Partially Observable Markov Decision Processes

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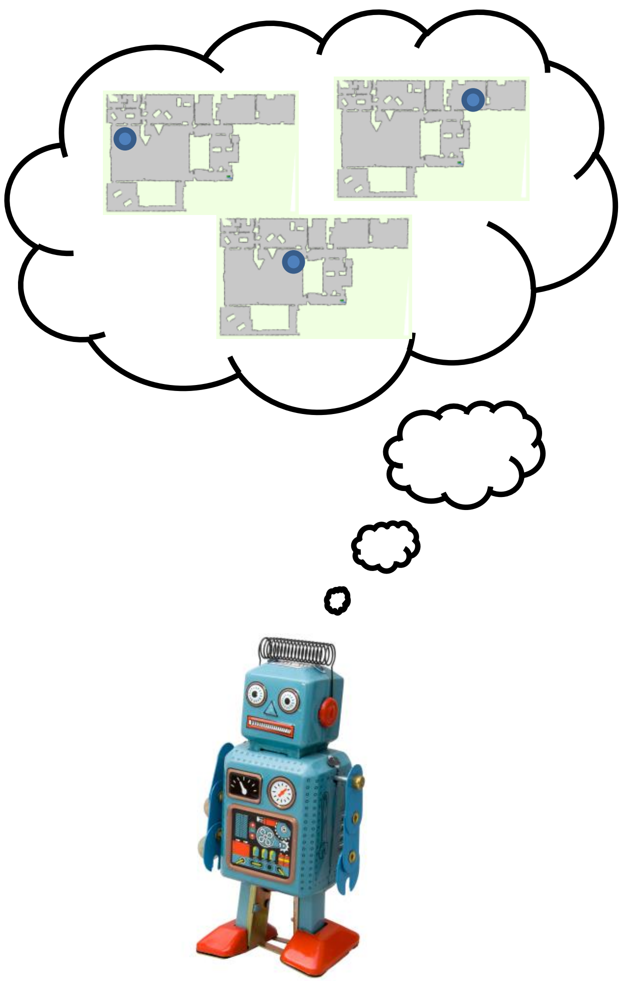
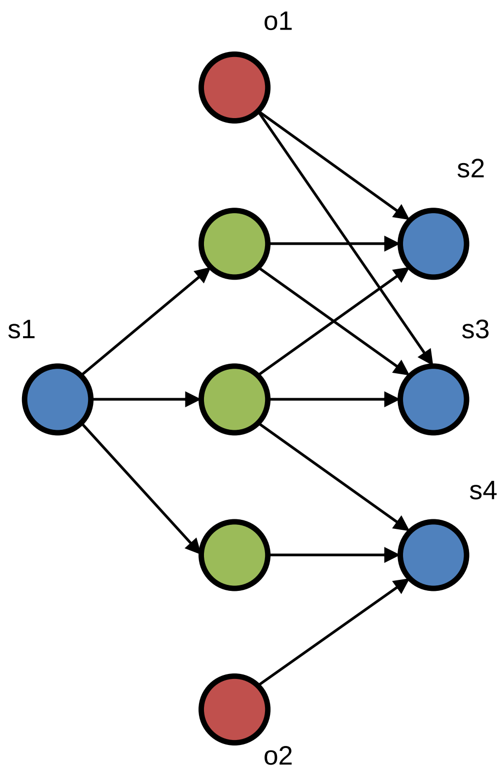
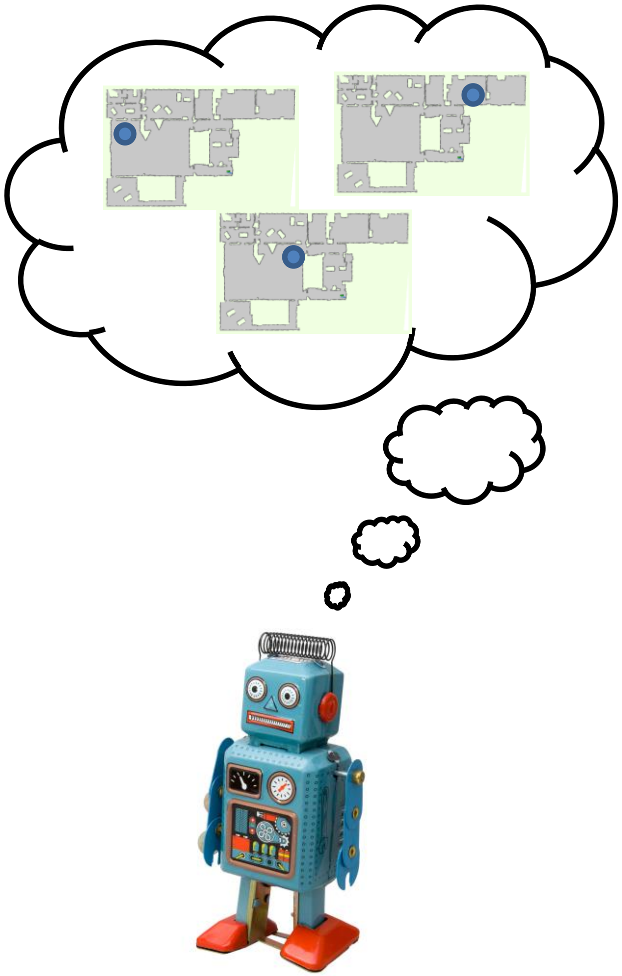
Partial Observability

