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# A4M33MAS - Multiagent Systems

## Belief-Desire-Intention (BDI) Architecture & Social Commitments

Michal Pechoucek & Michal Jakob

Department of Computer Science  
Czech Technical University in Prague



**O I** OTEVŘENÁ  
INFORMATIKA

In parts based on Michal Pechoucek: Multi-Agent Systems, Lecture course at State University of New York, University at Binghamton

# Towards Architectures for IA

01

- Reactive Architectures
- Deliberative Architectures

# Models of Practical Reasoning: BDI

*process of figuring out what to do – practical reasoning is a matter of weighing conflicting considerations for and against competing options, where the relevant considerations are provided by what the agent desires/values/cares about and what the agent believes (Bratman)*

- computational model of human decision process oriented towards an action, based on models of existing mental models of the agents
- human practical reasoning consists of two activities:
  - **deliberation**: deciding what state of affairs we want to achieve and
  - **means-ends reasoning** (planning): deciding how to achieve these states
- the outputs of deliberation process are **intentions**

# BDI Architecture

- **BELIEFS**
    - collection of information that the agents has about its the status of the environment, peer agents, self
  - **DESIRES**
    - set of long term goals the agent wants to achieve
  - **INTENTIONS**
    - agents immediate commitment to executing an action, either high-level or low level (depends on agents planning horizon)
- 
- BDI architecture connects: (i) reactive (ii) planning & (iii) logical representation. BDI architecture does not count on theorem proving

if  $\varphi \in \mathcal{L}_{agent}$  then  $\varphi, (\text{Bel } A \varphi), (\text{Des } A \varphi), (\text{Int } A \varphi) \in \mathcal{L}_{bdi}$

# BDI Inference Algorithm

- Basic algorithm:
  1. initial beliefs  $\rightarrow$  Bel
  2. while true do
  3. Read(get\_next\_percept)  $\rightarrow$  in
  4. Belief-revision(Bel, in)  $\rightarrow$  Bel
  5. Deliberate(Bel, Des)  $\rightarrow$  Int
  6. Plan(Bel, Int)  $\rightarrow$   $\pi$
  7. Execute( $\pi$ )
  8. end while

# BDI Modal Properties

01

- **BELIEFS**
  - KD45 system, modal logic where the B relation is serial, transitive and euclidean: satisfies K axioms, positive introspection axiom (4 axiom), negative introspection axiom (5 axiom), beliefs consistency axiom (D axiom).

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- KD system, modal logic requiring desired goals not to contradict (D axiom).

$$(\text{Des } A \varphi) \rightarrow \neg(\text{Des } \neg \varphi)$$

- **INTENTIONS**

- KD system, modal logic requiring intentions not to contradict (D axiom).

$$(\text{Int } A \varphi) \rightarrow \neg(\text{Int } A \neg \varphi)$$

# Properties of Intentions

- **Intention persistency:**

- agents track the success of their intentions, and are inclined to try again if their attempts fail

$$(\text{Int } A \varphi) \curvearrowright \varphi$$

- **Intention satisfiability:**

- agents believe their intentions are possible; that is, they believe there is at least some way that the intentions could be brought about.

$$(\text{Int } A \varphi) \Rightarrow \text{EF}\varphi$$

# Properties of Intentions

- **Intention-belief inconsistency:**

- agents do not believe they will not bring about their intentions; it would be irrational of agents to adopt an intention if believed was not possible

$$(Int\ A\ \varphi) \wedge (Bel\ A\ \neg EF\varphi)$$

- **Intention-belief incompleteness:**

- agent do not believe that their intention is possible to be achieved, may be understood as rational behavior

$$(Int\ A\ \varphi) \wedge (\neg Bel\ AEF\varphi)$$

- agents admit that their intentions may not be implemented.

$$(Int\ A\ \varphi) \wedge (Bel\ A\ EF\neg\varphi)$$

# Properties of Intentions

- **Intention side-effects:**

- Agents need not intend all the expected side effects of their intentions. Intentions are not closed under implication.

$$(\text{Bel } A \psi \Rightarrow \varphi) \wedge (\text{Int } A \psi) \wedge \neg(\text{Int } A \varphi)$$

\* *is thus classified as fully rational behavior*

- Example: I may believe that going to the dentist involves pain, and I may also intend to go to the dentist - but this does not imply that I intend to suffer pain!

