Monte Carlo Tree Search

PAH 2015

MCTS animation and RAVE slides by Michèle Sebag and Romaric Gaudel

Markov Decision Processes (MDPs)

- main formal model
  - $\Pi = \langle S, A, D, T, R \rangle$
  - states – finite set of states of the world
  - actions – finite set of actions the agent can perform
  - horizon – finite/infinite set of time steps $(1,2, \ldots)$
  - transition function
    - $T : S \times A \times S \times D \rightarrow [0,1]$
  - reward function
    - $R : S \times A \times S \times D \rightarrow \mathbb{R}$

Markov Decision Processes (MDPs)

- online planning ~ any-time algorithm
  - learn the next move
  - play it
  - iterate
- reward on final states (often win or lose)
- implicit (and compact) representation of large MDPs
  - cannot grow the full tree
  - cannot safely cut branches
  - cannot be greedy

Markov Decision Processes (MDPs)

- online planning
  - focus on current state
  - set of possible courses
  - decision making $\sim$ selection of one action

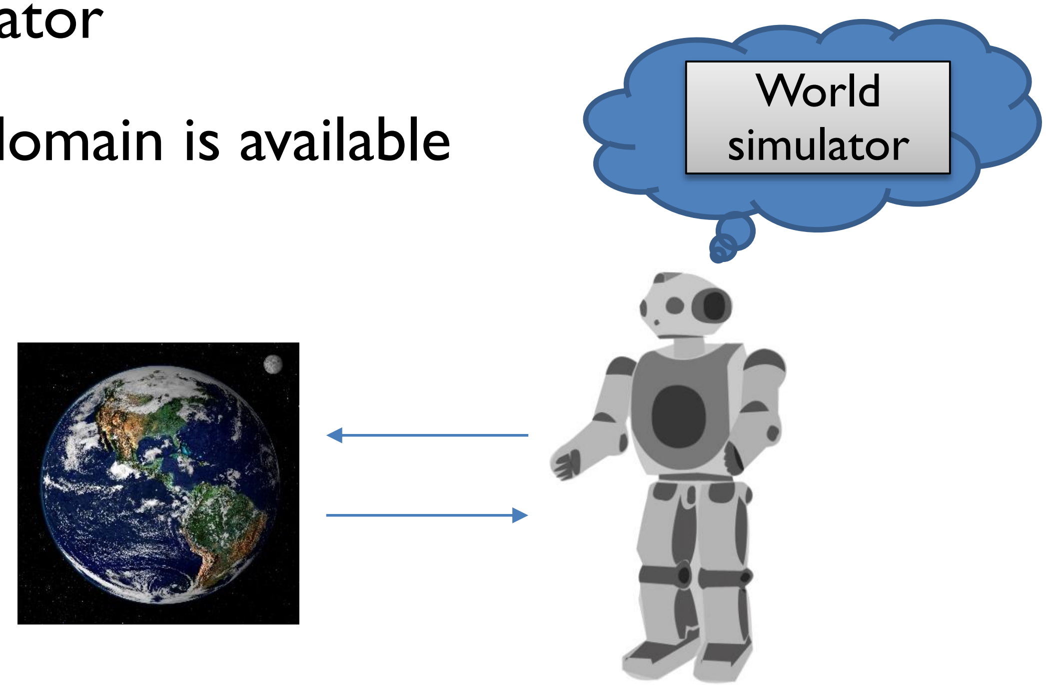
- online planning curse of dimensionality
  - number of applicable action is $O\left(\text{poly}(|\Pi|)\right)$
  - complexity because of the state-space size $O\left(\exp(|\Pi|)\right)$

MDPs – Using Monte Carlo Methods

- Monte Carlo sampling is a well known method for searching through large state space
- exploiting MC in sequential decision making has first been successfully designed in (Kocsis & Szepesvari, 2006)
- foundations in mathematical theory
  - Multi-Armed Bandit (MAB) Problem
  - Upper Confidence Bounds (UCB)
  - exploration/exploitation dilemma

Monte Carlo Methods

- Monte Carlo Simulation: a technique that can be used to solve a mathematical or statistical problem using repeated sampling to determine the properties of some phenomenon (or behavior)
- Monte-Carlo Planning: compute a good policy for an MDP by interacting with an MDP simulator
- when simulator of a planning domain is available or can be learned from data
  - even if not described as a MDP
  - queries has to be cheap (relatively)