- the world is not perfect
  - actions take some time to execute
  - actions may fail or yield unexpected results
  - the environment may change due to other agents
  - the agent does not have knowledge about whole situation
  - sensors are not precise



Partial Observability

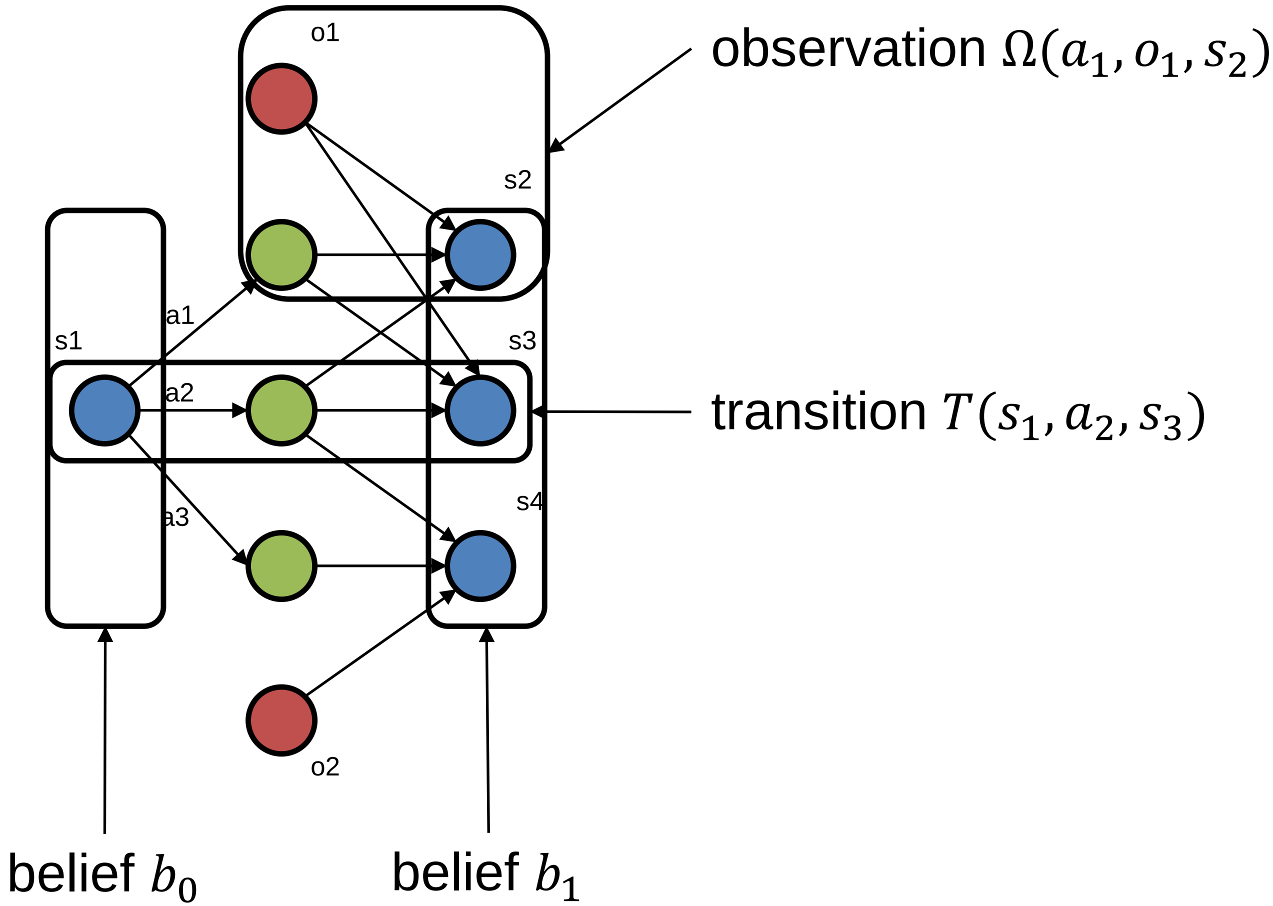




Partially Observable MDPs

- main formal model for scenarios with uncertain observations
- $\langle S, A, D, O, b_0, T, \Omega, R, \gamma \rangle$
  - states – finite set of states of the world
  - actions – finite set of actions the agent can perform
  - time steps
  - observations – finite set of possible observations
  - initial belief function $b_0 : S \rightarrow [0,1]$
  - transition function $T : S \times A \times S \rightarrow [0,1]$