# Rationality of Inevitables & Options

## 1. inevitables:

$$(\text{Int } A \text{ AG}\varphi) \Rightarrow (\text{Des } A \text{ AG}\varphi)$$

$$(\text{Des } A \text{ AG}\varphi) \Rightarrow (\text{Int } A \text{ AG}\varphi)$$

$$(\text{Bel } A \text{ AG}\varphi) \Rightarrow (\text{Des } A \text{ AG}\varphi)$$

$$(\text{Des } A \text{ AG}\varphi) \Rightarrow (\text{Bel } A \text{ AG}\varphi)$$

$$(\text{Int } A \text{ AG}\varphi) \Rightarrow (\text{Bel } A \text{ AG}\varphi)$$

$$(\text{Bel } A \text{ AG}\varphi) \Rightarrow (\text{Int } A \text{ AG}\varphi)$$

## 2. options:

$$(\text{Int } A \text{ EF}\varphi) \Rightarrow (\text{Des } A \text{ EF}\varphi)$$

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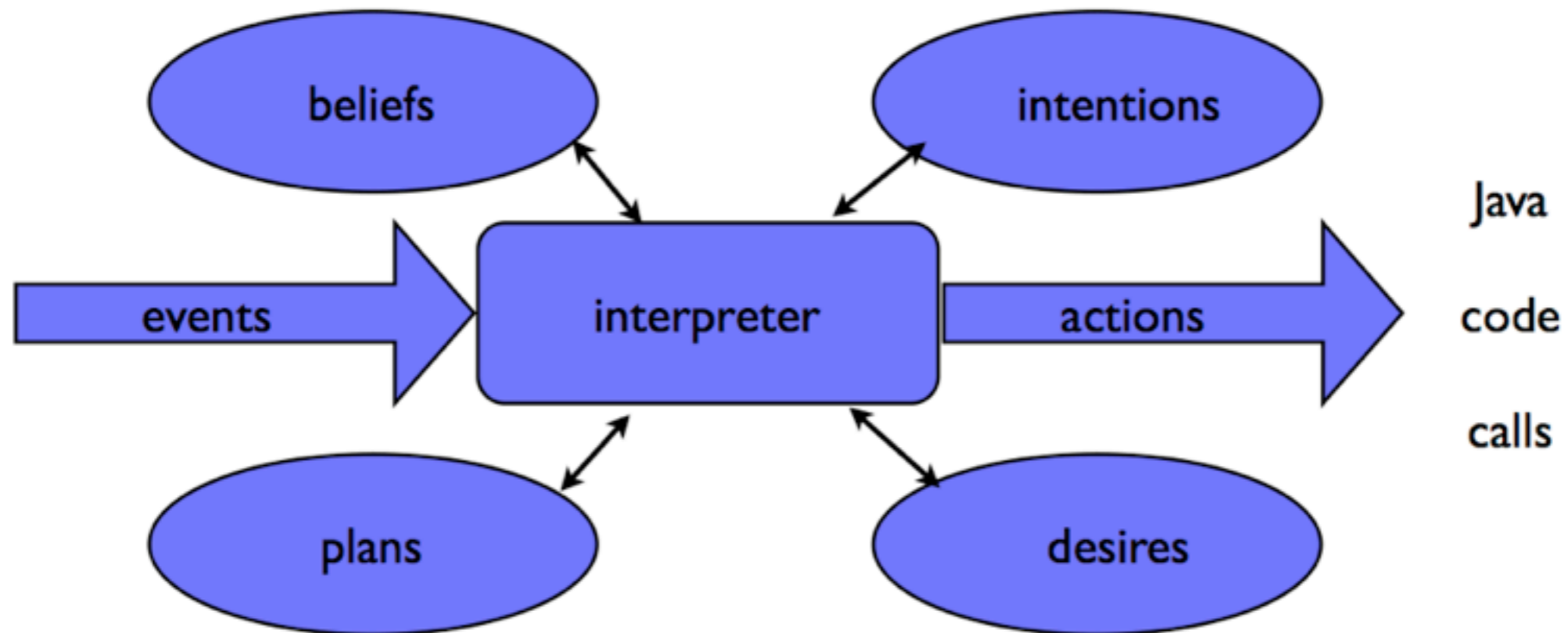
# Example: Model Checking AgentSpeak 01

