

Adversarial Search

based on Stuart Russel's slides (<http://aima.cs.berkeley.edu>)

Outline

- ◇ Games
- ◇ Perfect play
 - minimax decisions
 - α - β pruning
- ◇ Resource limits and approximate evaluation

Games vs. search problems

“Unpredictable” opponent \Rightarrow solution is a **strategy**
specifying a move for every possible opponent reply

Time limits \Rightarrow unlikely to find goal, must approximate

Plan of attack:

- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

Types of games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

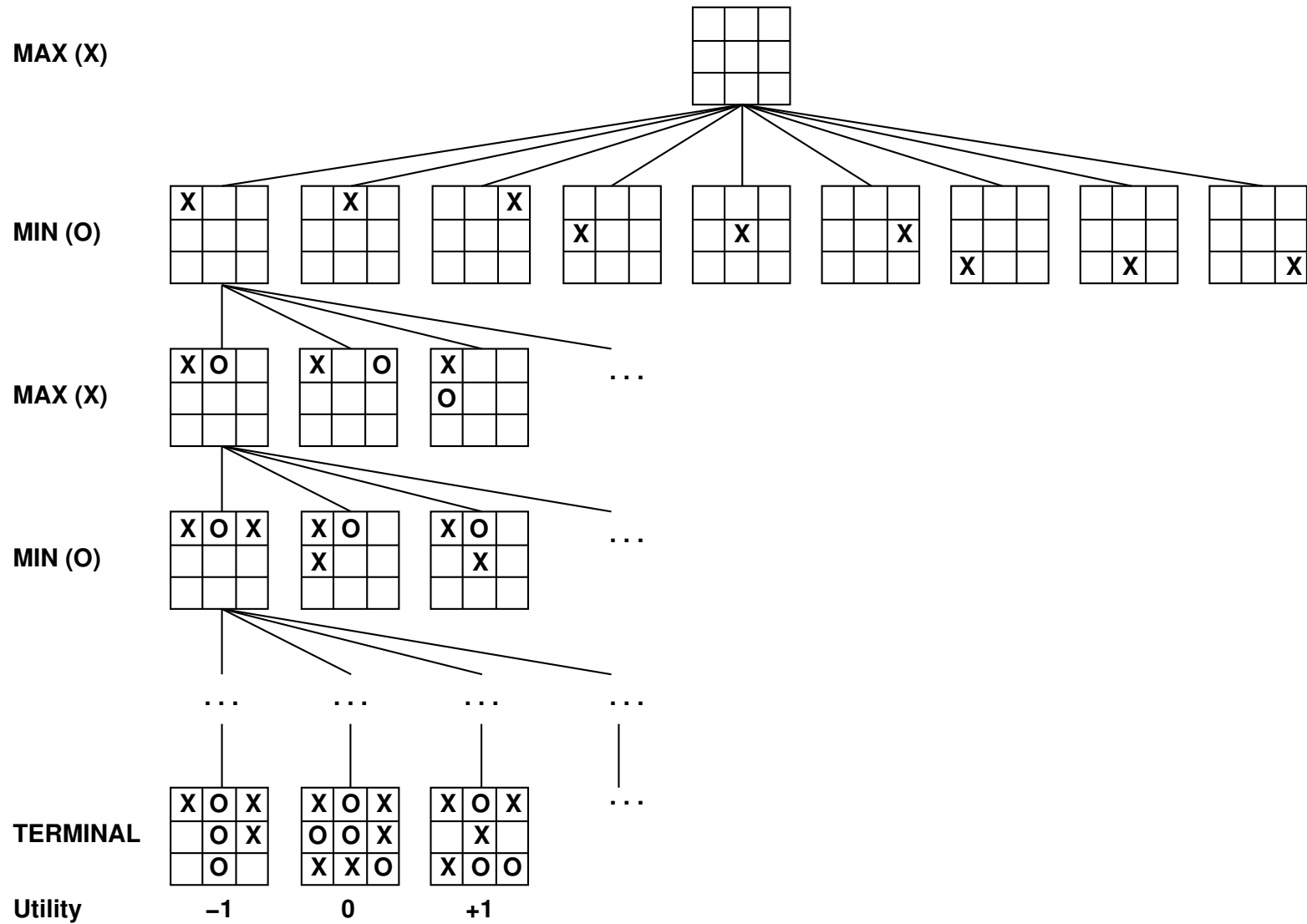
Game elements

$\text{PLAYER}(s)$, $\text{ACTIONS}(s)$, $\text{RESULT}(s, a)$, $\text{TERMINAL-TEST}(s)$