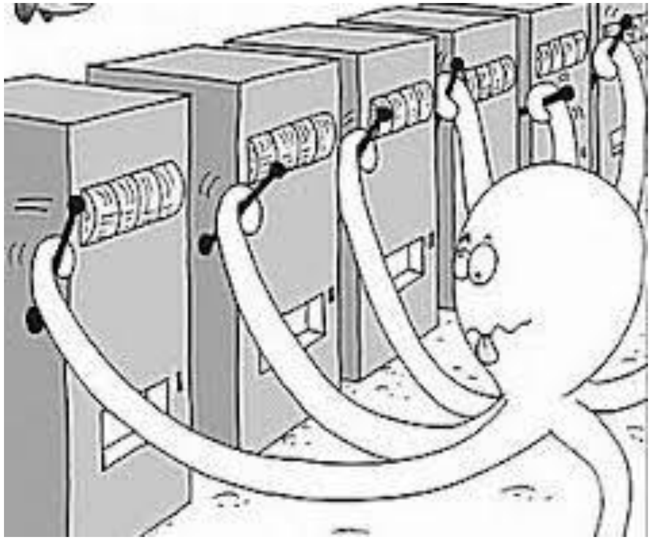


Monte Carlo Simulation

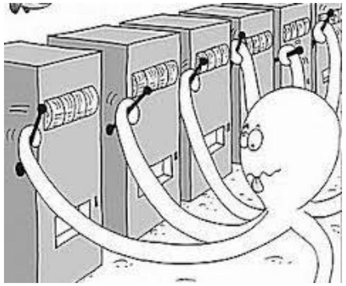
- Domains with Simulators
  - traffic
  - robotics
  - military missions
  - computer network
  - disaster relief and emergency planning
  - sports
  - board and video games
    - board (Go, Hex, Settlers of Catan, …), card (poker, Magic: The Gathering, …), RTS (Total War: Rome II, …)

Multi-Armed Bandit Problem

- sequential decision problem (over a single state)
- $k \geq 2$ stochastic actions (arms a_i)
  - each parameterized with an unknown probability distribution ν_i
  - each with a stored expectation μ_i
  - if executed (pulled) rewarded at random from ν_i
- objective
  - get maximal reward after N pulls
  - minimize regret of pulling wrong arm(s)



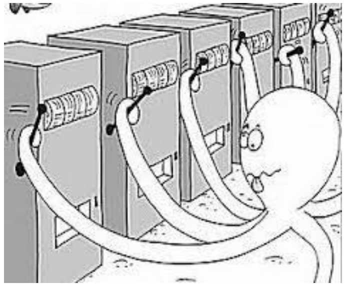
Multi-Armed Bandit Problem (variants)



- learning-while-acting
  - reward for each action
  - cumulative regret (exploration/exploitation dilemma)
  - algorithms: ϵ-greedy, UCB1
  - used in: Monte Carlo Tree Search, UCB1 applied to trees (UCT)
- online planning/learning-while-planning
  - reward only for final decision (N "free action tries" by simulator)
  - simple regret (only exploration)
  - algorithms: uniform sampling, ϵ-greedy, Sequential Halving
  - used in: Trial-based Heuristic Tree Search (THTS)

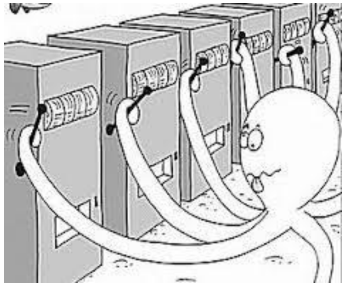
ϵ-greedy

- parameterized by ϵ



- flip a ϵ-biased coin
  - (ϵ): select arm a_i randomly with uniform probability and update μ_i
  - $(1-\epsilon)$: select estimated best arm a^* and update μ^*
- typically $\epsilon \approx \mathbf{0,1}$ (but this can vary depending on circumstances)
- exponential convergence to the optimal arm

Upper Confidence Bounds



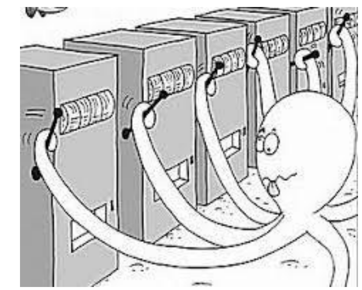
- UCB1 arm selection:
  - select arm a_i maximizing UCB1 formula:

$$\mu_i + \sqrt{\frac{2 \ln n}{n_i}}$$

  and update μ_i

  - n – times the state is visited; n_i – times the action is visited
  - μ_i – average reward from the previous plays
- exploration factor ensures to evaluate actions that are evaluated rarely
- only polynomial (but empirically fast) convergence to optimal arm

Sequential Halving



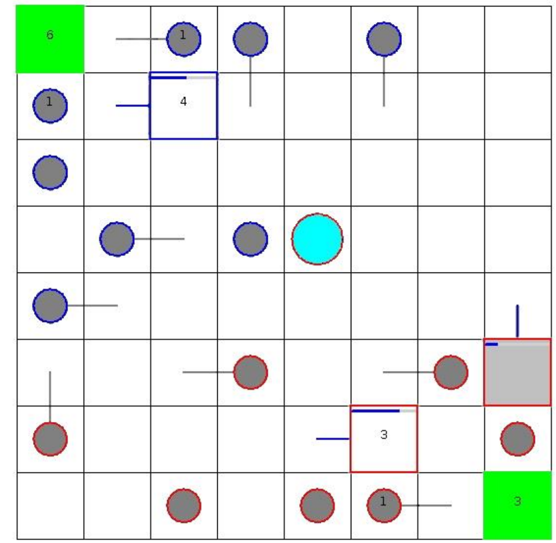
- parameterized by sampling budget T
- (1) begins with all arms as candidate arms S
- (2) sample/play all candidate arms in S t-times

$$t = \left\lfloor \frac{T}{|S|\lceil \log_2 k \rceil} \right\rfloor$$

  and update their μ_i

- (3) remove ⌊half⌋ of the candidate arms with lowest μ_i
- (4) until there is only one (resulting) candidate arm: goto (2)
- exponential convergence to the optimal arm (provided the budget is going to ∞; not any-time)