- **AgentSpeak(L)** is a BDI programming language introduced by Rao.
- A simple but powerful programming language for building rational agents. Based on Prolog.
- **Jason**:
  - implementation of AgentSpeak in Java
  - A development environment for AgentSpeak systems



# Example: Model Checking AgentSpeak 01

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# AgentSpeak Control Loop

01

1. agent receives events, which are either
  - external (from the environment, from perceptual data)
  - internally generated
2. tries to handle events by looking for plans that match the event and lead to the goal → desires (options)
3. chooses one plan from its desires to execute: becomes committed to it → intention
4. as it executes a plan may generate new events that require handling

# AgentSpeak: BDI primitives

- Beliefs: Symbolically represented (ground atoms or FOL formulas)

$$ag = \langle bs, ps \rangle: \text{BEL}_{\langle ag, C \rangle}(\varphi) \equiv \varphi \in bs.$$

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agent    beliefs    plans

(Bel A  $\varphi$ )

context

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- Manipulating beliefs:

+B	adding new belief
-B	dropping belief

# AgentSpeak: BDI primitives

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$$ag = \langle bs, ps \rangle: \text{BEL}_{\langle ag, C \rangle}(\varphi) \equiv \varphi \in bs.$$

- Manipulating beliefs:

+B            adding new belief

-B            dropping belief

- Manipulating goals/intentions:

+!D           adding new desire

-!D           dropping desire

- Plans:

**triggerCondition :**

**context** <-

**body.**

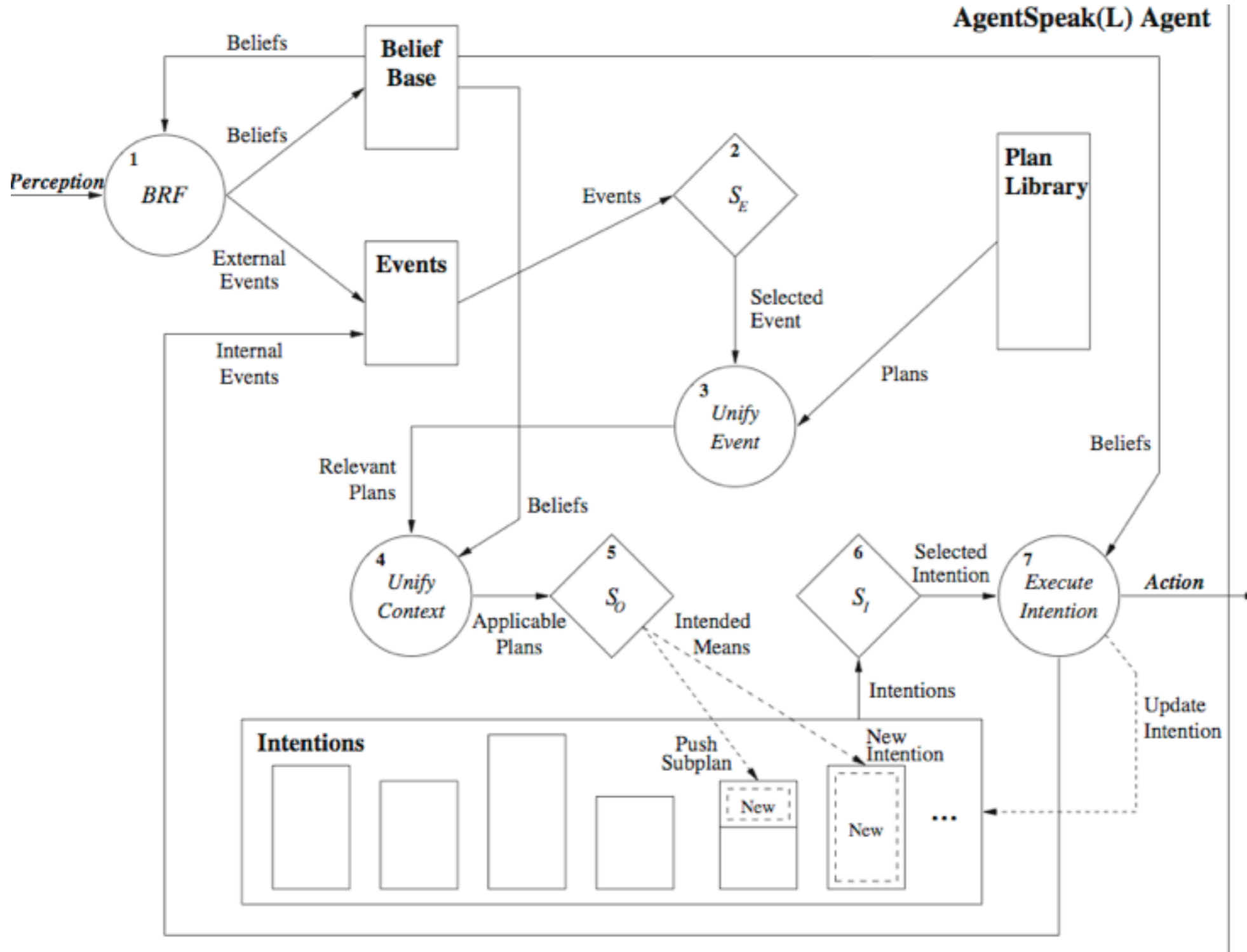
# AgentSpeak: Example

```
+green_patch(Rock)
  : not battery_charge(low)
  <- ?location(Rock,Coordinates);
     !traverse(Coordinates);
     !examine(Rock).
```

```
+!traverse(Coords)
  : safe_path(Coords)
  <- move_towards(Coords).
```

```
+!traverse(Coords) :
  : not safe_path(Coords)
  <- ...
```

