Geometry of Computer Vision and Graphics

T O M Á Š P A J D L A

C T U P R A G U E

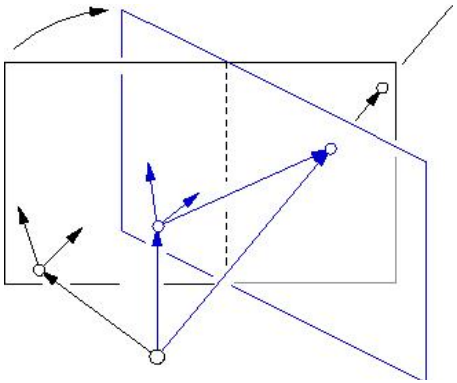
2 0 1 4

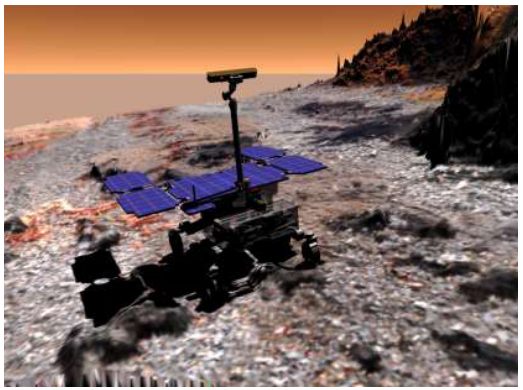
pajdla@cmp.felk.cvut.cz

T O M A S P A J D L A

Center for Machine Perception
Department of Cybernetics
Czech Technical University Prague
pajdla@cmp.felk.cvut.cz

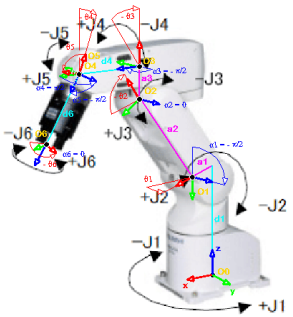
Research

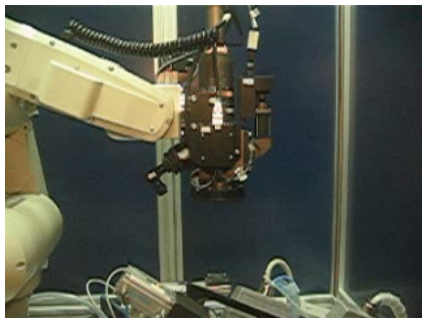


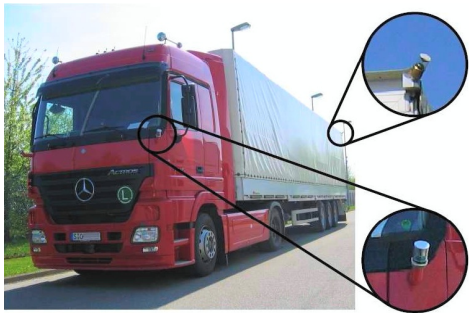


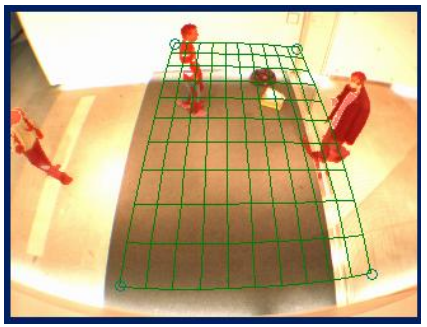
Applications

Neovision s.r.o., Daimler, Siemens, ...









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[[courses:a4m33gvg:start]] COURSE WARE

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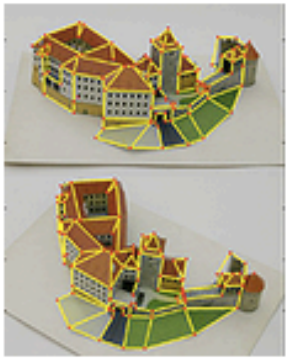
Annotation Schedule Students Submission system Forum

Geometry of Computer Vision and Graphics (Summer 2014)



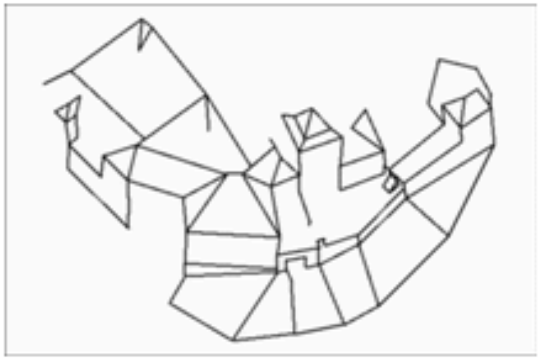


Table of Contents

He who loves practice without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast. — Leonardo Da Vinci (1452–1519)

And since geometry is the right foundation of all painting, I have decided to teach its rudiments and principles to all youngsters eager for art. — Albrecht Durer (1471–1528)

As for everything else, so for a mathematical theory: beauty can be perceived but not explained. — Arthur Cayley (1821–1895)

Edit

Content

We will explain the basics of Euclidean, Affine and Projective geometry and show how to measure distances and angles in a scene from its images. We will introduce a model of the perspective camera, explain how images change when moving a camera and show how to find the camera pose from images. We will demonstrate the theory in practical tasks of panorama construction, finding the camera pose, adding a virtual object to a real scene and reconstructing a 3D model of a scene from its images. We will be building on our previous knowledge of Linear algebra and will provide fundamentals of geometry for computer vision, computer graphics, image processing and object recognition in images.

