Algorithmic Game Theory

DeepStack

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Home exercise

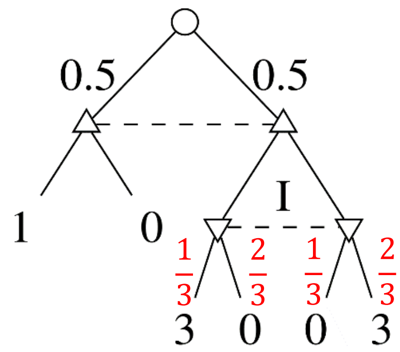
Prove or disprove the following conjecture.

Conjecture: For any n-player game and each $\epsilon > 0$, no-regret dynamic eventually converges to ϵ –Nash equilibrium.

Game decomposition

Perfect information example

Imperfect information example



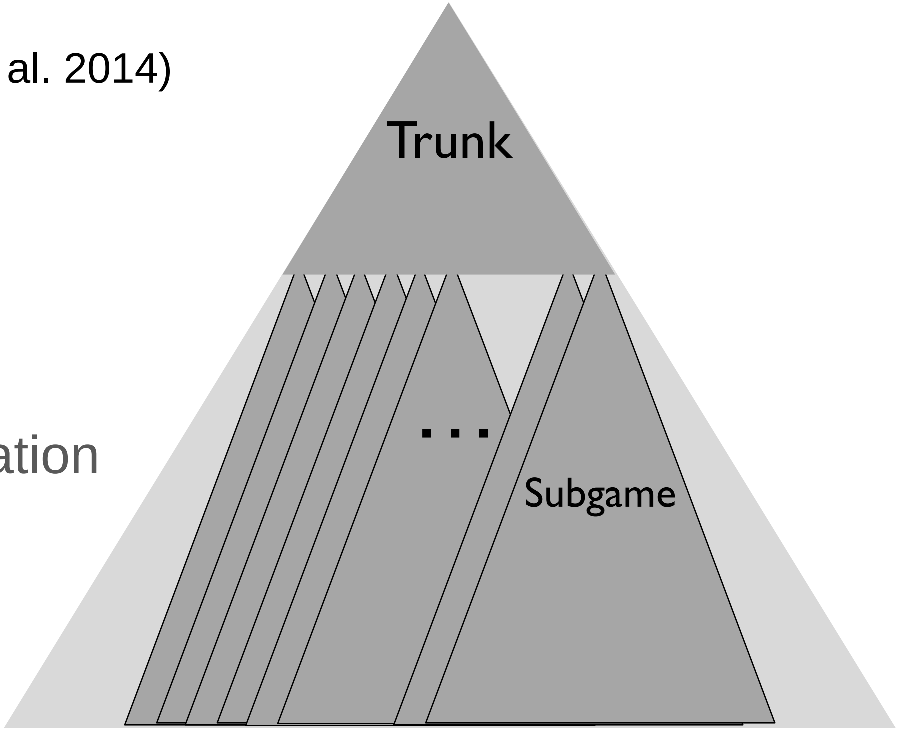
CFR-D

CFR with Decomposition (Burch et al. 2014)

Trades-of space for computation

- Store only the trunk
- Resolve subgames in each iteration

Resolve on demand in play



CDR-D

Augmented information set

- Set on undistinguishable histories for any player, not just the deciding one

Subgame (denoted S)

- forest of trees closed under descendance and belonging into augmented information sets

R(S)