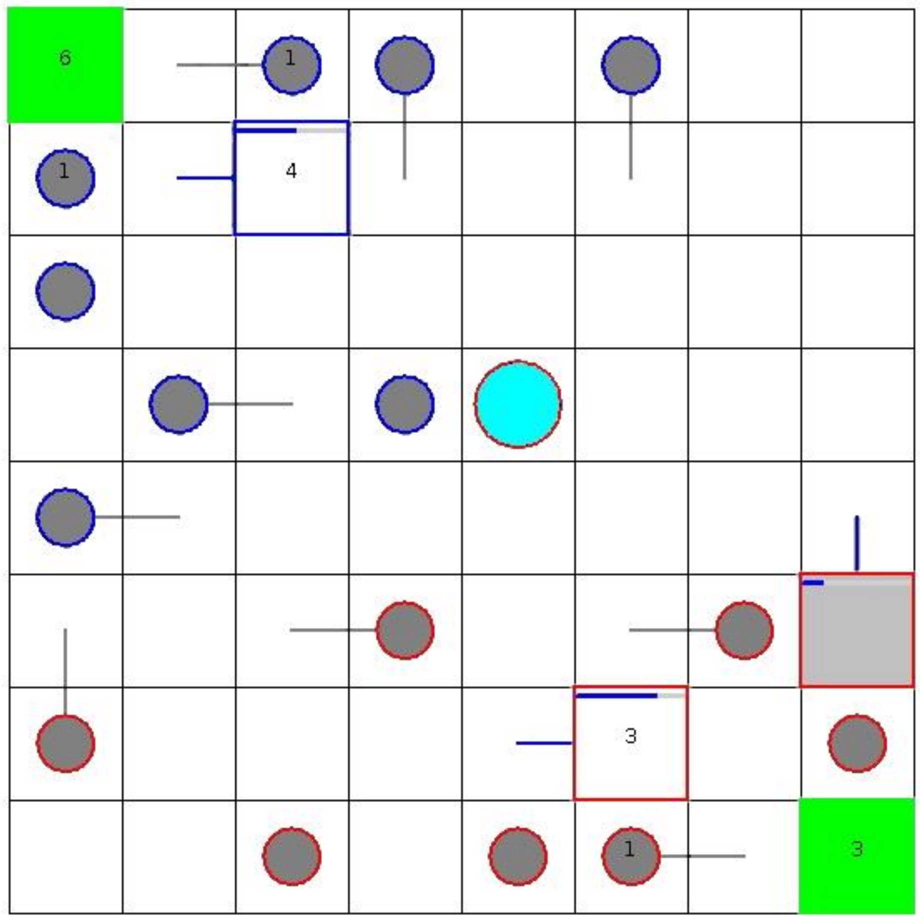
Combinatorial Multi-Armed Bandit Problem



- combination of actions (arms) has to be selected (some forbidden)
- reward defined over combinations of actions (c-actions)
- expectation of reward per c-action
- ☹ curse of dimensionality (action combinations), $O\left(\exp(|\Pi|)\right)$
- ☺ we can approximate
  - randomly generate candidate c-actions, pick the best one (NMC)
  - assume additive rewards for one c-action; linear-side inform. (LSI)

Combinatorial Multi-Armed Bandit Problem

- → sequential decision making (over different states): repeated MABs



Monte Carlo Tree Search (MCTS)

- sequential decision making (over different states)
- gradually grow the search tree
- two types of tree nodes
  - decision nodes (action selection) – the algorithm selects
  - chance nodes (world selection) – the world selects the outcome (in case of MDP model based on known probabilities)

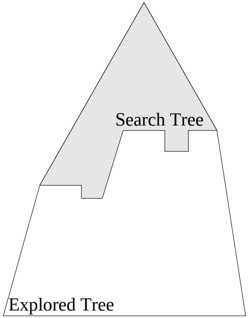
- returned solution: path (action from root) visited the most often

Monte-Carlo Tree Search

Kocsis Szepesvári, 06

Gradually grow the search tree:

- Iterate Tree-Walk
  - Building Blocks
    - Select next action
      Bandit phase
    - Add a node
      Grow a leaf of the search tree
    - Select next action bis
      Random phase, roll-out
    - Compute instant reward
      Evaluate
    - Update information in visited nodes
      Propagate
- Returned solution:
  - Path visited most often



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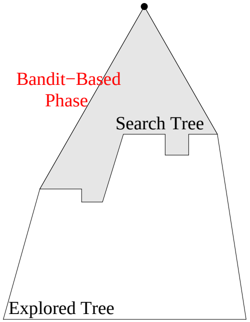

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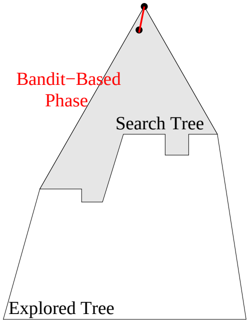

