



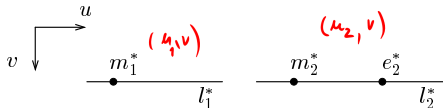
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► Rectification Homographies

Cameras ($\mathbf{P}_1, \mathbf{P}_2$) are rectified by a homography pair ($\mathbf{H}_1, \mathbf{H}_2$):

$$\mathbf{P}_i^* = \mathbf{H}_i \mathbf{P}_i = \mathbf{H}_i \mathbf{K}_i \mathbf{R}_i [\mathbf{I} \quad -\mathbf{C}_i], \quad i = 1, 2$$

rectified entities: \mathbf{F}^* , l_2^* , l_1^* , etc:



corresponding epipolar lines must be:

1. parallel to image rows \Rightarrow epipoles become $e_1^* = e_2^* = (1, 0, 0)$
2. equivalent $l_2^* = l_1^* \Rightarrow l_2^* \simeq l_1^* \simeq \underline{e}_1^* \times \underline{m}_1 = [\underline{e}_1^*]_{\times} \underline{m}_1 = \mathbf{F}^* \underline{m}_1$

both conditions together give the rectified fundamental matrix

$$\mathbf{F}^* \simeq \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$u_1 = u_2$$

A two-step rectification procedure

1. Find some pair of primitive rectification homographies $\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2$
2. Upgrade them to a pair of optimal rectification homographies from the class preserving \mathbf{F}^* .

► Geometric Interpretation of Linear Rectification

What pair of physical cameras is compatible with \mathbf{F}^* ?

- we know that $\mathbf{F} = (\mathbf{Q}_1 \mathbf{Q}_2^{-1})^\top [\mathbf{e}_1]_\times$
- we choose $\mathbf{Q}_1^* = \mathbf{K}_1^*$, $\mathbf{Q}_2^* = \mathbf{K}_2^* \mathbf{R}^*$; then

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$$(\mathbf{Q}_1^* \mathbf{Q}_2^{*-1})^\top [\mathbf{e}_1^*]_\times = (\mathbf{K}_1^* \mathbf{R}^{*\top} \mathbf{K}_2^{*-1})^\top \mathbf{F}^*$$

- we look for \mathbf{R}^* , \mathbf{K}_1^* , \mathbf{K}_2^* compatible with

$$(\mathbf{K}_1^* \mathbf{R}^{*\top} \mathbf{K}_2^{*-1})^\top \mathbf{F}^* = \lambda \mathbf{F}^*, \quad \mathbf{R}^* \mathbf{R}^{*\top} = \mathbf{I}, \quad \mathbf{K}_1^*, \mathbf{K}_2^* \text{ upper triangular}$$

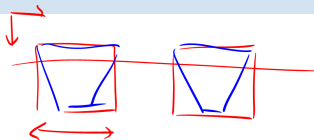
- we also want \mathbf{b}^* from $\mathbf{e}_1^* \simeq \mathbf{P}_1^* \mathbf{C}_2^* = \mathbf{K}_1^* \mathbf{b}^*$ \mathbf{b}^* in cam. 1 frame
- result:

$$\mathbf{R}^* = \mathbf{I}, \quad \mathbf{b}^* = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{K}_1^* = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{K}_2^* = \begin{bmatrix} k_{21} & k_{22} & k_{23} \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (29)$$

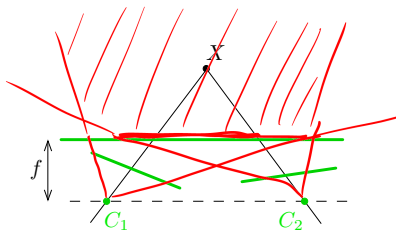
- rectified cameras are in canonical position with respect to each other
not rotated, canonical baseline
- rectified calibration matrices can differ in the first row only
- when $\mathbf{K}_1^* = \mathbf{K}_2^*$ then the rectified pair is called the standard stereo pair and the homographies standard rectification homographies