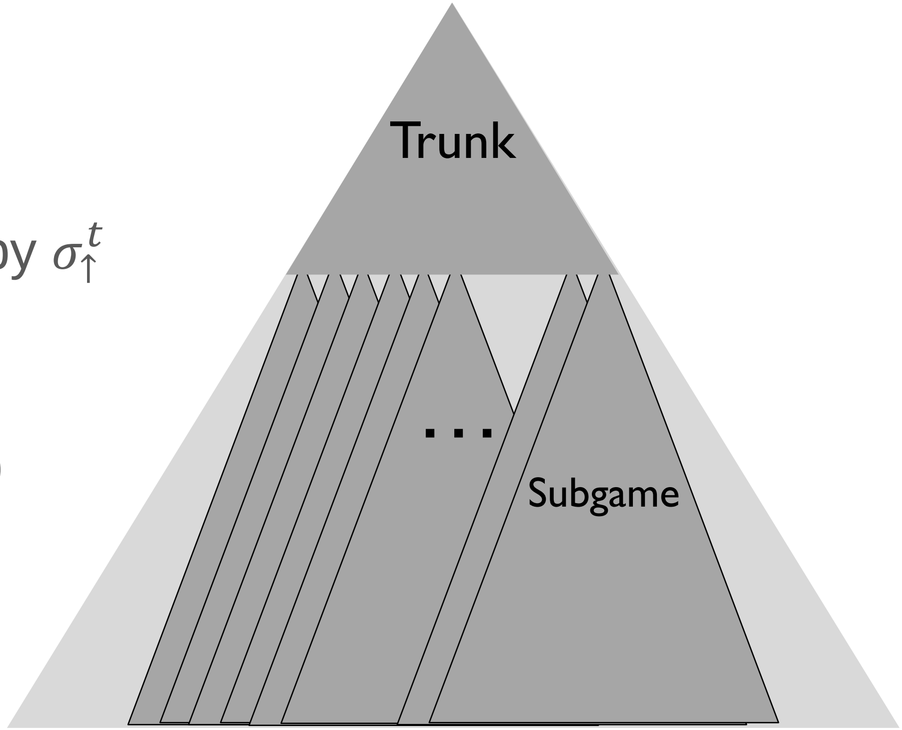
- set of augmented information sets in the root of a subgame

CFR-D: Solving Trunk Strategy

Initialize regrets to 0

For iteration $t = 1, \ldots, T$

- compute $\sigma_\uparrow^t$ from stored regrets
- update trunk average strategy by $\sigma_\uparrow^t$
- For each subgame S
  - $\sigma_S^t \leftarrow$SOLVE(S, $\sigma_\uparrow^t$)
  - For each augmented $I_p \in R(S)$
    - Compute value v_{I_p}
    - Update average value $\mathrm{cf}v_{I_p}$
- Update trunk regrets using v_{I_p}



CFR-D: Computing Trunk Strategy

CFR-D: Resolving Subgame

Assume blue player played D and the game reached S1

Unsafe resolving

Save resolving

No incentive to change trunk!

CFR-D More Complicated Resolving

CFR-D Resolving Game

When resolving for player 1

- Create new chance node as the root
- Create new nodes for player 2 grouped by her “information sets”
- Connect the root to nodes in proportion to player 1 trunk strategy
- For each player 2 node, add follow action leading to subgame
- For each player 2 node, add terminate action with CFV of IS

We need

- Distribution in the root IS generated by player 1 trunk strategy
- Counterfactual value achievable by player 2 in his root ISs

CFR-D Convergence properties

CFR-D achieves no regret in the trunk

It the counterfactual regret at each information set I at the root of a subgame is bounded by ϵ_S, then than the average regret over the whole game is

$$R^T_{full} \leq \frac{N_{TR}\sqrt{A}}{\sqrt{T}} + N_S \epsilon_S$$

Proof sketch: $\sigma^0[S \leftarrow \sigma_S^{0,*}], \sigma^1[S \leftarrow \sigma_S^{1,*}], \ldots$