The goal

1. Fundamentals of the geometry of the space and cameras using Linear algebra
2. Practical problems that can't be solved without the above theory
3. More advanced geometry in engineering

The method

1. Mathematical approach
2. Practical tasks
3. Examples, exercises, quizes

pajdla@cmp.felk.cvut.cz

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Lectures: Monday 14:30–16:00, KN:G–205

Lecturer: Tomáš Pajdla

Week	Date	Content
01	17.2.	Intro: Geometry of CV & G, LA L[2.1], image coordinate system L[5]
02	24.2.	Mathematical model of the perspective camera L[6]
03	03.3.	Camera calibration L[7] and pose L[4]
04	10.3.	Calibrated camera pose computation I L[7.2, 7.3]
05	17.3.	Calibrated camera pose computation II L[7.3–par.38], affine space I L[3]
06	24.3.	Homography L[8], affine space II L[3]
07	31.3.	Affine space III L[3] and projective plane L[9]
08	07.4.	Camera calibration from vanishing points L[11]
09	14.4.	Epipolar geometry L[12.1–12.3]
10	21.4.	*Easter Monday*
11	28.4.	3D reconstruction with a calibrated camera L[12.4]
12	05.5.	Calibrated camera motion computation L[12.5]
13	12.5.	Dual space and tensor product L[2.3]
14	19.5.	3D Reconstruction from Images – motivation & examples

Edit

Exercises: Monday 16:15–17:45, 18:00–19:30 in KN:E–230

Teachers: Martin Matoušek, Čeněk Albl,

Details about exercises (technical content and assessment) are in the separate section Labs.

Exercises: Monday 16:15–17:45, 18:00–19:30 in KN:E-230

Teachers: Martin Matoušek, Čeněk Albl,

Details about exercises (technical content and assessment) are in the separate section Labs.

Contacts

Lectures: Tomáš Pajdla	Labs: Zuzana Kúkelová	Labs: Martin Matoušek	Labs: Čeněk Albl
pajdla@cmp.felk.cvut.cz	kukelova@cmp.felk.cvut.cz	xmatousm@cmp.felk.cvut.cz	
KN, room G 104A	KN, room G 104	KN, room G 103	KN, room G 104
tel. (22435) 7348	tel. (22435) 5725	tel. (22435) 7305	tel. (22435) 5724
			

A4M33GVG - Geometrie počítačového vidění a grafiky (2+2c) - sudý a lichý kalendářní týden																
hodina	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
čas	7:30 - 9:00		9:15 - 10:45		11:00 - 12:30		12:45 - 14:15		14:30 - 16:00		16:15 - 17:45		18:00 - 19:30		20:30	
Pondělí									KN:G-205 - Pře T. Pajdla, Z. Kúkelová 1(27 stud.)		KN:E-230 - Cvi M. Matoušek, Č. Albl 102(11 stud.)		KN:E-230 - Cvi Č. Albl, M. Matoušek 101(16 stud.)			

Poř. číslo	Příjmení a jméno studenta	Ročník a kruh	Docházka																		Zápočet	Login / Email
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15					
1	Balga Matěj	1/--																				balgamat@fel.cvut.cz
2	Bužková Lucie	1/--																				buzkolu2@fel.cvut.cz
3	Cajthaml Jan	1/--																				cajthjan@fel.cvut.cz
4	Černohorská Veronika	1/--																				cernover@fel.cvut.cz
5	Doležal Jiří	1/--																				dolezji6@fit.cvut.cz
6	Hašlarová Kateřina	2/--																				haslakat@fel.cvut.cz
7	Hotovec Michal	1/--																				hotovmic@fel.cvut.cz
8	Langr Jan	1/--																				langrjan@fel.cvut.cz
9	Lendvorský Lukáš	1/--																				lendvluk@fit.cvut.cz
10	Lukášek Martin	1/--																				lukasma2@fel.cvut.cz
11	Musilová Renata	1/--																				musilren@fel.cvut.cz
12	Pokorný Jiří	1/--																				pokorj21@fel.cvut.cz
13	Polic Michal	1/--																				policmic@fel.cvut.cz
14	Šedivý Martin	1/--																				sedivma5@fel.cvut.cz
15	Šefčík Petr	1/--																				sefcipe2@fit.cvut.cz
16	Šerých Jonáš	2/45																				serycjon@fel.cvut.cz
17	Šimko Branislav	1/--																				simkobra@fel.cvut.cz
18	Slavíček Tomáš	2/--																				slavito3@fel.cvut.cz
19	Smetana Radek	3/--																				smetarad@fel.cvut.cz
20	Smrček Antonín	1/--																				smrceant@fel.cvut.cz
21	Špičková Martina	1/--																				spickma1@fel.cvut.cz
22	Týč Ladislav	1/--																				tycladis@fit.cvut.cz
23	Tyle Václav	1/--																				tylevacl@fel.cvut.cz
24	Ulrichová Pavlína	1/--																				ulricpav@fel.cvut.cz
25	Veit Matěj	3/--																				veitmate@fel.cvut.cz
26	Vlček Vladimír	1/--																				vlcekvl2@fel.cvut.cz
27	Vondráček Dominik	1/--																				vondrdom@fel.cvut.cz