$\text{UTILITY}(s, p)$

Zero(Constant)-sum games

Game tree (2-player, deterministic, turns)

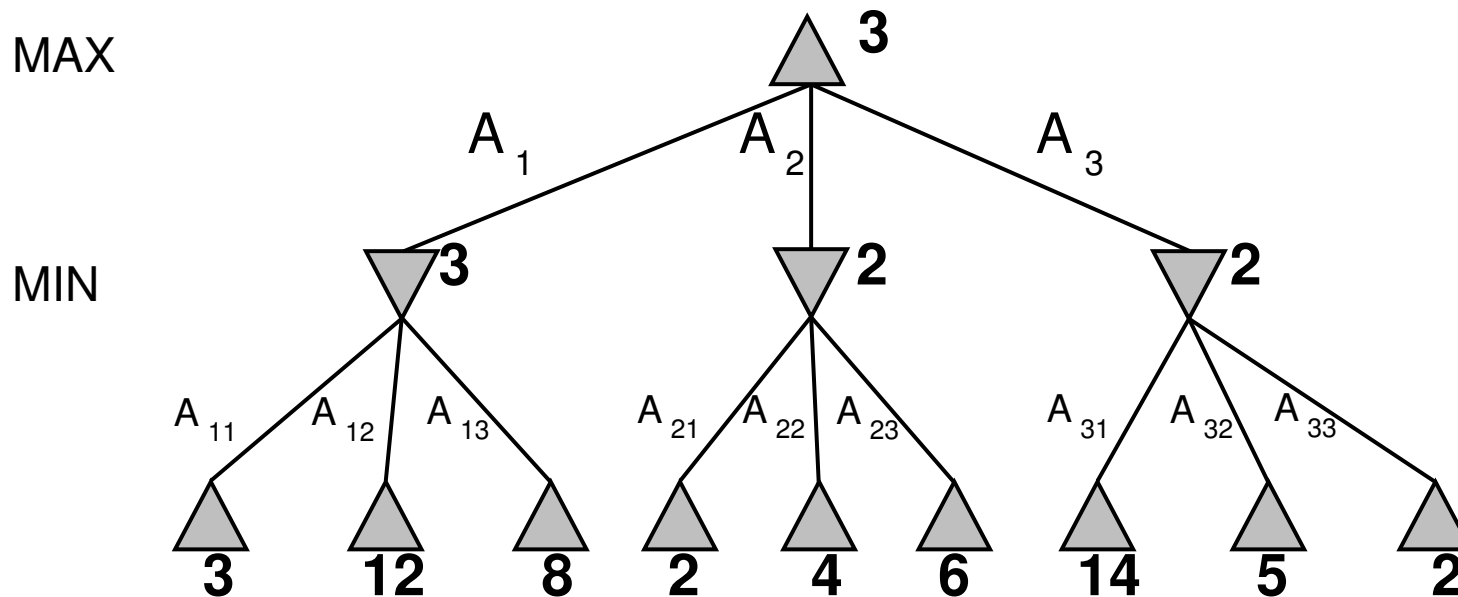


Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest **minimax value**
= best achievable payoff against best play

E.g., 2-ply game:



Properties of minimax

Complete??

Properties of minimax

Complete?? Only if tree is finite (chess has specific rules for this).
NB a finite strategy can exist even in an infinite tree!