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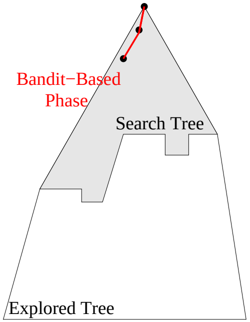

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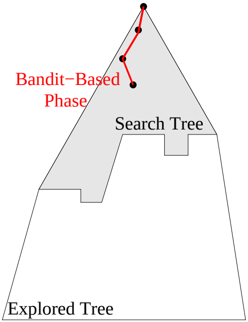

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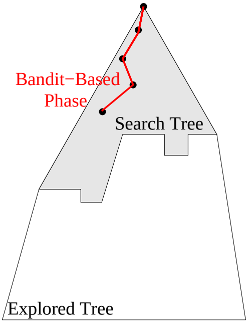

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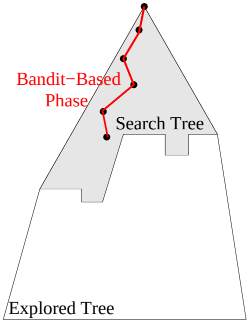

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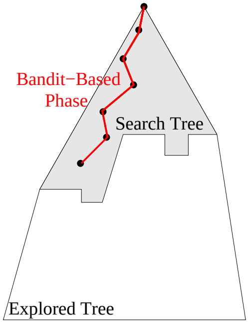

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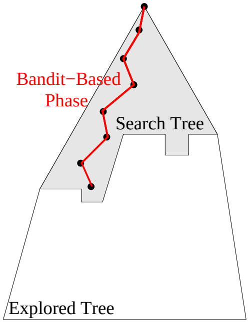

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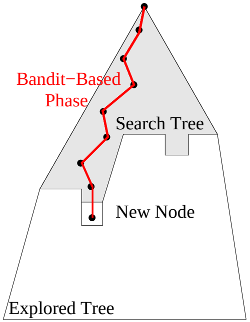

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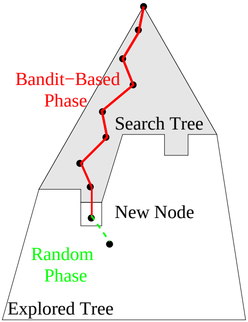

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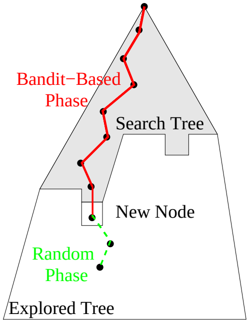

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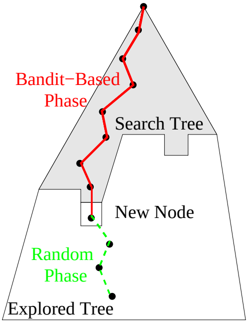

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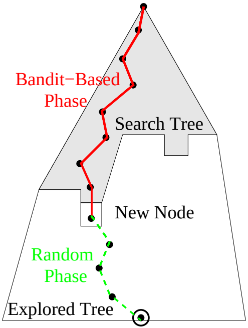
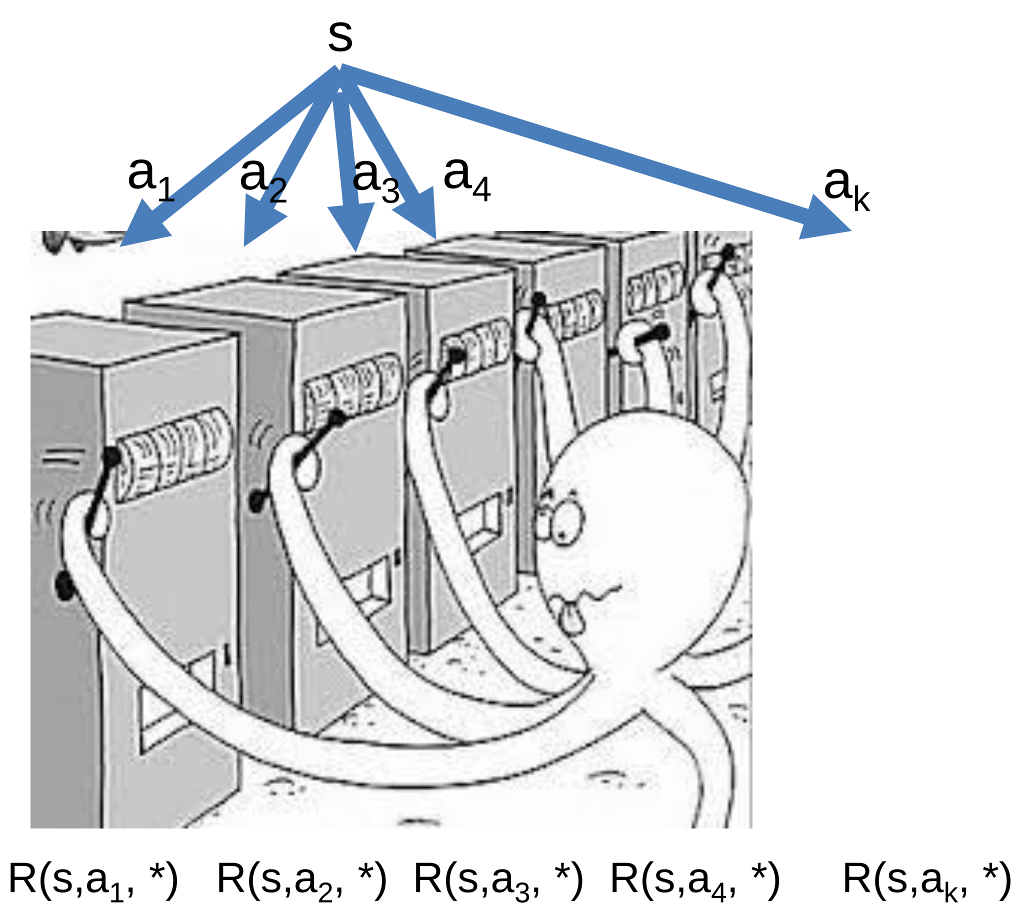