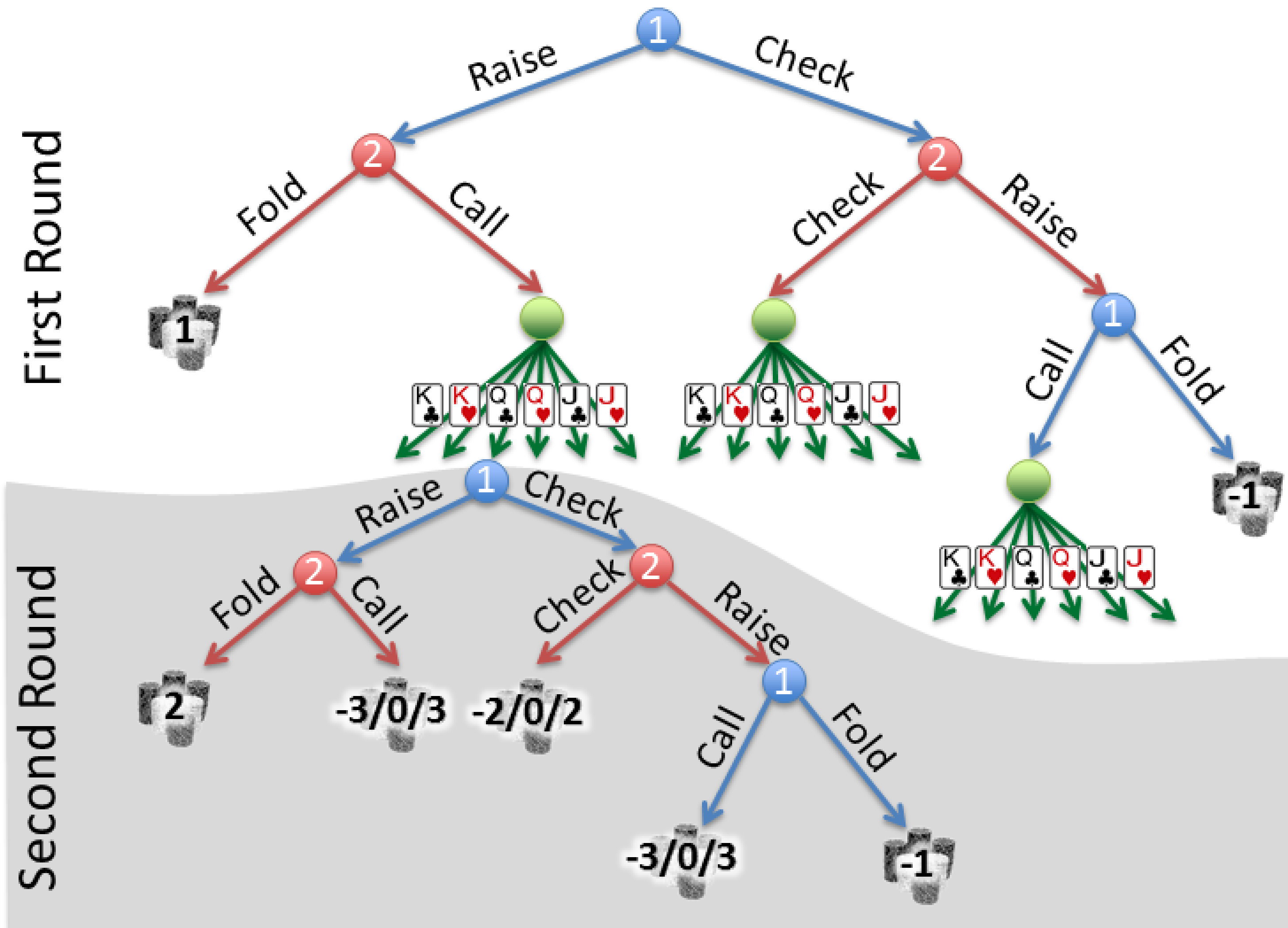
- CF regret in the trunk minimized by CFR
- CF regret in the subgame close to 0 for both players

CFR-D resolving forms a Nash equilibrium

If we run the recovery game for each player and each subgame until we reach regret below ϵ_R, the combined strategy has regret

$$R^T_{full} \leq \frac{N_{TR}\sqrt{A}}{\sqrt{T}} + N_S(3\epsilon_S + 2\epsilon_R)$$