► cont'd

- rectification is a homography (per image)
 - ⇒ rectified camera centers are equal to the original ones
- standard rectified cameras are in canonical orientation
 - ⇒ rectified image projection planes are coplanar
- standard rectification guarantees equal rectified calibration matrices
 - ⇒ rectified image projection planes are equal



standard rectification homographies reproject onto a common image plane parallel to the base-line



Corollary

- the standard rectified stereo pair has vanishing disparity for 3D points at infinity
 - but known \mathbf{F} alone does not give any constraints to obtain standard rectification homographies
 - for that we need either of these:
 1. projection matrices, or
 2. calibrated cameras, or
 3. a few points at infinity calibrating k_{1i} , k_{2i} , $i = 1, 2, 3$ in (29)

► Primitive Rectification

Goal: Given fundamental matrix \mathbf{F} , derive some simple rectification homographies $\mathbf{H}_1, \mathbf{H}_2$

1. Let the SVD of \mathbf{F} be $\mathbf{UDV}^\top = \mathbf{F}$, where $\mathbf{D} = \text{diag}(1, d^2, 0)$, $1 \geq d^2 > 0$
2. decompose $\mathbf{D} = \mathbf{A}^\top \mathbf{F}^* \mathbf{B}$, where (\mathbf{F}^* is given \rightarrow Slide 151)

$$\begin{aligned} \mathbf{m}_2^\top \mathbf{F} \mathbf{m}_1 &= 0 \\ \mathbf{m}_2^\top \hat{\mathbf{H}}_2^\top \mathbf{F}^* \hat{\mathbf{H}}_1 \mathbf{m}_1 &= 0 \end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & d & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -d & 0 \end{bmatrix}$$

$$\mathbf{F}^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

3. then

given \mathbf{F} find $\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2$

$$\mathbf{F} = \mathbf{UDV}^\top = \underbrace{\mathbf{UA}^\top}_{\hat{\mathbf{H}}_2^\top} \underbrace{\mathbf{F}^*}_{\mathbf{D}} \underbrace{\mathbf{BV}^\top}_{\hat{\mathbf{H}}_1}$$

and the primitive rectification homographies are

$$\hat{\mathbf{H}}_2 = \mathbf{AU}^\top, \quad \hat{\mathbf{H}}_1 = \mathbf{BV}^\top$$

⊗ P1; 1pt: derive some \mathbf{A}, \mathbf{B} from the admissible class

- rectification homographies do exist
- there are other primitive rectification homographies, these suggested are just simple to obtain

► Primitive Rectification Suffices for Calibrated Cameras

Obs: calibrated cameras: $d = 1 \Rightarrow \hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2$ are orthogonal

1. determine primitive rectification homographies ($\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2$) from the essential matrix
2. choose a suitable common calibration matrix \mathbf{K} , e.g.

$$\mathbf{K} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \quad f = \frac{1}{2}(f^1 + f^2), \quad u_0 = \frac{1}{2}(u_0^1 + u_0^2), \quad \text{etc.}$$

3. the final rectification homographies are

$$\mathbf{H}_1 = \mathbf{K}\hat{\mathbf{H}}_1, \quad \mathbf{H}_2 = \mathbf{K}\hat{\mathbf{H}}_2$$

- we got a standard camera pair and non-negative disparity

$$\mathbf{P}_i^+ \stackrel{\text{def}}{=} \underbrace{\mathbf{K}_i^{-1}} \mathbf{P}_i = \mathbf{R}_i [\mathbf{I} \quad -\mathbf{C}_i], \quad i = 1, 2 \quad \text{note we started from } \mathbf{E}, \text{ not } \mathbf{F}$$

$$\mathbf{H}_1 \mathbf{P}_1^+ = \cancel{\mathbf{K}} \hat{\mathbf{H}}_1 \mathbf{P}_1^+ = \mathbf{K} \underbrace{\mathbf{B}\mathbf{V}^\top \mathbf{R}_1}_{\mathbf{R}^*} [\mathbf{I} \quad -\mathbf{C}_1] = \mathbf{K}\mathbf{R}^* [\mathbf{I} \quad -\mathbf{C}_1]$$