Lecture and labs

1. Lectures and labs complement each other

2. The theory explained in the lecture is used in the labs to solve practical problems

3. Labs in front of the white board as well as using a computer

4. 4 tasks in 12 home works

    1. one week for solving a home work
    2. submissions end on Monday 6:00
    3. partially automatic evaluation (repeating submission allowed) evaluates the results of computation
    4. plots and text evaluated by teaching assistants
    5. Evaluations till Wednesday 23:59

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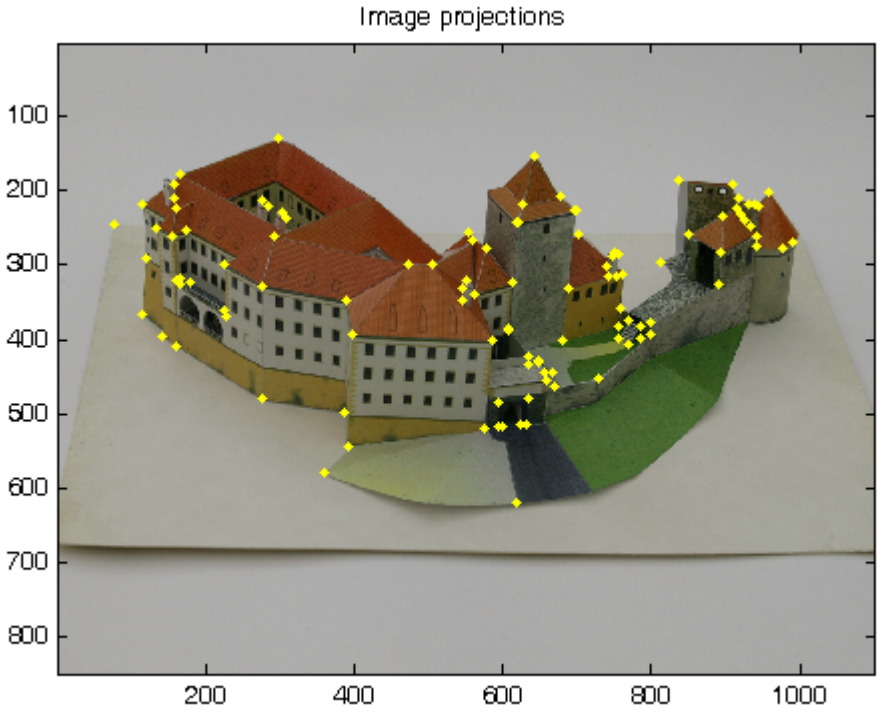
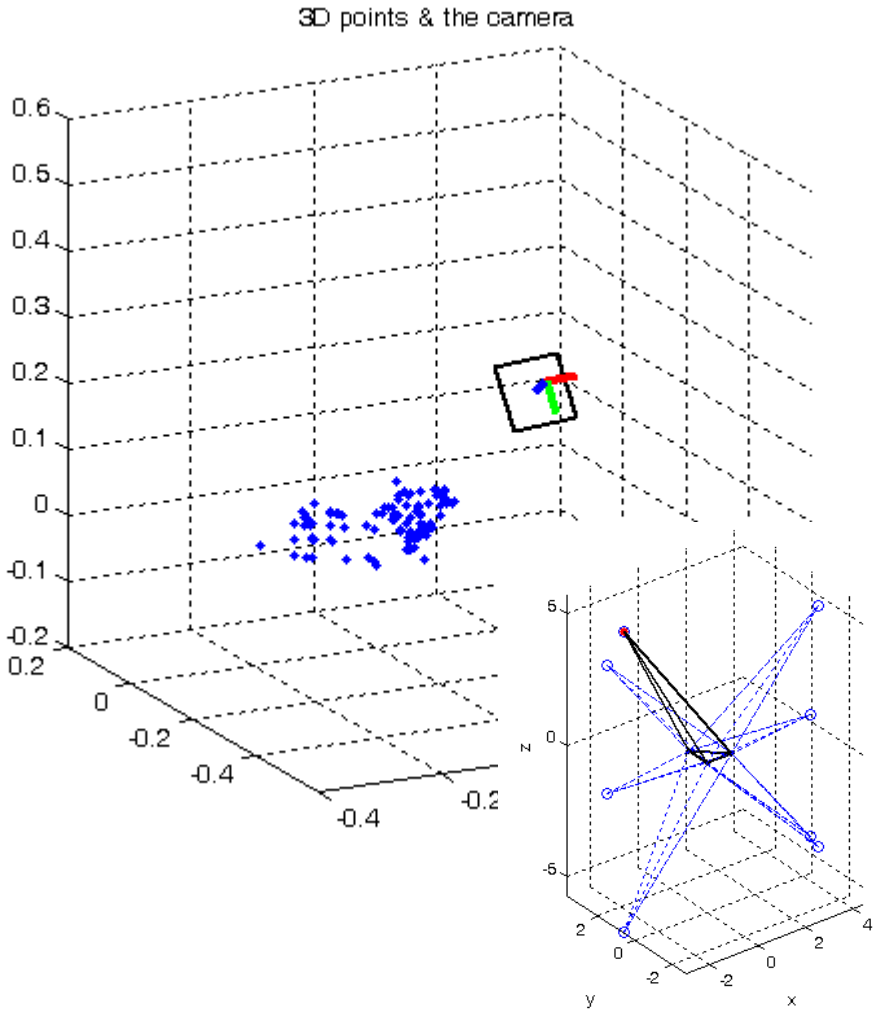
Task – Perspective camera in space