Optimal??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity?? $O(b^m)$

Space complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

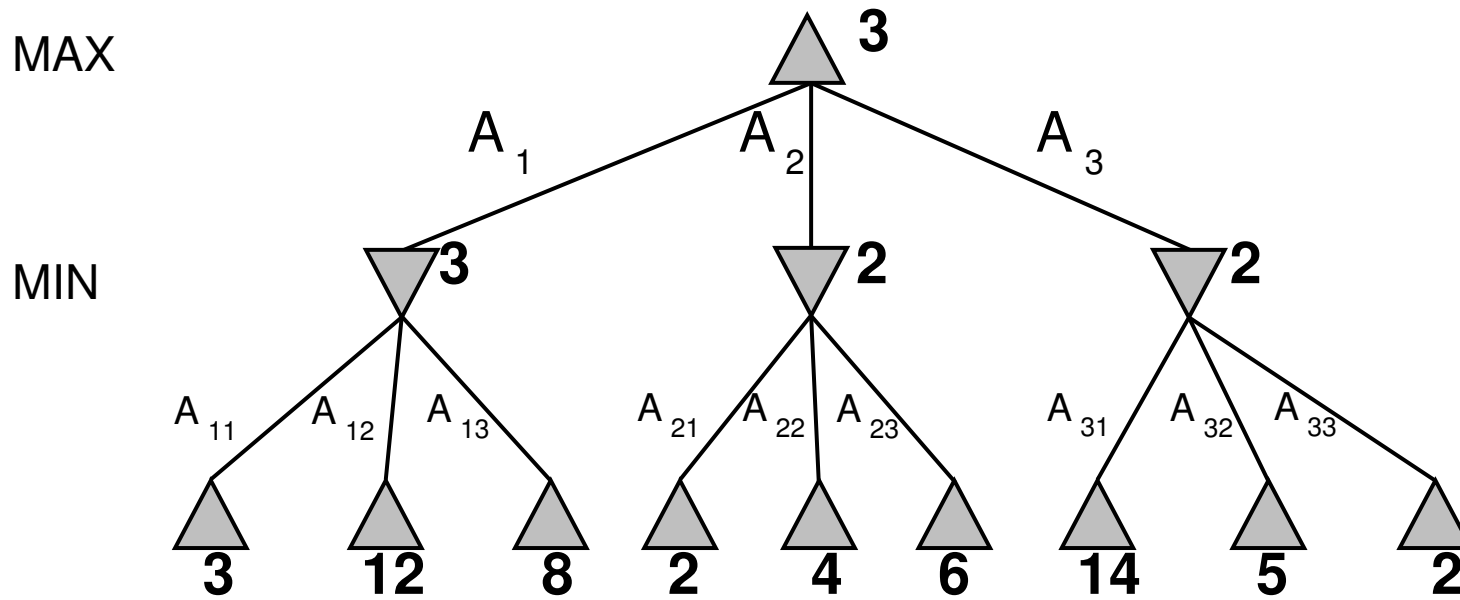
Time complexity?? $O(b^m)$

Space complexity?? $O(bm)$ (depth-first exploration)

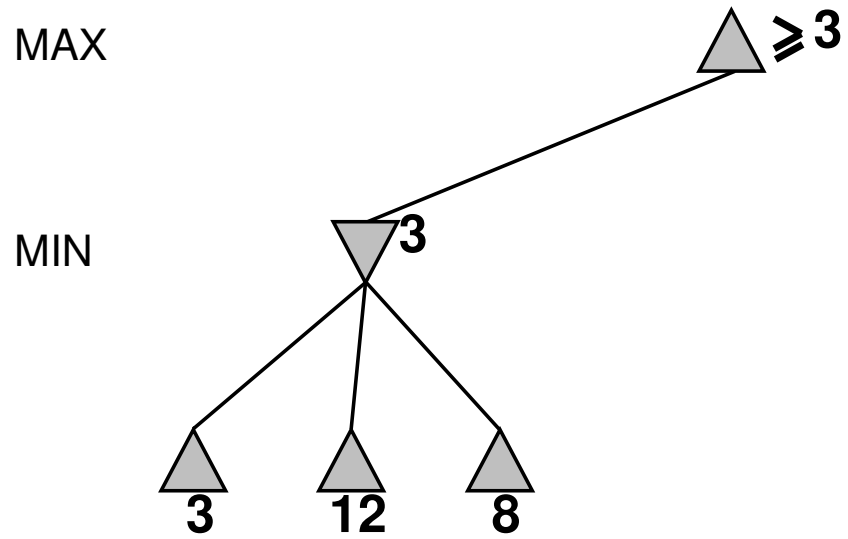
For chess, $b \approx 35$, $m \approx 100$ for “reasonable” games
⇒ exact solution completely infeasible

But do we need to explore every path?