UCT – Principle

- UCB1 applied on trees – UCT



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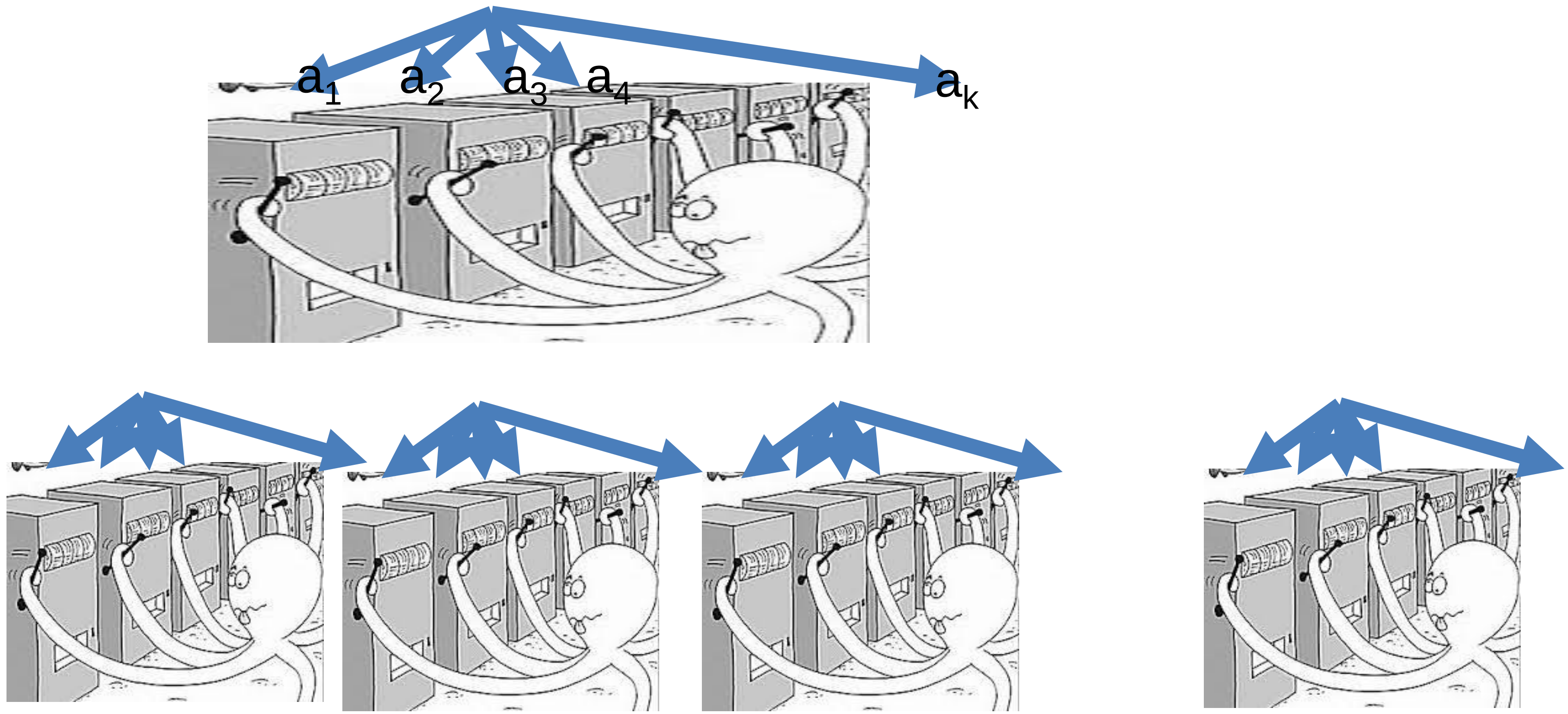
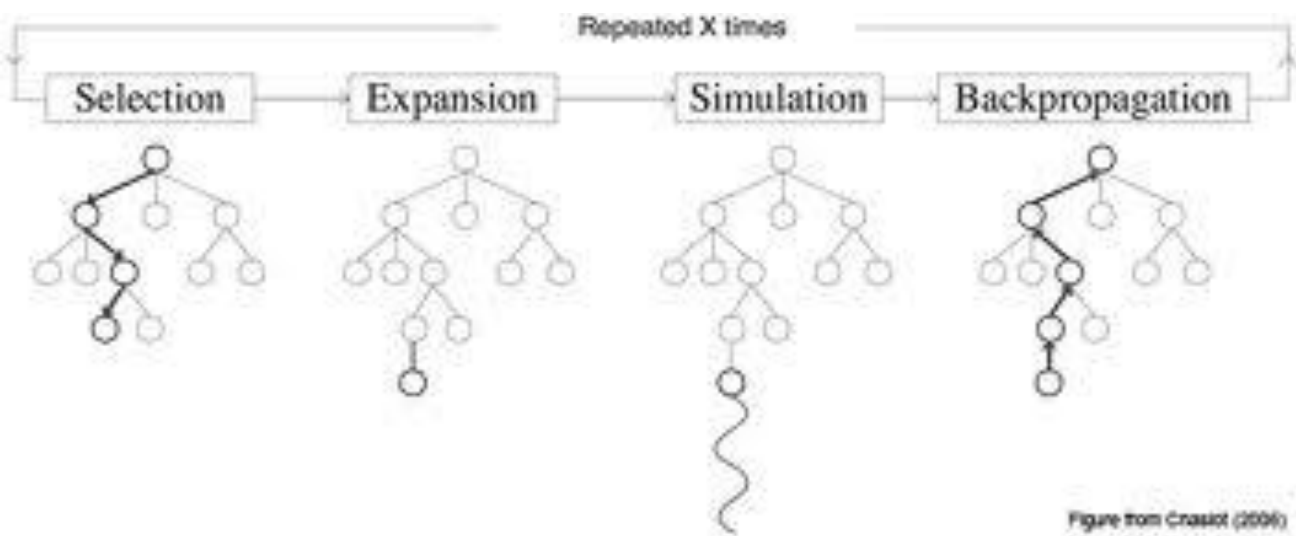