# AgentSpeak Reasoning Lifecycle





# AgentSpeak: Example

01

# AgentSpeak: Example

## Agent I

```
+auction(N) : true  
  <- place_bid(N,6).
```

# AgentSpeak: Example

## Agent 1

```
+auction(N) : true  
  <- place_bid(N,6).
```

## Agent 2

```
myself(ag2).  
bid(ag2,4).  
ally(ag3).
```

```
+auction(N) : myself(I) & ally(A) & not(alliance(A,I))  
  <- ?bid(I,B); place_bid(N,B).
```

```
+auction(N) : myself(I) & ally(A) & (alliance(A,I))  
  <- place_bid(N,0).
```

```
+alliance(A,I) : myself(I) & ally(A)  
  <- ?bid(I,B);  
    .send(A,tell,bid(I,B));  
    .send(A,tell,alliance(A,I)).
```

# AgentSpeak: Example

01

## Agent3

```
myself(ag3).
bid(ag3,3).
ally(ag2).
threshold(3).

+auction(N) : threshold(T) & .gte(T,N)
             <- !bid_normally(N).

+auction(N) : myself(I) & winner(I)
             & ally(A) & not(alliance(I,A))
             <- !bid_normally(N).

+auction(N) : myself(I) & not(winner(I))
             & ally(A) & not(alliance(I,A))
             <- !alliance(I,A);
             !bid_normally(N).

+auction(N) : alliance(I,A)
             <- ?bid(I,B); ?bid(A,C);
             .plus(B,C,D); place_bid(N,D).

+!bid_normally(N) : true
                 <- ?bid(I,B); place_bid(N,B).

+!alliance(I,A) : not(alliance(I,A))
                <- .send(A,tell,alliance(I,A)).
```

# AgentSpeak: Example

01

# AgentSpeak: Example

□( ¬(Bel ag3 winner(ag3)) ∧  
(Des ag3 alliance(ag3, ag2)) ⇒  
◇(Int ag3 alliance(ag3, ag2)) )

# AgentSpeak: Example

- (  $\neg(\text{Bel ag3 winner(ag3)}) \wedge$   
 (Des ag3 alliance(ag3, ag2))  $\Rightarrow$   
 ◇(Int ag3 alliance(ag3, ag2)) )
  
- ◇( (Bel ag2 alliance(ag3, ag2))  $\wedge$   
 (Bel ag3 alliance(ag3, ag2)) )

# AgentSpeak: Example

- (  $\neg$ (Bel ag3 winner(ag3)) $\wedge$   
 (Des ag3 alliance(ag3, ag2))  $\Rightarrow$   
 $\diamond$ (Int ag3 alliance(ag3, ag2)) )
  
- $\diamond$ ( (Bel ag2 alliance(ag3, ag2)) $\wedge$   
 (Bel ag3 alliance(ag3, ag2)) )
  
- ( (Bel ag2 alliance(ag3, ag2)) $\wedge$   
 (Bel ag3 alliance(ag3, ag2))  $\Rightarrow$   
 $\diamond$ □winner(ag3) )