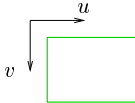
$$\mathbf{H}_2 \mathbf{P}_2^+ = \cancel{\mathbf{K}} \hat{\mathbf{H}}_2 \mathbf{P}_2^+ = \mathbf{K} \underbrace{\mathbf{A}\mathbf{U}^\top \mathbf{R}_2}_{\mathbf{R}^*} [\mathbf{I} \quad -\mathbf{C}_2] = \mathbf{K}\mathbf{R}^* [\mathbf{I} \quad -\mathbf{C}_2]$$

- one can prove that $\mathbf{B}\mathbf{V}^\top \mathbf{R}_1 = \mathbf{A}\mathbf{U}^\top \mathbf{R}_2$ with the help of (11)
- points at infinity project to $\mathbf{K}\mathbf{R}^*$ in both images \Rightarrow they have zero disparity



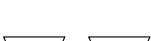


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► The Degrees of Freedom in Epipolar Rectification

Proposition 1 Homographies \mathbf{A}_1 and \mathbf{A}_2 are rectification-preserving if the images stay rectified, i.e. if $\mathbf{A}_2^{-\top} \mathbf{F}^* \mathbf{A}_1^{-1} \simeq \mathbf{F}^*$, which gives

$$\mathbf{A}_1 = \begin{bmatrix} l_1 & l_2 & l_3 \\ 0 & s_v & t_v \\ 0 & q & 1 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} r_1 & r_2 & r_3 \\ 0 & s_v & t_v \\ 0 & q & 1 \end{bmatrix},$$


where $s \neq 0$, u_0 , l_1 , $l_2 \neq 0$, l_3 , r_1 , $r_2 \neq 0$, r_3 , q are 9 free parameters.

general	transformation		canonical	type
l_1, r_1	horizontal scales		$l_1 = r_1$	algebraic
l_2, r_2	horizontal skews		$l_2 = r_2$	algebraic
l_3, r_3	horizontal shifts		$l_3 = r_3$	algebraic
q	common special projective			geometric
s_v	common vertical scale			geometric
t_v	common vertical shift			algebraic
9 DoF			9 - 3 = 6 DoF	

- q is rotation about the baseline
- s_v changes the focal length

proof: find a rotation \mathbf{G} that brings \mathbf{K} to upper triangular form via RQ decomposition: $\mathbf{A}_1 \mathbf{K}_1^* = \hat{\mathbf{K}}_1 \mathbf{G}$ and $\mathbf{A}_2 \mathbf{K}_2^* = \hat{\mathbf{K}}_2 \mathbf{G}$

Corollary for Proposition 1 Let $\bar{\mathbf{H}}_1$ and $\bar{\mathbf{H}}_2$ be (primitive or other) rectification homographies. Then $\mathbf{H}_1 = \mathbf{A}_1 \bar{\mathbf{H}}_1$, $\mathbf{H}_2 = \mathbf{A}_2 \bar{\mathbf{H}}_2$ are also rectification homographies.

Proposition 2 Pairs of rectification-preserving homographies $(\mathbf{A}_1, \mathbf{A}_2)$ form a group with group operation $(\mathbf{A}'_1, \mathbf{A}'_2) \circ (\mathbf{A}_1, \mathbf{A}_2) = (\mathbf{A}'_1 \mathbf{A}_1, \mathbf{A}'_2 \mathbf{A}_2)$.