Demonstrates (DLT, P3P)

Afinne space, coordinate systems, solving systems of linear equations, eigenvalues and numerical solution to algebraic equation in one unknown, perspective projection

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Task: Homography

Homography between scene and image plane

Homography between images from a rotating camera

Demonstrates (homografie)

Linear mapping, the change of coordinates induced by the change of bases

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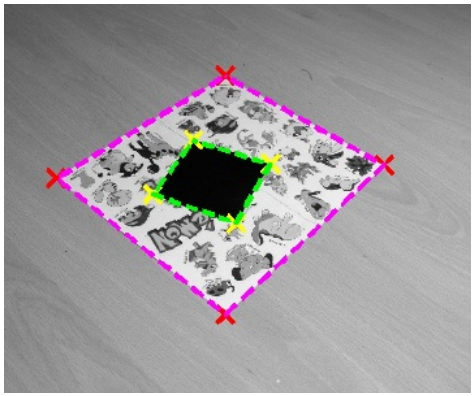
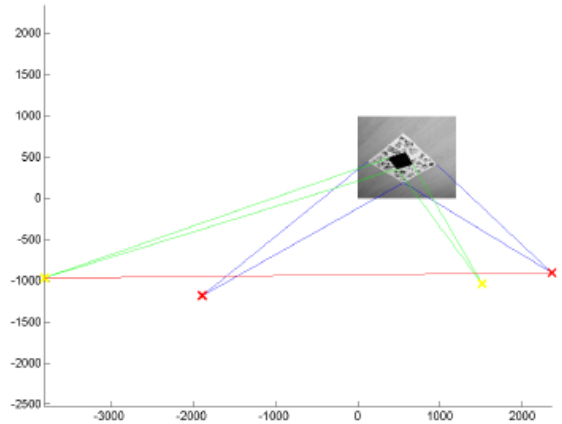
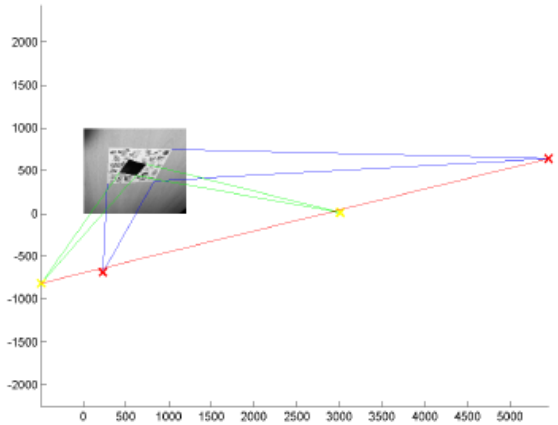
Task: Virtual object







Demonstrates (vanishing points, auto-calibration)

Linear mapping, change of coordinates, angular measurements in different using scalar products, points and lines at infinity