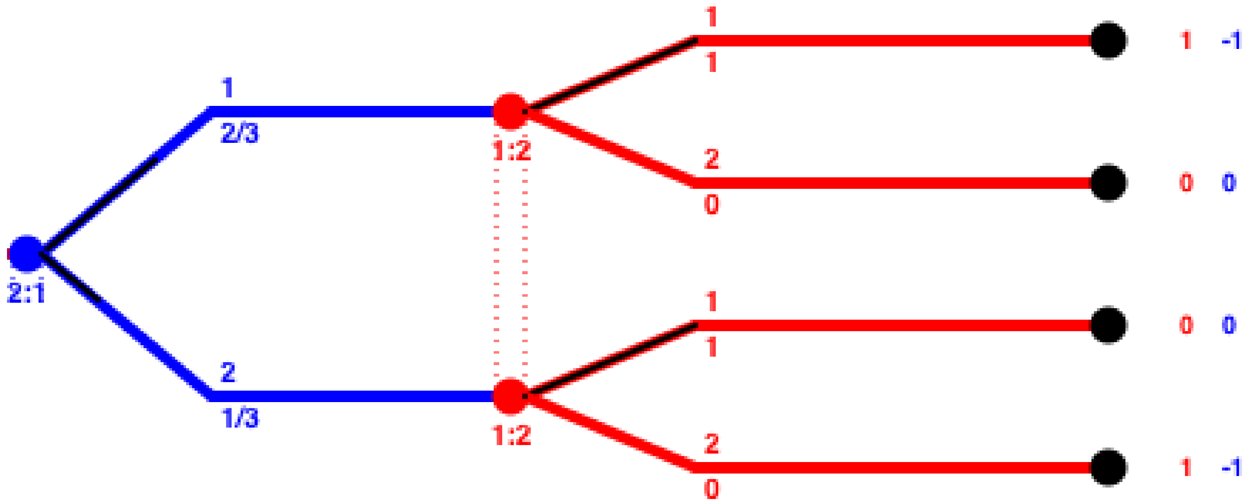
Public Tree



First Round
Second Round
1
Raise
Check
2
Fold
Call
Check
Raise
1
K Q J
Call
Fold
-1
Raise
Check
Fold
Call
Check
Raise
2
-3/0/3
-2/0/2
Call
Fold
-3/0/3
-1

Public Tree

Matching pennies



Phantom Tic-Tac-Toe

Visibility-based pursuit-evasion games

Augmented IS in poker public node

Resolving poker subgame

To resolve, we need

$\forall I_1 \in R(S) \;\; \pi_1(I_1)$

$\forall I_2 \in R(S) \;\; cfv_2(I_2)$

In poker it means

$\pi_1(I_1)$ - probability that player 1 holds each hand = range

$cfv_2(I_2)$ - how much player 2 can win with each hand

In root (after chance reals hole cards)

$\pi_i(I_i)$ - uniform

$cfv_i(I_i)$ - pre-computed offline

DeepStack: updating maintained values

Assuming DeepStack is player 1

Own action

- replace player 2's *cfv*s by the once computed in the resolve game
- update player 1's range based on the played strategy