# Social Commitments

# Agents Individual/Social Commitments

- Commitments: knowledge structure, declarative programming concept based on intentions (intentions are special kinds of comms).
  - specify relationships among different intentional states of the agents
  - specify social relations among agents, based on their comms to joint actions

*The commitment is an agent's state of 'the mind' where it commits to adopting the single specific intention or a longer term desire.*

- We distinguish between:
  - specific, commonly used commitments
  - individual commitments
  - general commitments
  - social commitments

# Individual Commitments

- $A$  can get committed to its intention  $\varphi$  in several different ways:  $\square\square$ 
  - **blind commitment**: also referred to as fanatical commitment, the agent is intending the intention until it believes that it has been achieved (persistent intention)

$$(\text{Commit } A \varphi) \equiv \text{AG}((\text{Int } A \varphi) \rightsquigarrow (\text{Bel } A \varphi))$$

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- **single-minded commitment**: besides above it intends the intention until it believes that it is no longer possible to achieve the goal

$$(\text{Commit } A \varphi) \equiv \text{AG}((\text{Int } A \varphi) \rightsquigarrow ((\text{Bel } A \varphi) \vee (\text{Bel } A \neg \text{EF}\varphi)))$$

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- **open-minded commitment**: besides above it intends the intention as long as it is sure that the intention is achievable

$$(\text{Commit } A \varphi) \equiv \text{AG}((\text{Int } A \varphi) \curvearrowright ((\text{Bel } A \varphi) \vee \neg(\text{Bel } A \text{EF}\varphi)))$$

# General Commitments

- Commitment is defined as  $(\text{Commit } A \varphi \psi \lambda)$ , where
- Convention is defined as  $\lambda = \{\langle \rho_k, \gamma_k \rangle\}_{k \in \{1, \dots, l\}}$ 
  - provided  $\curvearrowright$  stands for *until*,  $A$  stands for *always in the future*,  $\text{Int}$  is *agent's intention* and  $\text{Bel}$  is *agent's belief* then for  $\lambda = \langle \rho, \gamma \rangle$  the commitment has the form:

$$(\text{Commit } A \varphi \psi \lambda) \equiv \psi \wedge A((\text{Int } A \varphi) \wedge \text{decommitment\_rule} \quad ) \curvearrowright \gamma)$$

$$(\text{Commit } A \varphi \psi \lambda) \equiv \psi \wedge A((\text{Int } A \varphi) \wedge ((\text{Bel } A \rho) \Rightarrow A(\text{Int } A \gamma)) \curvearrowright \gamma) \curvearrowright \gamma)$$

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$$\begin{aligned}
 (\text{Commit } A \varphi \psi \lambda) &\equiv \\
 &\psi \wedge A((\text{Int } A \varphi) \wedge \\
 &\quad ((\text{Bel } A \rho_1) \Rightarrow A(\text{Int } A \gamma_1)) \curvearrowright \gamma_1) \\
 &\quad \dots \\
 &\quad ((\text{Bel } A \rho_k) \Rightarrow A(\text{Int } A \gamma_k)) \curvearrowright \gamma_k) \\
 &\quad \curvearrowright \bigvee_i \gamma_i)
 \end{aligned}$$

# Joint (Social) Commitment

01

- Form of a commitment that represents how a group of agents is committed to a joint action (goal, intention, ...)



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$$(\text{Commit } A \varphi \psi \lambda) \equiv \psi \wedge A((\text{Int } A \varphi) \wedge ((\text{Bel } A \rho) \Rightarrow A(\text{Int } A \gamma)) \curvearrowright \gamma) \curvearrowright \gamma)$$

$$(\text{J-Commit } \Theta \varphi \psi \lambda) \equiv$$

$$\begin{aligned} & \forall A : (A \in \theta) \Rightarrow \\ & \psi \wedge A((\text{Int } A \varphi) \wedge \\ & \quad ((\text{Bel } A \rho) \Rightarrow A(\text{Int } A \gamma) \curvearrowright \gamma) \\ & \quad \curvearrowright \gamma) \end{aligned}$$