  - observation probability $\Omega : A \times O \times S \rightarrow [0,1]$
  - reward function $R : S \times A \rightarrow \mathbb{R}$
  - discount factor $0 \leq \gamma < 1$

Partially Observable MDPs - probabilities



Partially Observable MDPs - beliefs

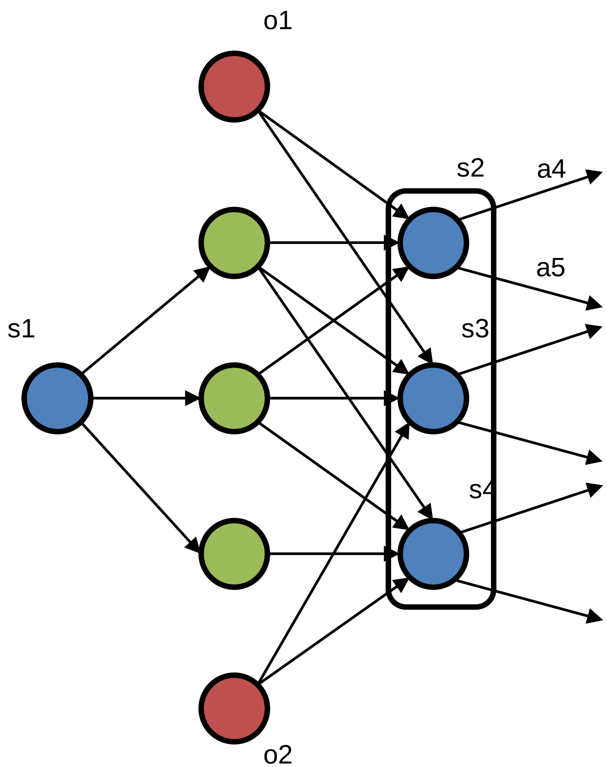
- beliefs represent a probability distribution over states
- beliefs are uniquely identified by the history
  - b_1 - probability distribution over states after playing one action
  - $b_t \leftarrow \Pr(s_t | b_0, a_0, o_1, \ldots, o_{t-1}, a_{t-1}, o_t)$
- we can exploit dynamic programming (define transformation of beliefs, belief update)
  - $b_t(s') = \mu\Omega(a, o, s'). \sum_{s \in S} T(s, a, s') b_{t-1}(s)$
  - where
    - o is the last observation
    - a is the last action
    - μ is the normalizing constant