Chance action

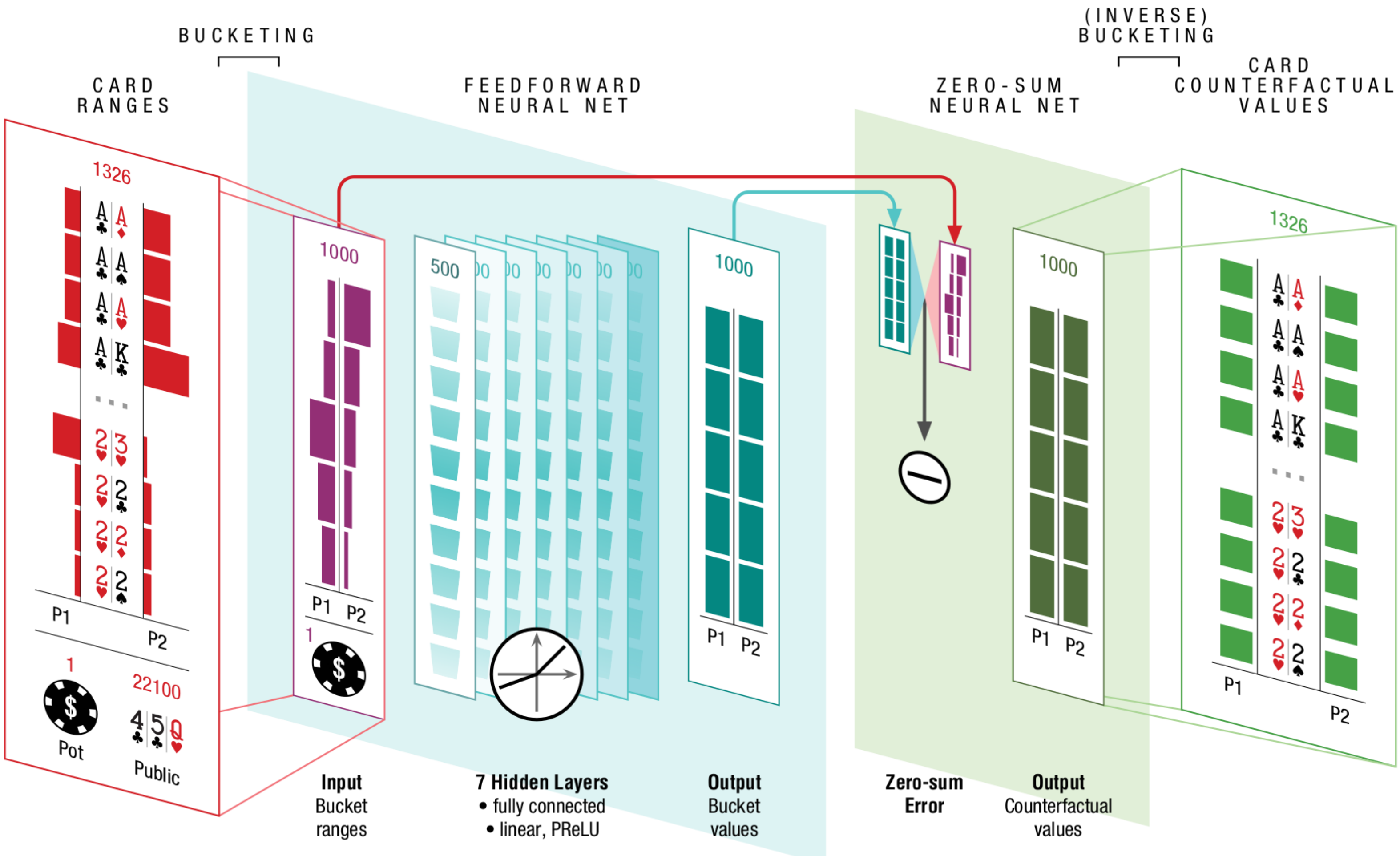
- replace player 2's *cfv*s from the last resolve above chance
- keep player 1's range unchanged

Opponent's action

- no update required!

DeepStack: Limited look-ahead

DeepStack: Neural Network



DeepStack: Training

Turn Network (right after dealing turn card)

- 10M pseudo-random ranges, pots, random boards
- Solve by CFR^{+} until the end of the game
- Extract CFVs for training, train Turn NN

Flop Network (right after dealing flop cards)

- 10M pseudo-random ranges, pots, random boards
- Solve by DeepStack (CFR-D) using the pre-trained Turn NN
- Extract CFVs for training, train Turn NN

Pre-flop Network

- 10M pseudo-random ranges, pots
- Enumerating 22100 possible flops and averaging