Proof:

- closure by Proposition 1
- associativity by matrix multiplication
- identity belongs to the set
- inverse element belongs to the set by $\mathbf{A}_2^\top \mathbf{F}^* \mathbf{A}_1 \simeq \mathbf{F}^* \Leftrightarrow \mathbf{F}^* \simeq \mathbf{A}_2^{-\top} \mathbf{F}^* \mathbf{A}_1^{-1}$

Optimal and Non-linear Rectification

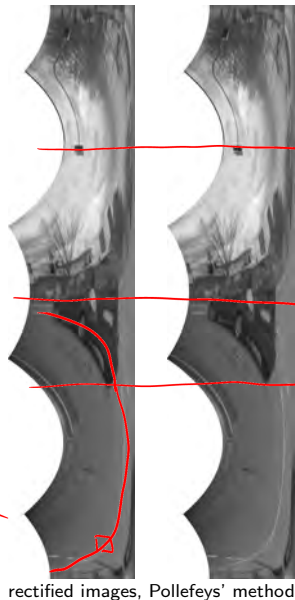
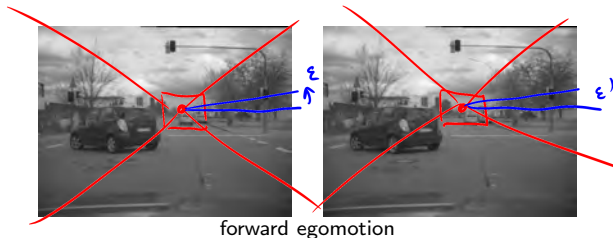
Optimal choice for the free parameters

- by minimization of residual image distortion, eg. [Gluckman & Nayar 2001]

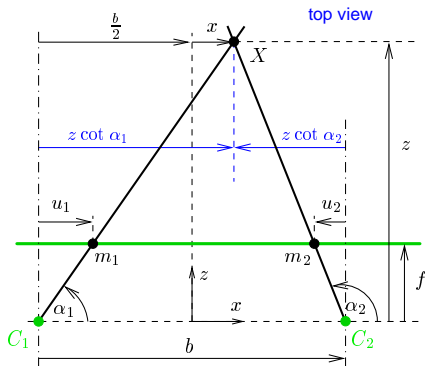
$$\mathbf{A}_1^* = \arg \min_{\mathbf{A}_1} \iint_{\Omega} (\det J(\mathbf{A}_1 \hat{\mathbf{H}}_1 \underline{\mathbf{x}}) - 1)^2 d\underline{\mathbf{x}}$$

image domain

- by minimization of image information loss [Matoušek, ICIG 2004]
- non-linear rectification suitable for forward motion [Pollefeys et al. 1999], [Geyer & Daniilidis 2003]



► Binocular Disparity in Standard Stereo Pair



- Assumptions: single image line, standard camera pair

$$b = z \cot \alpha_1 - z \cot \alpha_2$$

$$u_1 = f \cot \alpha_1$$

$$u_2 = f \cot \alpha_2$$

$$b = \frac{b}{2} + x - z \cot \alpha_2$$

- $X = (x, z)$ from **disparity** $d = u_1 - u_2$:

$$z = \frac{bf}{d}, \quad x = \frac{b}{d} \frac{u_1 + u_2}{2}, \quad y = \frac{bv}{d}$$

$$z' = bf/2$$

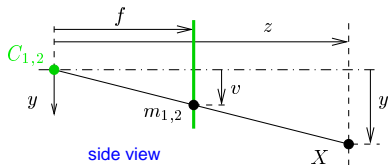
f, d, u, v in pixels, b, x, y, z in meters

Observations

- constant disparity surface is a frontoparallel plane
- distant points have small disparity
- relative error in z is large for small disparity

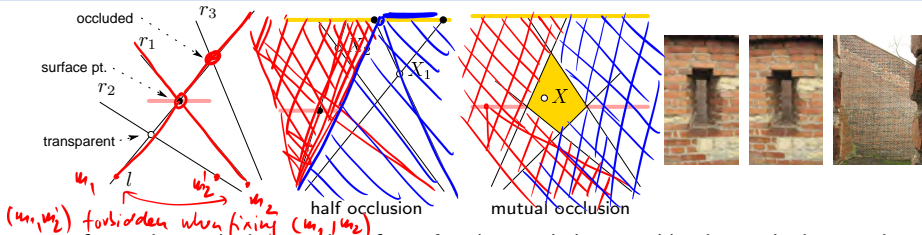
$$\frac{1}{z} \frac{dz}{dd} = -\frac{1}{d}$$