UCT – Phases

- UCB1 applied on trees – UCT
  - cumulative or simple regret?
  - why?
- using bandits in sequential decision making
- 4 phases from MCTS



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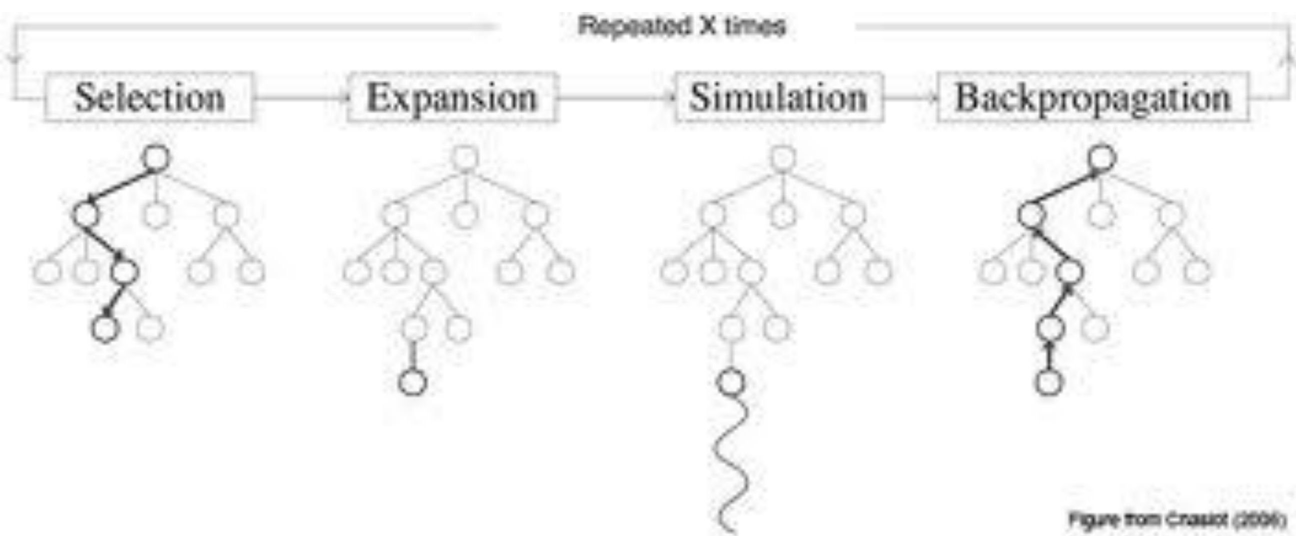
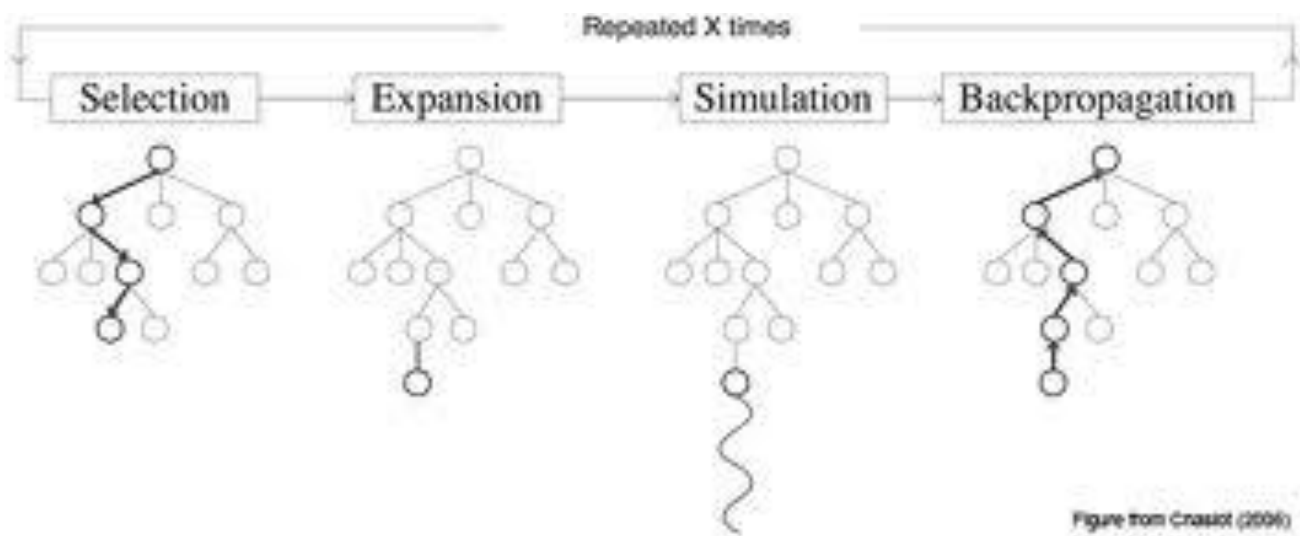