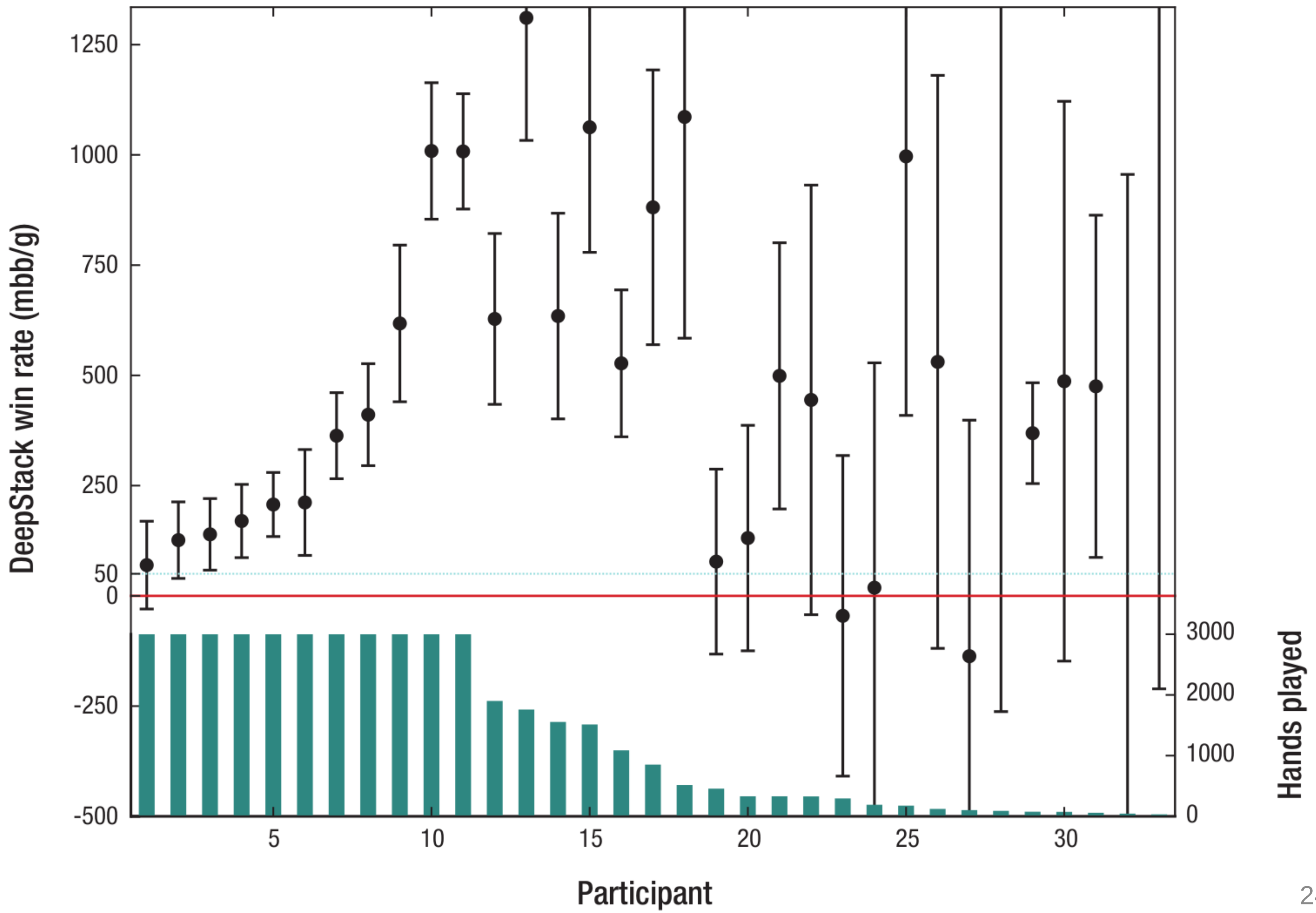
DeepStack: Convergence

Theorem: If the error of CFVs returned by the value function is less then ϵ and T iterations of resolving are used for each decision, than the exploitability of the player strategy is less than

$$k_1\epsilon + \frac{k_2}{\sqrt{T}}$$

where k_1, k_2 are game-specific constants.

DeepStack: Results



References

Burch, N., & Bowling, M. (2013). CFR-D: Solving Imperfect Information Games Using Decomposition. arXiv Preprint arXiv:1303.4441, 1–15. Retrieved from http://arxiv.org/abs/1303.4441

Moravčík, M., Schmid, M., Burch, N., Lisý, V., Morrill, D., Bard, N., Davis T., Waugh K., Johanson M., Bowling, M. (2017). DeepStack: Expert-Level Artificial Intelligence in No-Limit Poker. https://doi.org/10.1126/science.aam6960