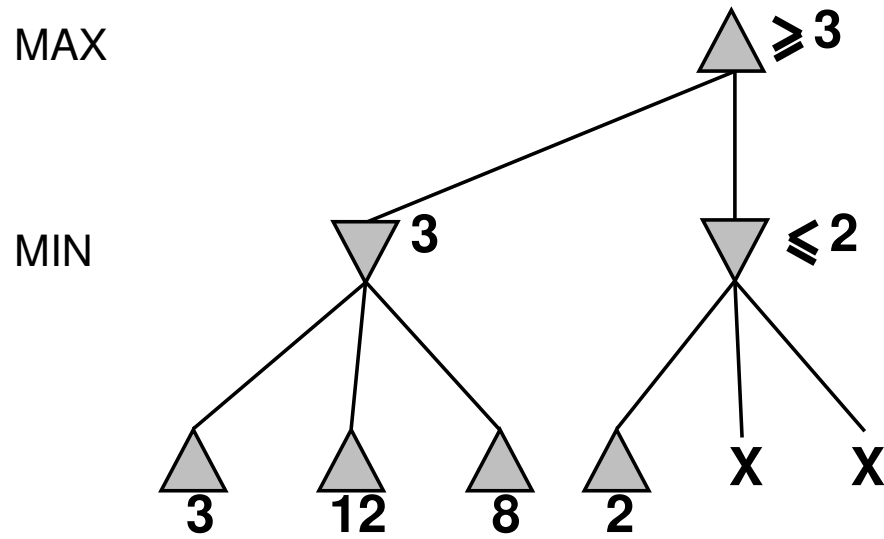
α - β pruning example



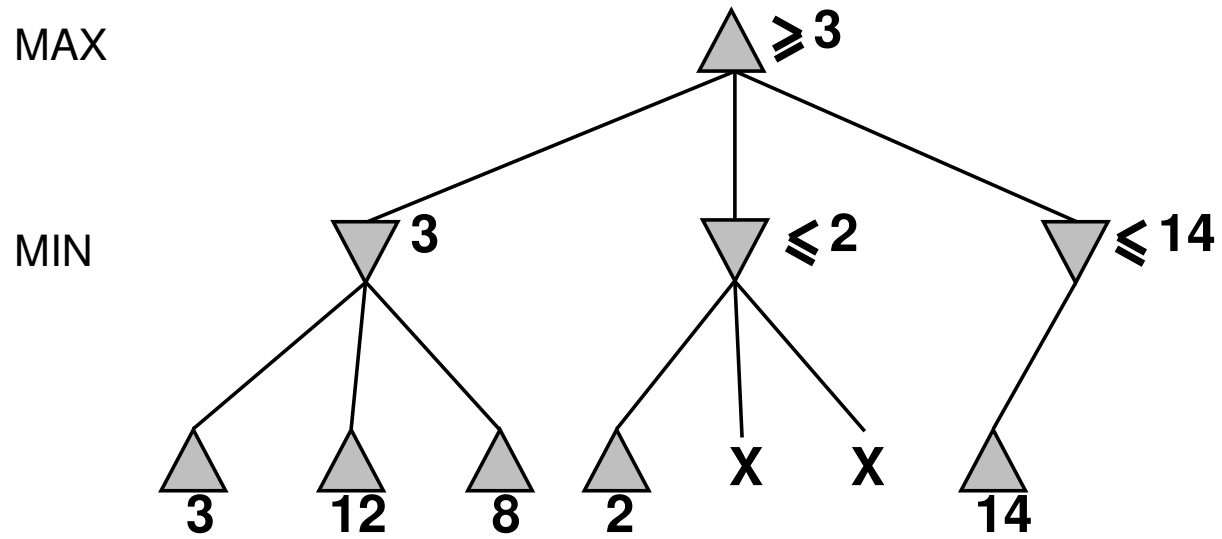
α - β pruning example



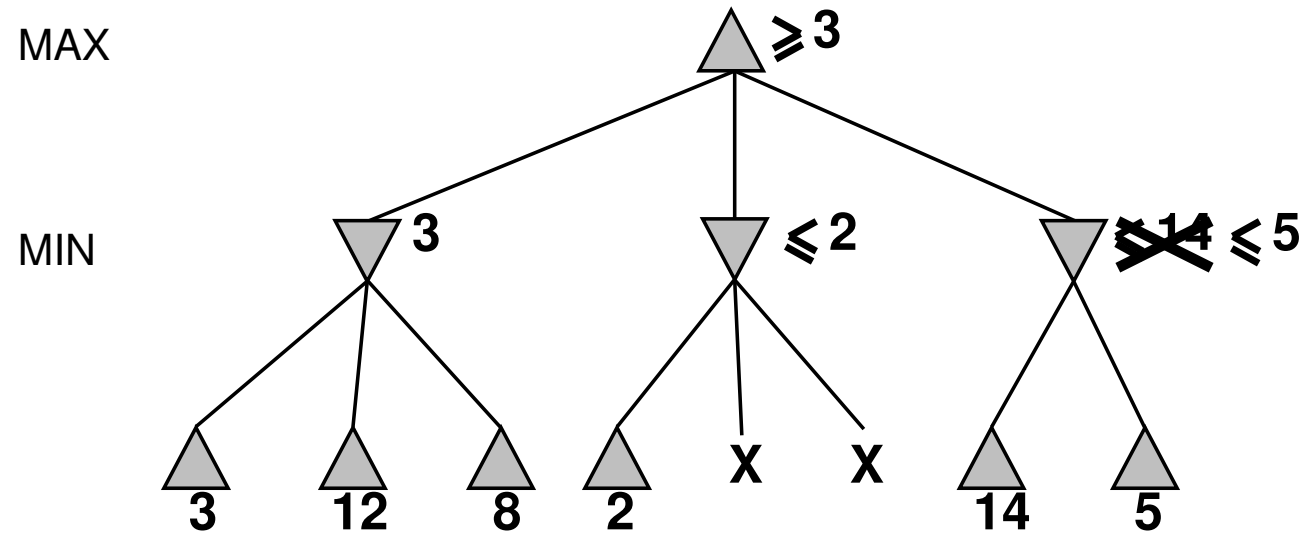
α - β pruning example



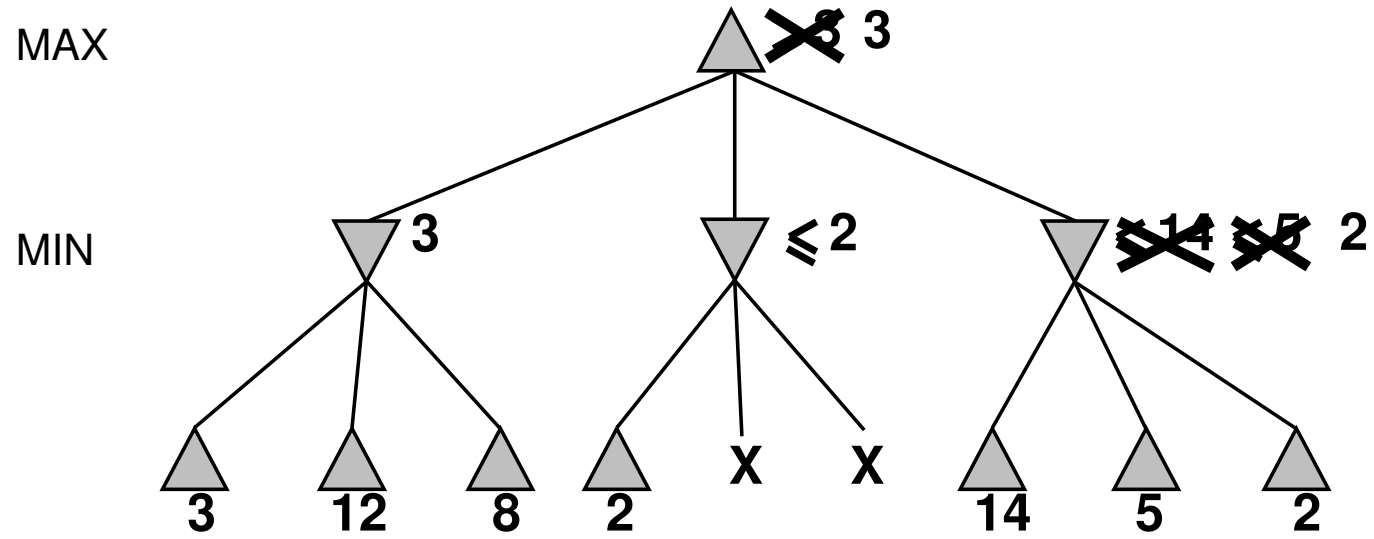
α - β pruning example



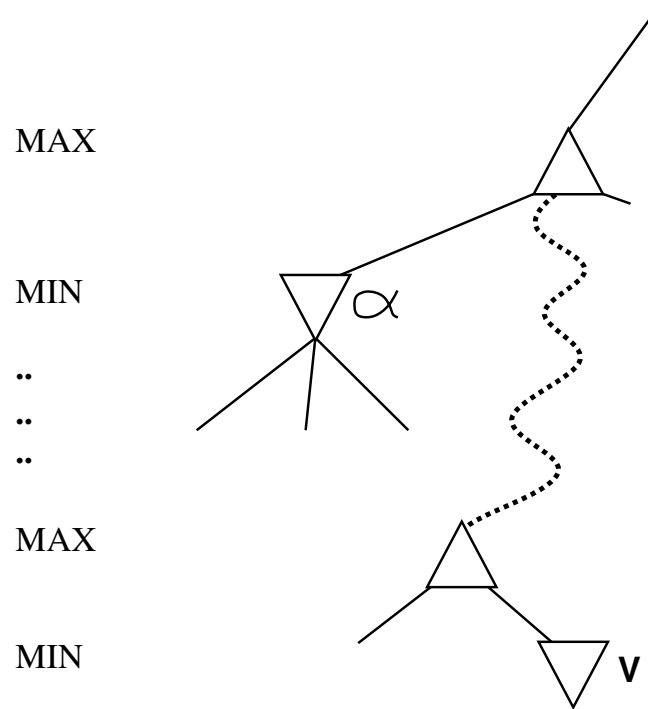
α - β pruning example



α - β pruning example



Why is it called α - β ?