Partially Observable MDPs - values

- beliefs determine new values
  - $V(b) = \max_{a \in A}[R(b,a) + \gamma \sum_{b' \in B} T(b,a,b')V(b')]$
- what we have done …
  - we have transformed a POMDP to a continuous state MDP
  - belief state is a simplex
    - $|S| - 1$ dimensions
- in theory we can use all the algorithms for MDPs (value iteration)
  - but B is infinite

Solving Continuous State MDPs

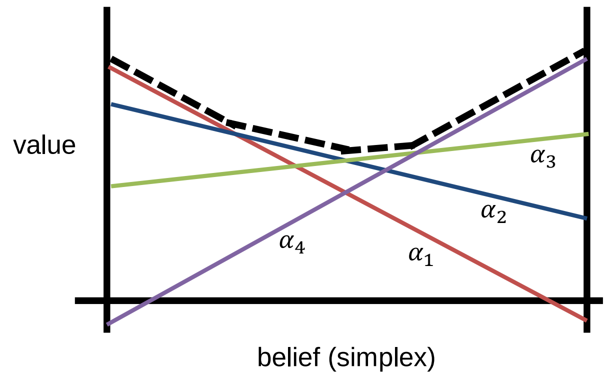
- in value iteration we take max of actions
- the belief space can be partitioned depending on the fact, which action is the best one

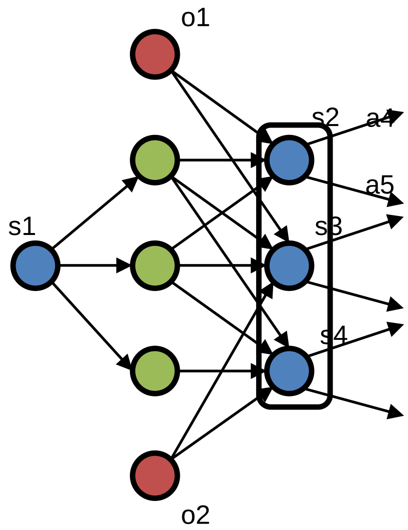


s2	s3	s4	V(a4)	V(a5)
0.2	0.1	0.7	3	2
0.7	0.1	0.2	1	7

Solving Continuous State MDPs

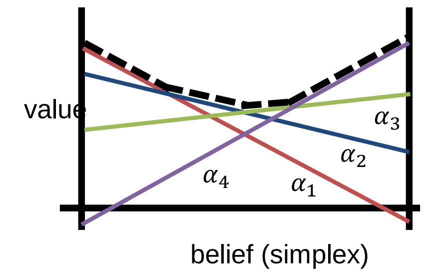
- values can be compactly represented as a finite set of α vectors; $V = \{\alpha_0, \dots, \alpha_m\}$
  - α vector is an $|S|$ dimensional hyper-plane
    - a linear function representing utility values after selecting some fixed action
  - defines the value function over a bounded region of the belief
  - $V(b) = \max_{\alpha \in V} \sum_{s \in S} \alpha(s) b(s)$
  - V is a piece-wise linear convex function





Solving Continuous State MDPs

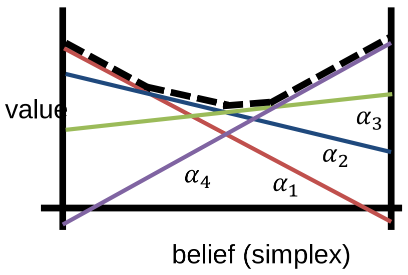
- **Q: Can we modify value iteration algorithm to work with α functions?**



- exact value iteration for POMDPs
  - $V^t(b) = \max_{a\in A}[\sum_{s\in S} R(s,a)b(s) +$
  - $+\gamma \sum_{o\in O} \max_{\alpha'\in V^{t-1}} \sum_{s\in S}\sum_{s'\in S} T(s,a,s')\Omega(o,s',a)\alpha'(s')b(s)]$
- the above formula compute values (we need α-vectors)
  - $\alpha^{a,*}(s) = R(s,a)$
  - $\alpha_i^{a,o}(s) = \gamma \sum_{s'\in S} T(s,a,s')\,\Omega(o,s',a)\alpha_i'(s') \qquad \forall \alpha'_i \in V'$
  - $V^a = \alpha^{a,*} \oplus \alpha^{a,o_1} \oplus \alpha^{a,o_2} \oplus \cdots$
  - $V = \cup_{a\in A} V^a$

