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**GRAPHICAL MARKOV MODELS (WS2011)**  
**5. SEMINAR**

**Assignment 1.** Let  $y^i, i \in I$  be a finite set of points in  $\mathbb{R}^n$ . The task is to find the point  $x$  closest to all of them, i.e.

$$x^* = \arg \min_{x \in \mathbb{R}^n} \max_{i \in I} \|x - y^i\|^2.$$

Interpret the task and its solution geometrically. Derive an algorithm for solving this task.

**Assignment 2.** When discussing empirical risk minimisation based learning (section 9), we have assumed that there exists an HMM such, that all examples in the training data are correctly recognised by this HMM. Discuss how to modify the learning task in case that this assumption is not valid.

*Hint:* Study how this situation is handled for “simple” linear classifiers. Try to generalise for the case of sequences and HMMs.

**Assignment 3.** Derive an EM-algorithm for learning the following conditional independent probability model (sometimes called “naive Bayes probabilistic model”)

$$p(x, y, k) = p(x|k)p(y|k)p(k),$$

where  $x \in X, y \in Y, k \in K$  and all three sets are finite. The training data provided for learning are independent realisations of pairs  $(x, y)$  with the empirical distribution  $p^*(x, y)$  (the corresponding values of  $k$  are not observed).

**Assignment 4.** (Discussion) Get familiar with the definition of Bayesian networks. Compare them with Markov Random Fields and Gibbs Random Fields defined on undirected graphs.



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