α is the best value (to MAX) found so far off the current path

If v is worse than α , MAX will avoid it \Rightarrow prune that branch

Define β similarly for MIN

Properties of α - β

Pruning does not affect final result

Good move ordering improves effectiveness of pruning

With “perfect ordering,” time complexity = $O(b^{m/2})$

⇒ doubles solvable depth

A simple example of the value of reasoning about which computations are relevant (a form of **metareasoning**)

Unfortunately, 35^{50} is still impossible!

Resource limits

Standard approach:

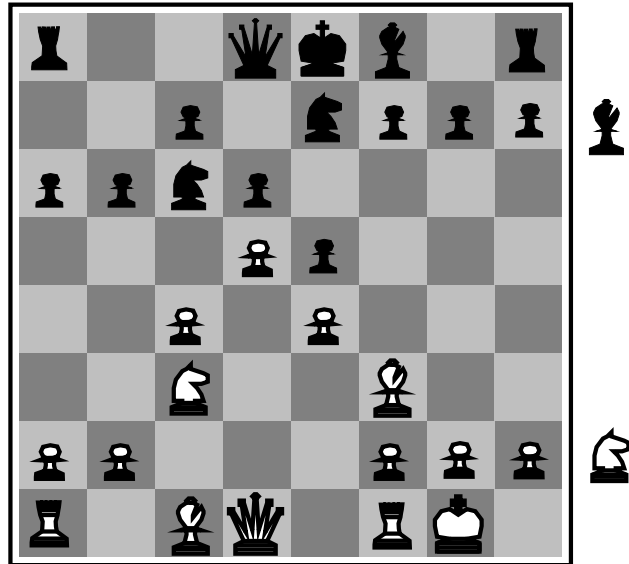
- Use CUTOFF-TEST instead of TERMINAL-TEST
e.g., depth limit (perhaps add quiescence search)
- Use EVAL instead of UTILITY
i.e., evaluation function that estimates desirability of position