UCT – Phases

- UCB1 applied on trees – UCT
  - cumulative or simple regret?
  - why? → "it just works"
- using bandits in sequential decision making
- 4 phases from MCTS



UCT – Selection

- selection (UCB1)
  - for each action a_i applicable in s UCB selects the one that maximizes

$$c\sqrt{\frac{\ln n}{n_i}} + \sum_{s' \in S} T(s, a_i, s')[R(s, a_i, s') + \gamma V(s')]$$

  - n – times the state is visited; n_i – times the action is visited
  - $V(s)$ – average reward from the previous iterations
  - c - exploration constant (linear to expected utility)
- exploration factor ensures to evaluate actions that are evaluated rarely

UCT – Expansion, Simulation, Backup

- expansion (MCTS)
  - in a selection node where not all actions were yet sampled, expand (uniformly) randomly one of the new nodes
- simulation (MCTS)
  - (uniformly) randomly select actions in decision nodes
  - using the simulator based on the probabilities in the MDP simulate world behavior in the chance nodes MDP
- backup (MCTS)
  - updating μ_i^s for all search tree nodes along the trial based on the rewards (incl. the simulation)

Beyond UCT

- UCT is far from optimal algorithm
  - there exist simple examples where vanilla UCT performs bad
- number of reasons
  - learning the best action is different from learning the best (contingency) plan
  - situation that occur in states does not exactly correspond to multi-armed bandit (mathematically)
- there are modifications and improvements
  - RAVE (Gelly & Silver, 2007) → rapid action value estimate
  - THTS (Keller & Helmert, 2013) → MaxUCT, UCT*
  - many others …

Beyond UCT many others

- numbers of possible of improvements
- vanilla UCT is not that fast
- MCTS/UCT requires large number of iterations to converge
- depth-limited rollouts
- reducing branching factor (some actions are dominated → remove)
- different action selection principles
- improving rollout policy (biased simulators, "clever" decision nodes)
- incorporate prior knowledge
- parallelization

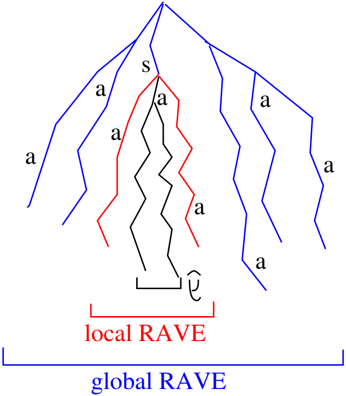
RAVE: Rapid Action Value Estimate

Gelly Silver 07

Motivation

- It needs some time to decrease the variance of $\hat{\mu}_{s,a}$
- Generalizing across the tree ?

$RAVE(s, a) =$
average $\{\hat{\mu}(s', a), s \ parent \ of \ s'\}$



Rapid Action Value Estimate, 2

Using RAVE for action selection

In the action selection rule, replace $\hat{\mu}_{s,a}$ by

$$\alpha \hat{\mu}_{s,a} + (1-\alpha)\left(\beta RAVE_\ell(s,a) + (1-\beta) RAVE_g(s,a)\right)$$

$\alpha = \frac{n_{s,a}}{n_{s,a}+c_1}$ $\beta = \frac{n_{parent(s)}}{n_{parent(s)}+c_2}$

Using RAVE with Progressive Widening

- PW: introduce a new action if $\lfloor \sqrt[b]{n(s)+1} \rfloor > \lfloor \sqrt[b]{n(s)} \rfloor$
- Select promising actions: it takes time to recover from bad ones
- Select argmax $RAVE_\ell(parent(s))$.

