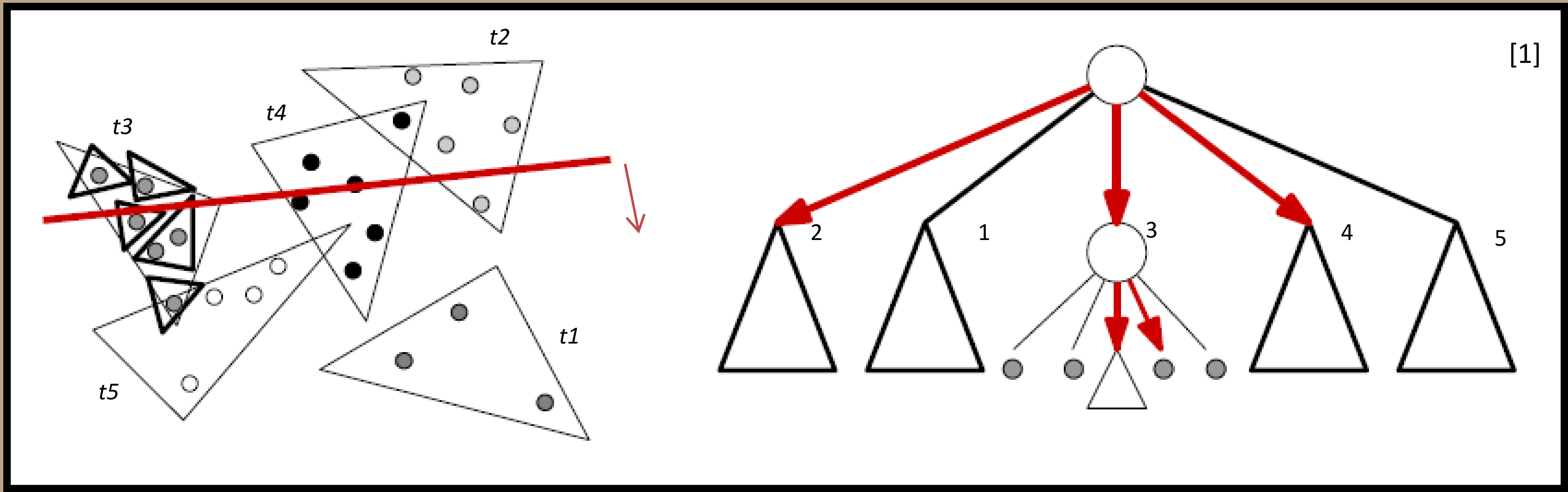
2D example



2D example

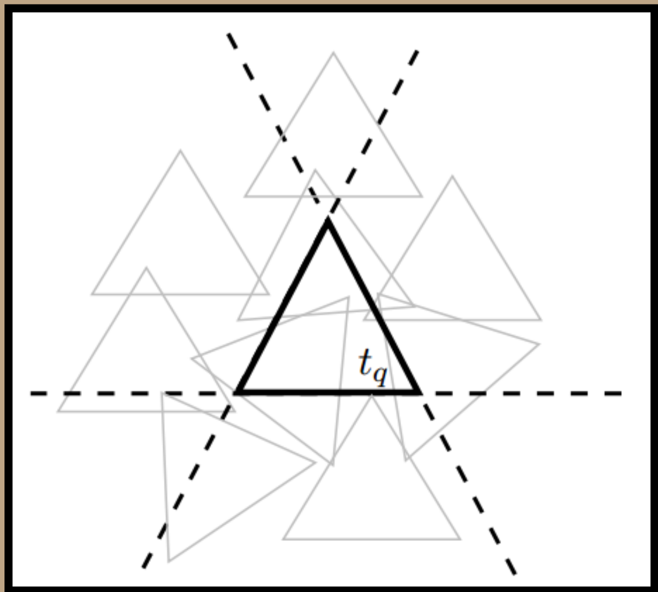


2D example



Which modifications do we need if we want to use triangles instead of half-planes?

None



Tree properties

Theorem:

For any set S of n points in the plane and any parameter r with $1 \leq r \leq n$,
ψS of size r and crossing number $O(\sqrt{r})$ exists.
Moreover, for any $\varepsilon > 0$ such ψS can be constructed in time $O(n^{1+\varepsilon})$. [2]

- *crossing number* … maximum triangles crossed by any line

Theorem:

Given a set S of n points in the plane, for any $\varepsilon > 0$, **a triangular range-counting query can be answered in $O(n^{1/2+\varepsilon})$ time** using a partition tree.

The tree can be **built** in **$O(n^{1+\varepsilon})$** time and uses **$O(n)$ space**. [2]

References

[1] M. de Berg, O. Cheong, M. van Kreveld, M. Overmars
Computional geometry, Algorithm and applications

[2] J. Matoušek, Efficient partition trees, Discrete & Computational Geometry

[3] Dr. André Schulz, Advanced Data Structures lecture
http://courses.csail.mit.edu/6.851/spring10/scribe/lec06.pdf

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