Suppose we have 100 seconds, explore 10^4 nodes/second

$\Rightarrow 10^6$ nodes per move $\approx 35^{8/2}$

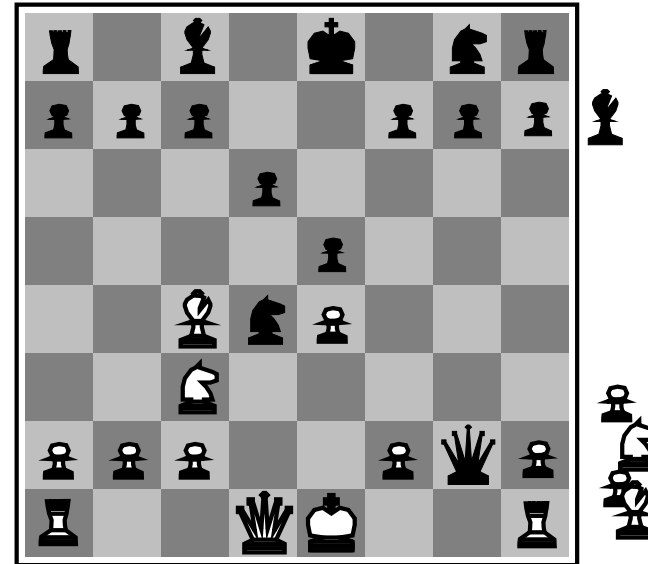
$\Rightarrow \alpha\text{-}\beta$ reaches depth 8 \Rightarrow pretty good chess program

Evaluation functions



Black to move

White slightly better



White to move

Black winning

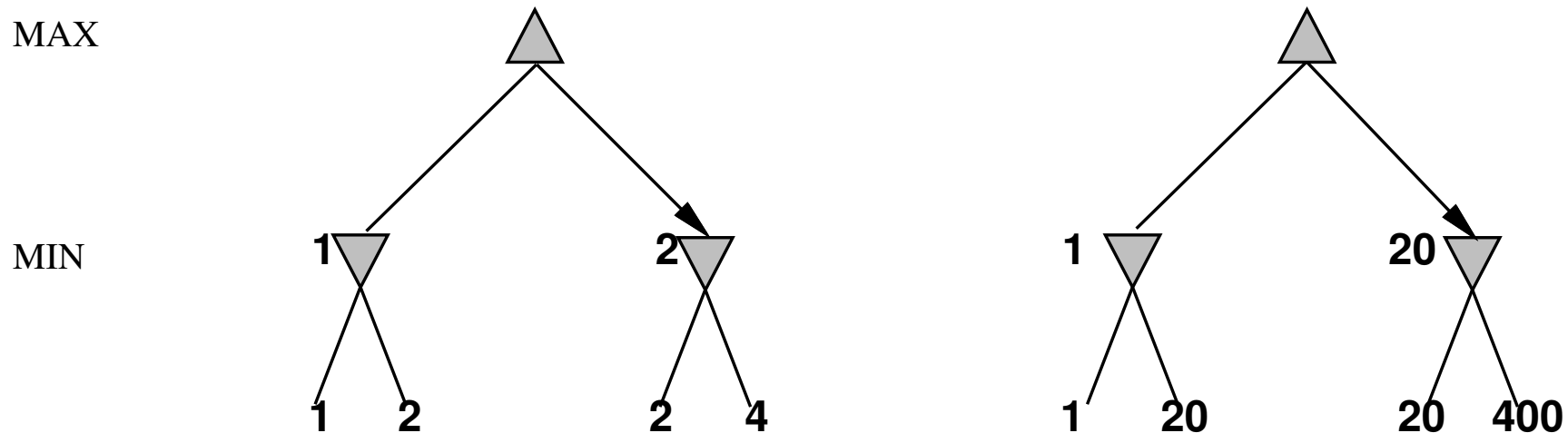
For chess, typically **linear** weighted sum of **features**

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

e.g., $w_1 = 9$ with

$f_1(s) = (\text{number of white queens}) - (\text{number of black queens}), \text{ etc.}$

Digression: Exact values don't matter



Behaviour is preserved under any monotonic transformation of $EVAL$

Only the order matters:

payoff in deterministic games acts as an **ordinal utility** function

Deterministic games in practice

Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

Othello (Reversi): human champions refuse to compete against computers, who are too good.

~~Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.~~

Not any more! See, <https://en.wikipedia.org/wiki/AlphaGo>

Poker game: <http://www.fel.cvut.cz/en/aktuality/deepstack-poker.html>

Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI

- ◇ perfection is unattainable \Rightarrow must approximate
- ◇ good idea to think about what to think about
- ◇ optimal decisions depend on information state, not real state

Games are to AI as grand prix racing is to automobile design

See also the complementary slides, for algorithm details.