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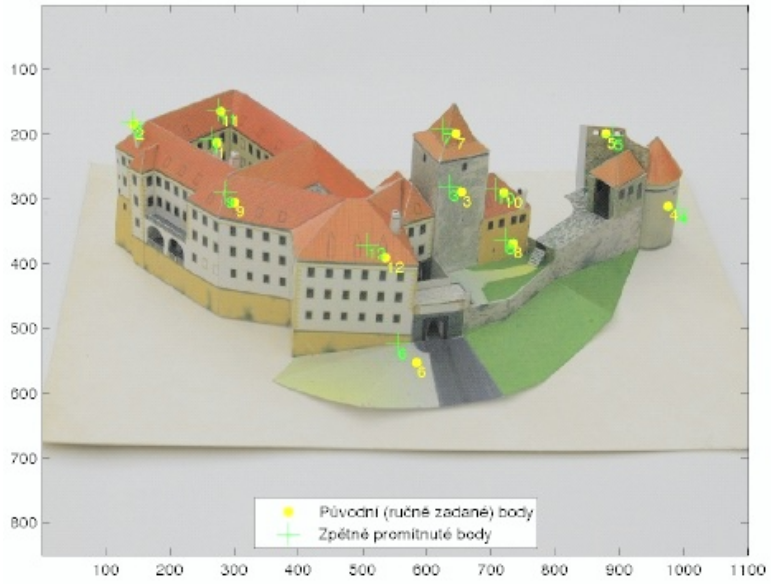
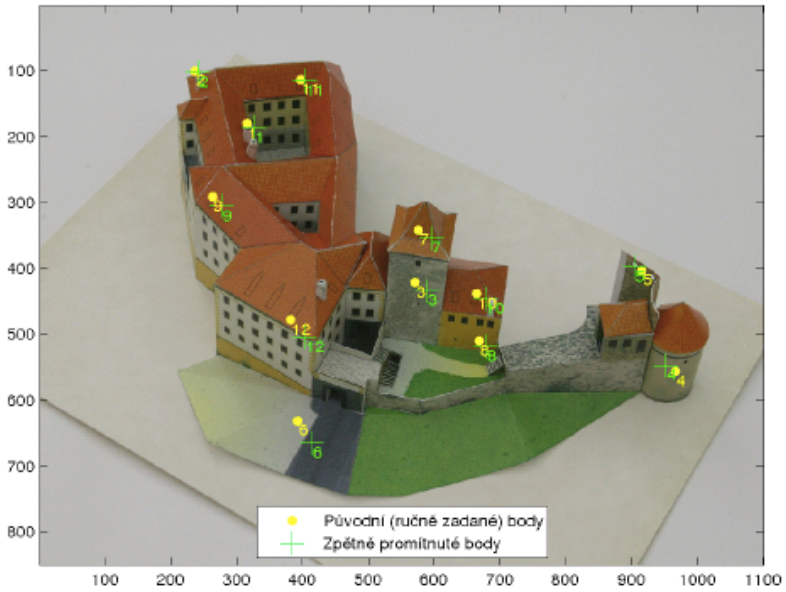
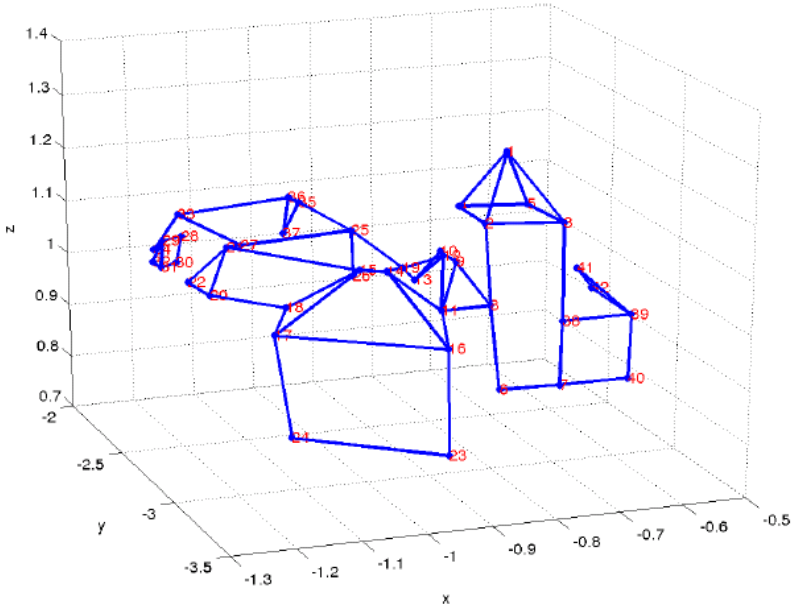
Task: 3D Scene reconstruction







Demonstrates (3D reconstruction from 2 images)

Linear mapping, choosing a coordinate system,
projective space, epipolar geometry,
3D reconstruction

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Assesment (zápočet)

Grading: (Grading 2013)

1. All home works must be submitted and accepted (0 in the column ~HW)
2. At least 50% of points in total from the home works (at least 0,5 in the column HW).
3. At least 50% of points in total from the tests (at least 0,5 in the column T).
4. Regular submission of home works ends on 27 May 2013. Later submissions are possible only by agreement with teaching assistants.
5. All the above conditions have to be fulfilled and the results have to be recorded in the Submission system before coming to the exam.

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Exam

The exam consists of a written and an oral part. It is required to achieve at least 50% of points from the written exam to be admitted to the oral exam.

Exam content:

1. **Linear algebra:** linear space, basis, coordinates, linear dependence/independence, matrices, rank, determinant, eigenvalues and eigenvectors, solving systems of linear equations, Frobenius theorem and linear independence, linear function, affine function, linear mapping and its matrix, computing roots of a polynomial via eigenvalues of its companion matrix, dual space, dual basis, change of the dual basis corresponding to a change of a basis, vector product and derived linear mappings.
2. **Course material:** GVG-2014-Lecture.pdf.

