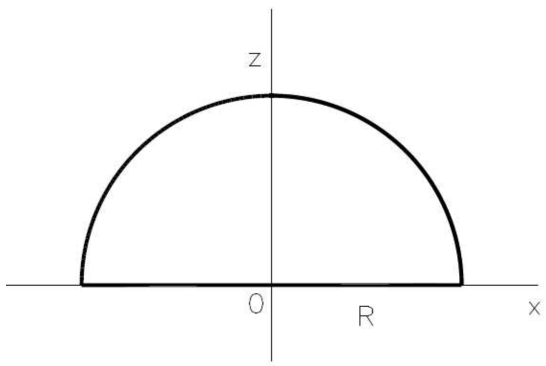
Homework 5 for the Physics for OI

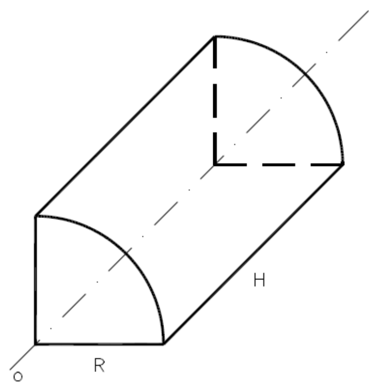
This homework is focused on coordinate systems, center of mass and moment of inertia.

Your tasks:

a) To calculate coordinates (especially z-coordinate) of a center of mass of hemisphere of radius R with its bottom in the xy plane and centered around z-axis (see the picture).
b) To calculate the moment of inertia of the hemisphere from the a) if the axis of rotation would be the z-axis.
c) To calculate the moment of inertia of a sphere with its center at the origin of the coordinate system if the axis of rotation would be the z-axis (the hemisphere from the a) was completed to a sphere).
d) To calculate the moment of inertia of a quarter of cylinder (see the picture) of radius R and height H if the axis of rotation is “o”.
e) To calculate the moment of inertia of a cylinder (the quarter of cylinder was completed to a cylinder) of radius R and height H if the axis of rotation is “o”.



Hemisphere



Quarter of cylinder

Additional instructions and hints:

The density of each body is constant and equal to ρ. Evaluate each result both in the pure format (as a function of the ρ, R, or H) and also divided by the mass of the body (the result will be just a function of radius R)
Use triple integration in cylindrical coordinates for the d) and e). Assume that the “o” axis is identical with the z-axis.
Recalculation between Cartesian and cylindrical coordinates:

$$x = r\sin\phi$$

$$y = r\cos\phi$$

$$z = z$$

Mass of a cylinder calculated in cylindrical coordinates, “o” axis is z-axis.

$$m = \rho V = \rho \int_0^H \int_0^{2\pi} \int_0^R r\,dr\,d\phi\,dz$$

Use triple integration in spherical coordinates for the a), b) and c).

Recalculation between Cartesian and spherical coordinates:

$$x = r\sin\theta\cos\varphi$$
$$y = r\sin\theta\sin\varphi$$
$$z = r\cos\theta$$

Mass of a sphere with its center at the origin of coordinate system calculated in spherical coordinates

$$m = \rho V = \rho\int_0^{\pi}\int_0^{2\pi}\int_0^{R} r^2 \sin\theta \, dr\, d\varphi\, d\theta$$

You can use either *Integrate* function from the Mathematica or chained application of the direct integration sign.