- increasing baseline increases disparity and reduces the error



► Understanding Basic Occlusion Types

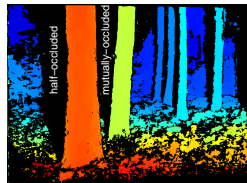
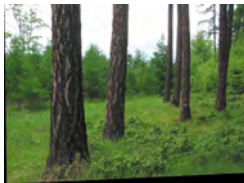
assumption: opaque objects



- surface point at the intersection of rays l and r_1 occludes a world point at the intersection (l, r_3) and implies the world point (l, r_2) is transparent, therefore

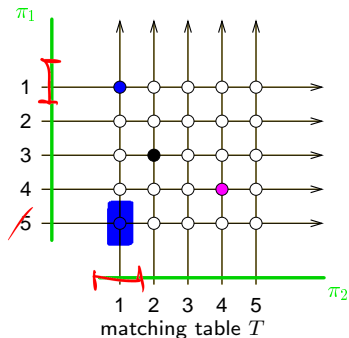
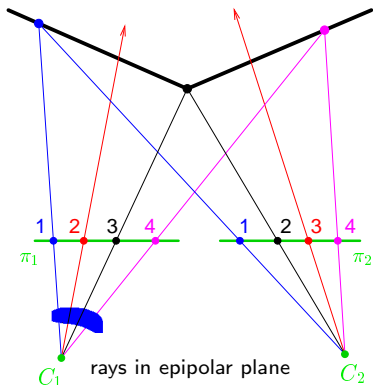
(l, r_3) and (l, r_2) are excluded by (l, r_1)

- in half-occlusion, every world point such as X_1 or X_2 is excluded by a binocularly visible surface point
 \Rightarrow decisions on correspondences are not independent
- in mutual occlusion this is no longer the case: any X in the yellow zone is not excluded
 \Rightarrow decisions in the zone are independent on the rest



► Matching Table

Based on the observation on mutual exclusion we expect each pixel to match at most once.



matching table

- rows and columns represent optical rays
- nodes: possible correspondence pairs
- full nodes: correspondences
- numerical values associated with nodes: descriptor similarities

[see next](#)

Image Point Descriptors And Their Similarity

Descriptors: Tag image points by their (viewpoint-invariant) physical properties:

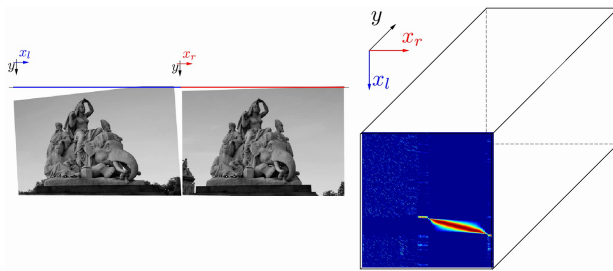
- texture window
- reflectance profile under a moving illuminant
- photometric ratios
- dual photometric stereo
- polarization signature
- ...

[Moravec 77]

[Wolff & Angelopoulou 93-94]

[Ikeuchi 87]

- similar points are more likely to match
- we will compute image similarity for all 'match candidates' and get the matching table