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Rules

1. **Lecture:** It is very **difficult** to pass the course without attending lectures.
2. **Labs:** It is **impossible** to pass the course without attending labs.
3. **Home works:** Home works are assigned at a lab where they can be discussed with teaching assistants. Students work out homweorks **independently** (rules in Czech). The deadline for submitting a homework is on the next Monday 6:00 in the morning. Late submissions are penalized (10% for each commenced **day** of delay but not more than 50% of points).
4. **Assesment:** see above.
5. **Tests:** Students work out test **independently**.
6. **The final grade:** depends on the exam (40%), tests (30%), and home works (30%). The ratios may be slightly adjusted according to the relative difficulty of the home works and tests.

pajdla@cmp.felk.cvut.cz

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Literature

1. P. Olšák. Úvod do algebry, zejména lineární. ČVUT 2007.
2. P. Pták. Introduction to Linear Algebra. Vydavatelství ČVUT, Praha, 2007.
3. R. Hartley and A.Zisserman. Multiple View Geometry in Computer Vision. Cambridge University Press 2003.
4. Maple – A0B01MVM Matematika v Maple Installation

pajdla@cmp.felk.cvut.cz

The course info!

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Lecture 1

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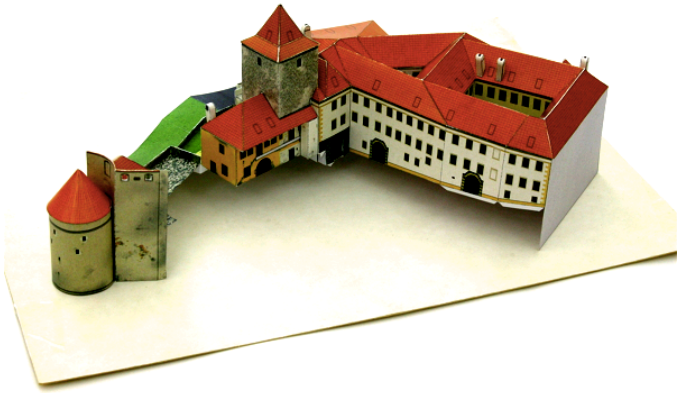
3D RECONSTRUCTION from PHOTOS

SCENE

DIGITAL CAMERA



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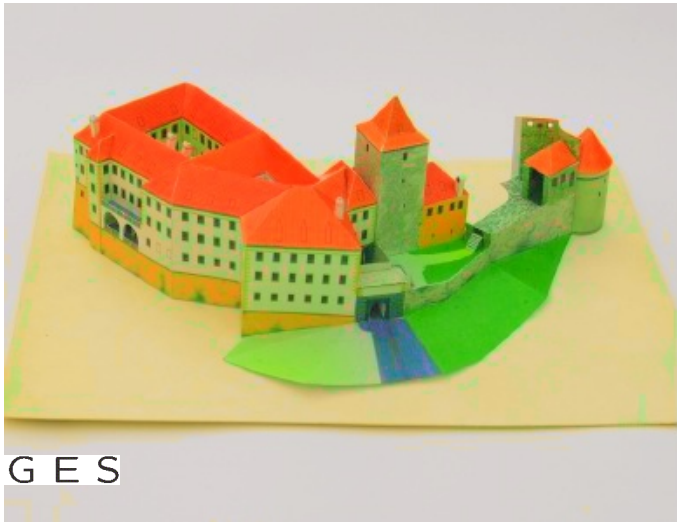