# Joint (Social) Commitment

- Form of a commitment that represents how a group of agents is committed to a joint action (goal, intention, ...)
  - for a convention in the form of  $\lambda = \{\langle \rho_k, \gamma_k \rangle\}_{k \in \{1, \dots, l\}}$

$$(J\text{-Commit } \Theta \varphi \psi \lambda) \equiv \forall A : (A \in \theta) \Rightarrow \psi \wedge A((\chi_1 \wedge \chi_2) \curvearrowright \chi_3)$$

where

$$\chi_1 = (\text{Int } A \varphi)$$

$$\chi_2 = ((\text{Bel } A \rho_1) \Rightarrow A((\text{Int } A \gamma_1) \curvearrowright \gamma_1)) \wedge ((\text{Bel } A \rho_2) \Rightarrow A((\text{Int } A \gamma_2) \curvearrowright \gamma_2)) \wedge \dots \wedge ((\text{Bel } A \rho_n) \Rightarrow A((\text{Int } A \gamma_n) \curvearrowright \gamma_n))$$

$$\chi_3 = \gamma_1 \vee \gamma_2 \vee \dots \vee \gamma_n$$

# Blind Social Commitment

- each agent is trying to accomplish the commitment until achieved

$$\lambda_{blind} = \{\langle (\text{Bel } A \varphi), (\text{M-Bel } \Theta \varphi) \rangle\}$$

$$\psi_{blind} = \neg(\text{Bel } A \varphi)$$

$$\begin{aligned}
 (\text{J-Commit } \Theta \varphi \psi \lambda) &\equiv \forall A : (A \in \Theta) \Rightarrow \\
 &(\neg(\text{Bel } A \varphi) \wedge (A((\text{Int } A \varphi) \wedge \\
 &\quad ((\text{Bel } A \varphi) \Rightarrow A((\text{Int } A (\text{M-Bel } \Theta \varphi)) \\
 &\quad \curvearrowright (\text{M-Bel } \Theta \varphi)))) \\
 &\curvearrowright (\text{M-Bel } \Theta \varphi)).
 \end{aligned}$$

# Minimal Social Commitment

- minimal social commitment, also related to as **joint persistent goal**:
  - initially agents do not believe that goal is true but it is possible
  - every agent has the goal until *termination condition* is true
  - until termination: if agent believes that the goal is either *true or impossible* than it will want the goal that it becomes a mutually believed, but keep committed
  - the termination condition is that it is *mutually believed* either goal is true or impossible to be true.

$$\psi_{soc} = \neg(\text{Bel } A \varphi) \wedge (\text{Bel } A \text{ EF}\varphi)$$

$$\lambda_{soc} = \left\{ \begin{array}{l} \langle (\text{Bel } A \varphi), (\text{M-Bel } \Theta \varphi) \rangle, \\ \langle (\text{Bel } A \text{ AG}\neg\varphi), (\text{M-Bel } \Theta \text{ AG}\neg\varphi) \rangle \end{array} \right\}$$

# Minimal Social Commitment

01

$(\text{J-Commit } \Theta \varphi \psi_{soc} \lambda_{soc}) \equiv$

$\forall A, A \in \Theta : [\neg(\text{Bel } A \varphi) \wedge (\text{Bel } A \text{ EF}\varphi)] \wedge$

$A \left[ \begin{array}{l} (\text{Int } A \varphi) \wedge \\ ((\text{Bel } A \varphi) \Rightarrow A((\text{Int } A(\text{M-Bel } \Theta \varphi))) \curvearrowright \chi) \wedge \\ ((\text{Bel } A \text{ AG}\neg\varphi) \Rightarrow A((\text{Int } A(\text{M-Bel } \Theta \text{ AG}\neg\varphi))) \curvearrowright \chi) \end{array} \right] \curvearrowright \chi$

where  $\chi \equiv ((\text{M-Bel } \Theta \varphi) \vee (\text{M-Bel } \Theta \text{ AG}\neg\varphi))$

# Mutual Belief ?

## Definition 1:

$$(M\text{-Bel } \Theta \varphi) \equiv \forall A, A \in \Theta (\text{Bel } A (M\text{-Bel } \Theta \varphi))$$

## Definition 2:

$$(\text{E-Bel}^0 \Theta \varphi) \equiv \forall A, A \in \Theta (\text{Bel } A \varphi)$$

$$(\text{E-Bel}^k \Theta \varphi) \equiv \forall A, A \in \Theta (\text{E-Bel}^{k-1} \Theta \varphi)$$

$$(M\text{-Bel } \Theta \varphi) \equiv \forall m \in \mathbb{N} (\text{E-Bel}^m \Theta \varphi)$$



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