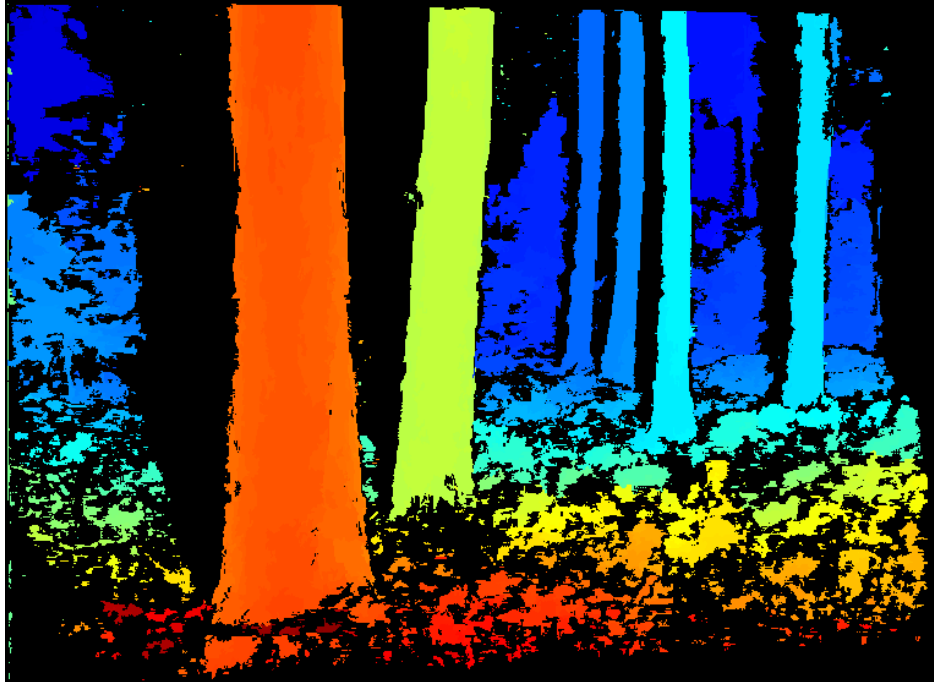


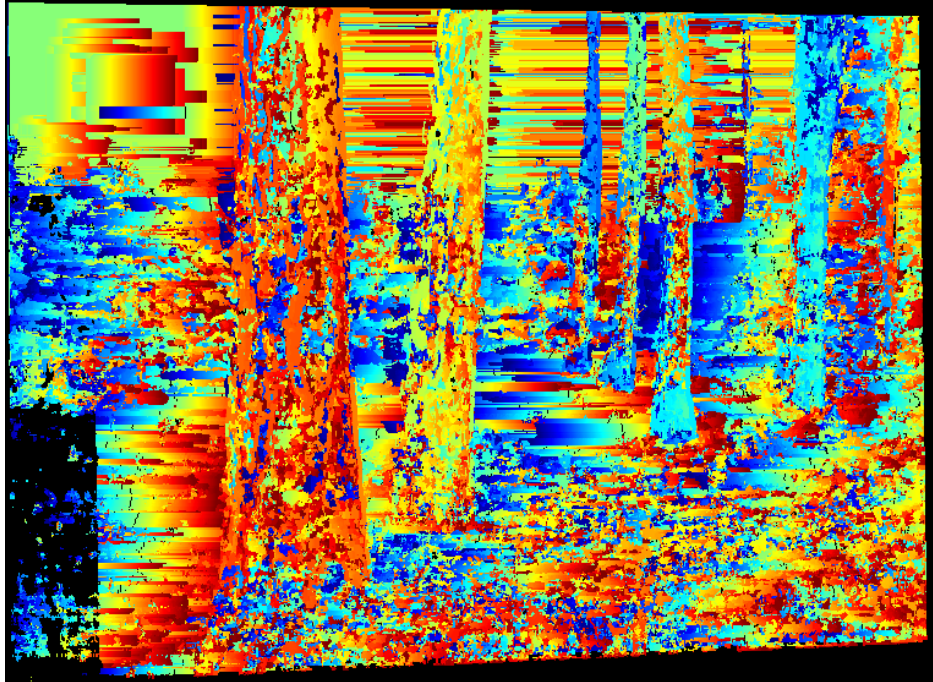
video

Thank You

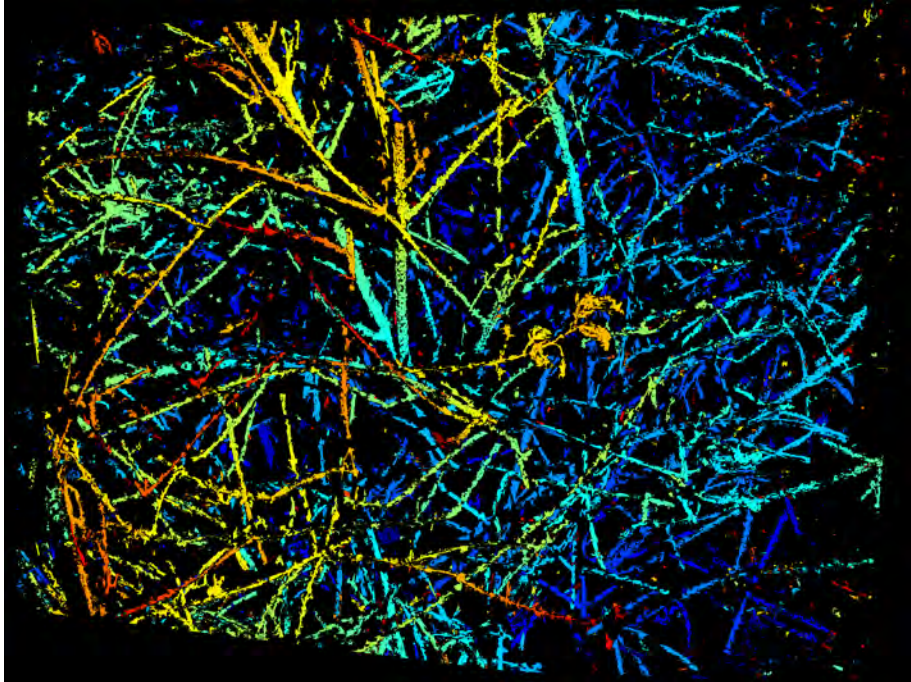