DIGITAL IMAGES

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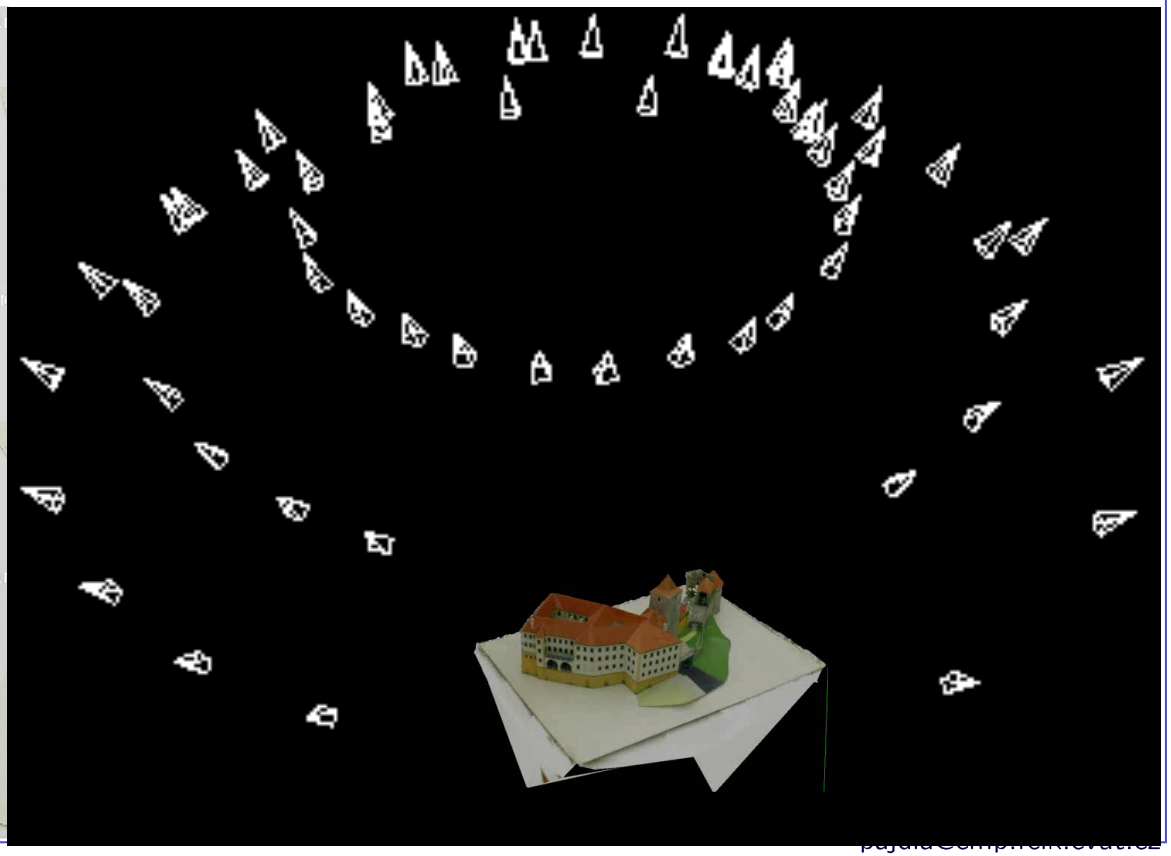
3D REKONSTRUKCE z FOTOGRAFIÍ



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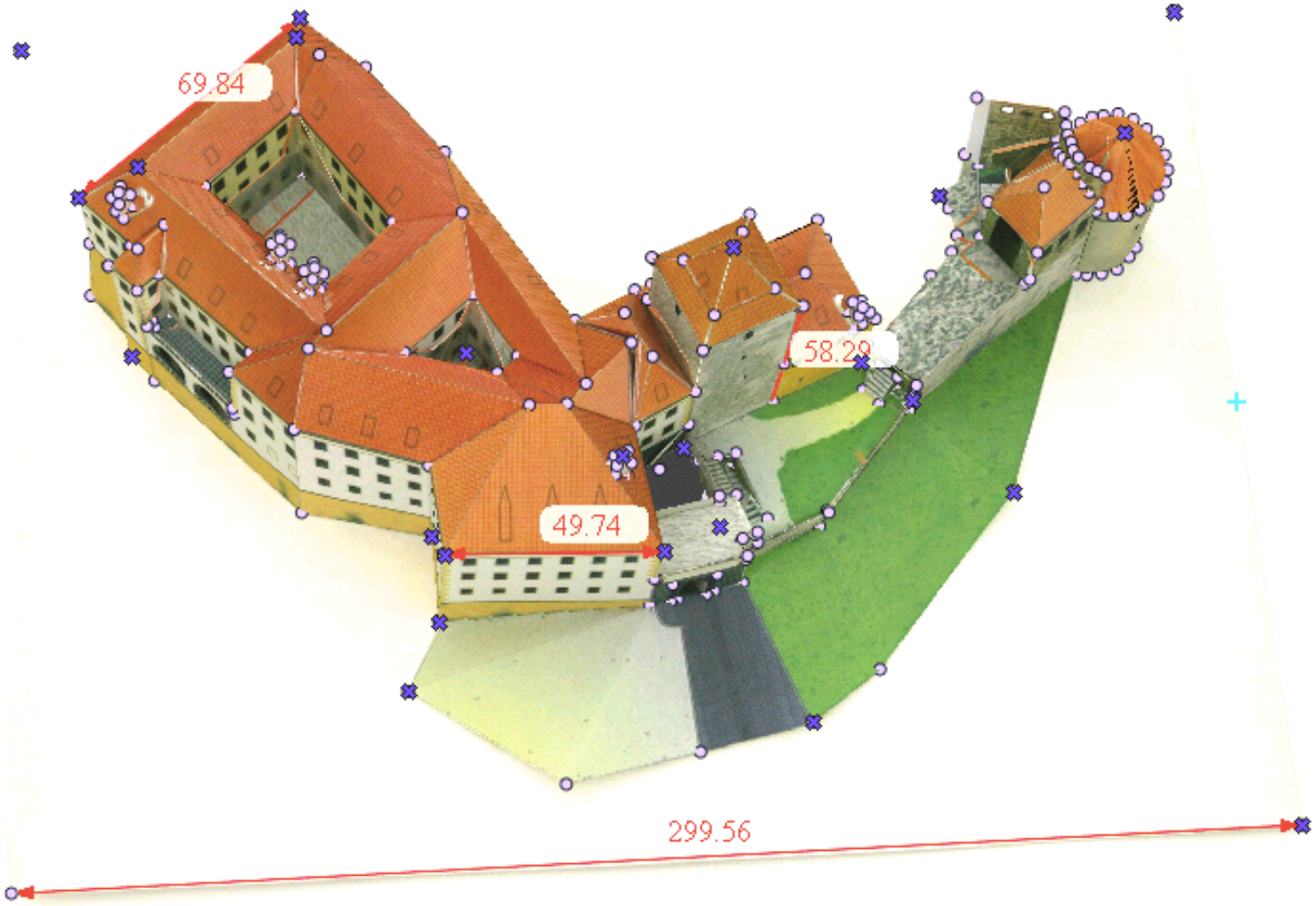