Trial-based Heuristic Tree Search (THTS)

- a common framework based on five ingredients:
  - heuristic function
  - backup function
  - action selection
  - outcome selection
  - trial length
- subsuming: MCTS, UCT, FIND-and-REVISE, AO* (AND/OR graph solver), Real-Time Dynamic Programming (RTDP), various heuristic functions (e.g., iterative deepening search)
- providing: MaxUCT, UCT*, …
- UCT* in PROST 2014 is currently best performing IPPC planner

Trial-based Heuristic Tree Search (THTS)

- Heuristic function
  - action value initialization (Q-value)

$$h: S \times A \mapsto \mathbb{R}$$

  - state value initialization (V-value)

$$h: S \mapsto \mathbb{R}$$

- Action selection
  - UCB1, ϵ-greedy, …
- Outcome selection
  - Monte Carlo sampling; outcome based on biggest potential impact

Trial-based Heuristic Tree Search (THTS)

- optimal policy derived from the Bellman optimality equation:

$$V^*(s) = \begin{cases} 0 & \text{if } s \text{ is terminal} \\ \max_{a \in A} Q^*(a,s) & \text{otherwise} \end{cases}$$

$$Q^*(a,s) = R(a,s) + \sum_{s' \in S} P(s'|a,s) \cdot V^*(s')$$

- Full Bellman backup ~ Bellman optimality equation, *k* trials
- Monte Carlo backup

$$V^k(s) = \begin{cases} 0 & \text{if } s \text{ is terminal} \\ \frac{\sum_{a \in A} n_{a,s} \cdot Q^k(a,s)}{n_s} & \text{otherwise} \end{cases}$$

$$Q^k(a,s) = R(a,s) + \frac{\sum_{s' \in S} n_{s'} \cdot V^k(s')}{n_{a,s}}$$

Trial-based Heuristic Tree Search (THTS)

Algorithm 1: The THTS schema.

THTS(MDP M, timeout T)**:**
- $n_0 \leftarrow$ getRootNode(M)
- **while** not solved(n_0) and time() $< T$ **do**
  - visitDecisionNode(n_0)
- **return** greedyAction(n_0)

visitDecisionNode(Node n_d)**:**
- **if** n_d was never visited **then** initializeNode(n_d)
- $N \leftarrow$ selectAction(n_d)
- **for** $n_c \in N$ **do**
  - visitChanceNode(n_c)
- backupDecisionNode(n_d)

visitChanceNode(Node n_c)**:**
- $N \leftarrow$ selectOutcome(n_c)
- **for** $n_d \in N$ **do**
  - visitDecisionNode(n_d)
- backupChanceNode(n_c)

Trial-based Heuristic Tree Search (THTS)

- maintains explicit tree of alternating decision and chance nodes
- selection phase
  - alternating visitDecisionNode and visitChangeNode
  - selection by selectAction and selectOutcome
  - tree traversing (down)
- expansion phase
  - when unvisited node encountered
  - added child node for each action
  - heuristics used to initialize the estimates
  - allows selection phase for new nodes

Algorithm 1: The THTS schema.

```
THTS(MDP M, timeout T):
    n0 ← getRootNode(M)
    while not solved(n0) and time() < T do
        visitDecisionNode(n0)
    return greedyAction(n0)

visitDecisionNode(Node nd):
    if nd was never visited then initializeNode(nd)
    N ← selectAction(nd)
    for nc ∈ N do
        visitChanceNode(nc)
    backupDecisionNode(nd)

visitChanceNode(Node nc):
    N ← selectOutcome(nc)
    for nd ∈ N do
        visitDecisionNode(nd)
    backupChanceNode(nc)
```

Trial-based Heuristic Tree Search (THTS)

- selection and expansion phases alternate until the trial length
- backup phase (backupDecisionNode & backupChanceNode)
  - all selected nodes are updated in reverse order
  - when another selected, but not yet visited → selection phase
  - a trial ends when the backup is called on the root node
  - tree backing (up)
- the process is repeated until the timeout T allows for another trial
- highest expectation action is returned greedyAction

Algorithm 1: The THTS schema.

```
THTS(MDP M, timeout T):
    n_0 ← getRootNode(M)
    while not solved(n_0) and time() < T do
        visitDecisionNode(n_0)
    return greedyAction(n_0)

visitDecisionNode(Node n_d):
    if n_d was never visited then initializeNode(n_d)
    N ← selectAction(n_d)
    for n_c ∈ N do
        visitChanceNode(n_c)
    backupDecisionNode(n_d)

visitChanceNode(Node n_c):
    N ← selectOutcome(n_c)
    for n_d ∈ N do
        visitDecisionNode(n_d)
    backupChanceNode(n_c)
```

MaxUCT

- backup function
  - action-value by Monte Carlo backup ($Q^k(s)$)
  - state-value by Full Bellman backup ($V^*(s)$)
- action selection → UCB1
- outcome selection → Monte Carlo sampling (MDP based)
- heuristic function → N/A
- trial length → UCT (horizon length, i.e. to leafs)

UCT*

- backup function
  - Partial Bellman backup
    (weighted proportionally to subtree probability)
- action selection → UCB1
- outcome selection → Monte Carlo sampling (MDP based)
- heuristic function → Iterative Deepening Search (depth: 15)
- trial length → explicit tree length +1
  (only initialized new nodes using heuristics)
- resembles classical heuristic Breadth-First-Search (rather than UCT Depth-First-Search)