Exact Value Iteration for POMDPs

- exact baseline algorithm, however has several disadvantages
- complexity
  - exponential in size of observations $|O|$
  - base of the exponent is $|V|$
  - it is important to remove dominated alpha-vectors
  - useful only for very small domains



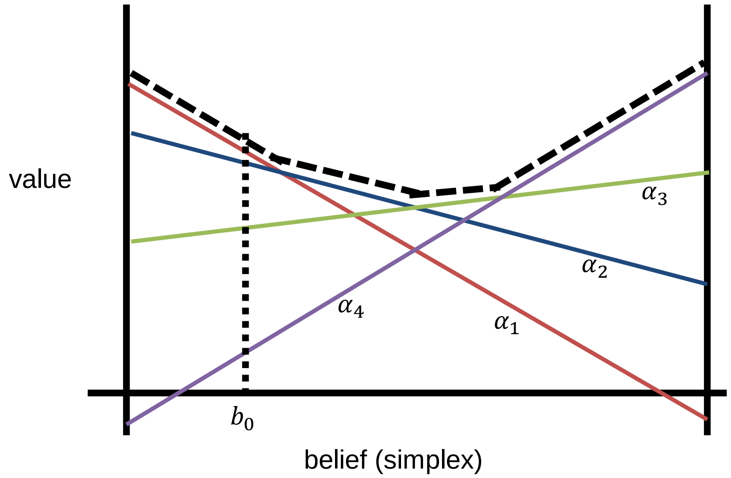
- Tiger example

Exact Value Iteration for POMDPs

- can we do better than full value iteration?
- only a fraction of all belief state is actually achievable in POMDP
  - we can sample the belief state

Point Based Value Iteration for POMDPs

- instead of the complete belief space we use a limited set
  - $B = \{b_0, \dots, b_q\}$
- the algorithm keeps only a single alpha vector for one belief point
- anytime algorithm altering 2 main steps
  - belief point value update
  - belief point set expansion

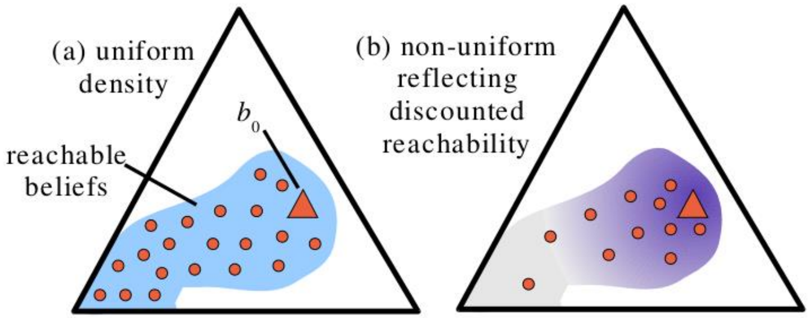


Point Based Value Iteration for POMDPs

- belief value update
  - $V_b^a = \alpha^{a,*} + \gamma \sum_{o \in O} \arg \max_{\alpha \in \alpha_i^{a,o}} (\alpha . b)$
  - $V \leftarrow \arg \max_{V_b^a, \forall a \in A} V_b^a . b \quad \forall b \in B$
- removes the exponential complexity
- VI state ends after h iterations
  - finite horizon / the error is smaller than ε
- belief point set expansion
  - sampling new beliefs from existing beliefs
  - trying to uniformly cover reachable belief space

Point Based Value Iteration for POMDPs

- further improvements
- exploiting heuristics
  - for setting initial values
  - selecting belief points



- current scalability
  - up to 10^5 states of POMDP
- further reading
  - Shani, Pineau, Kaplow: A survey of point-based POMDP solvers (2012)

Beyond (PO)MDPs

- many other models
- specific variants of MDPs / generalization
  - AND/OR graphs
  - influence diagrams
  - dynamic Bayesian networks
- multiple agents
  - decentralized (PO)MDPs - DEC-(PO)MDPs
    - theoretical framework for multi-agent planning
  - partially observable stochastic games (POSG)
    - theoretical framework for interaction of rational agents