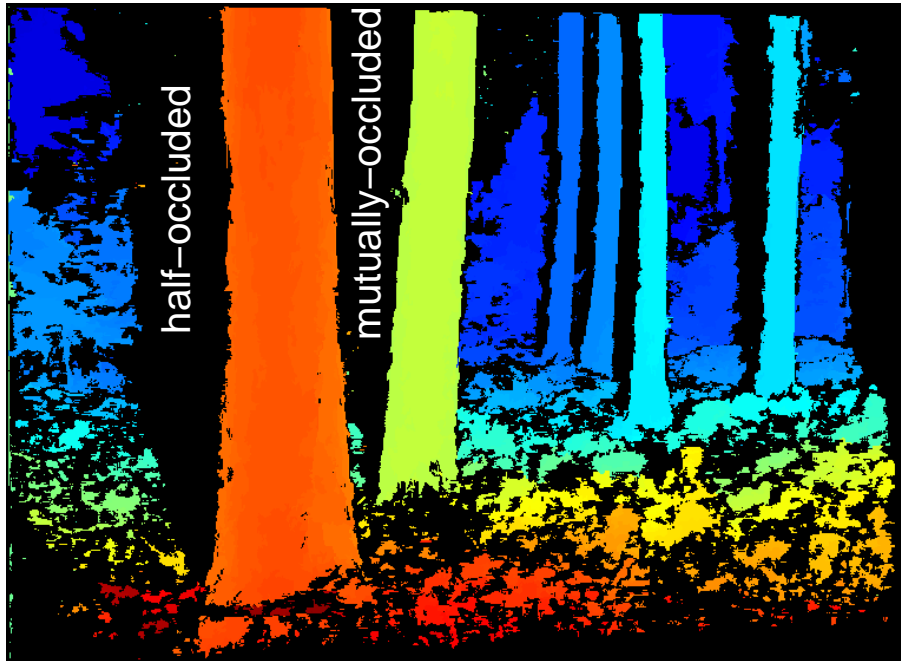












half-occluded

mutually-occluded



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