3D MODEL

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3D REKONSTRUKCE z FOTOGRAFIÍ

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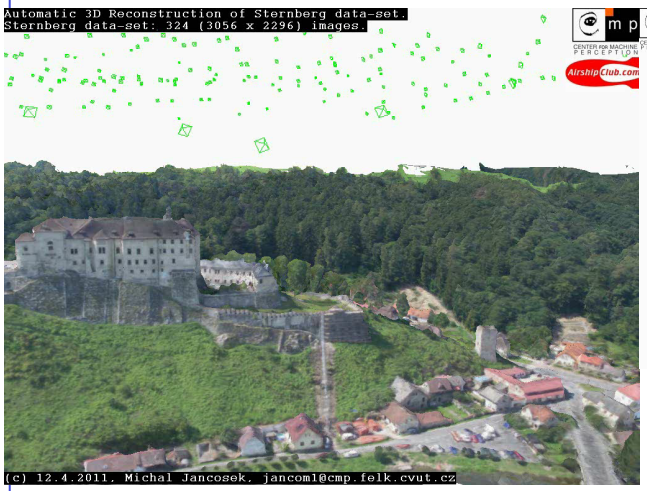
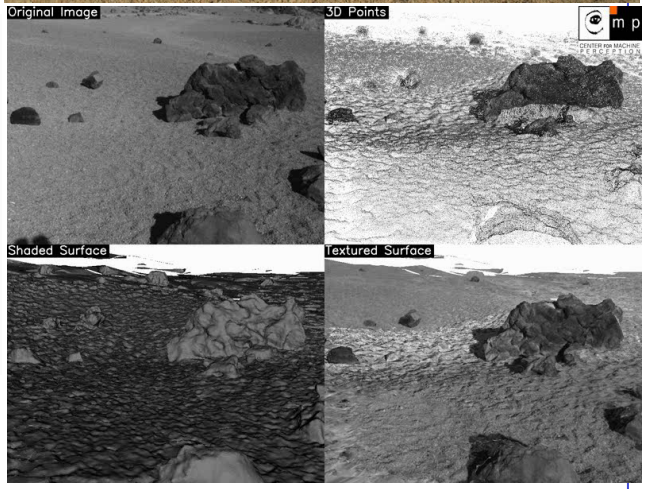
3D RECONSTRUCTION





landscape

objects

robotics

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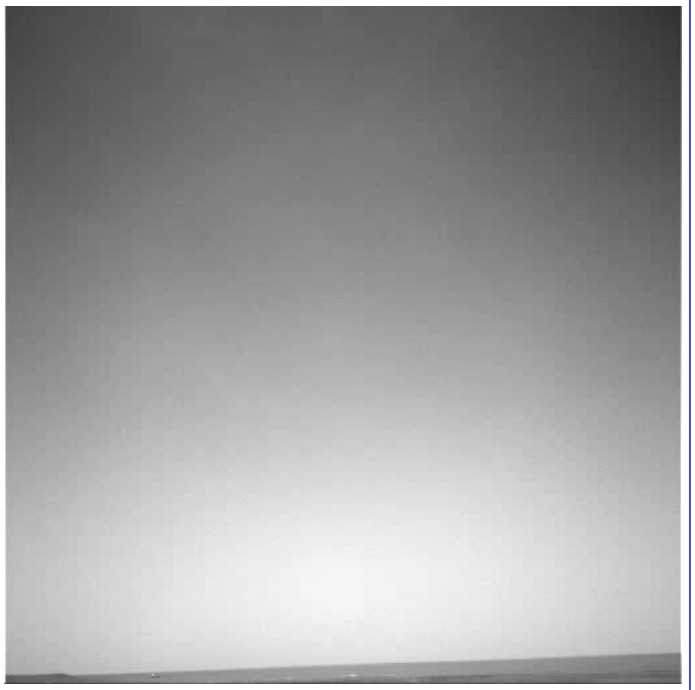
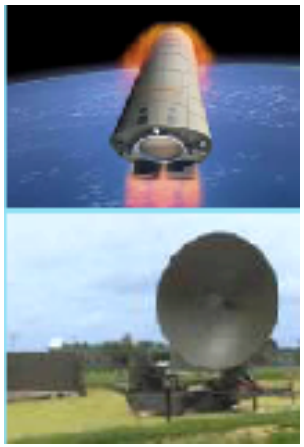
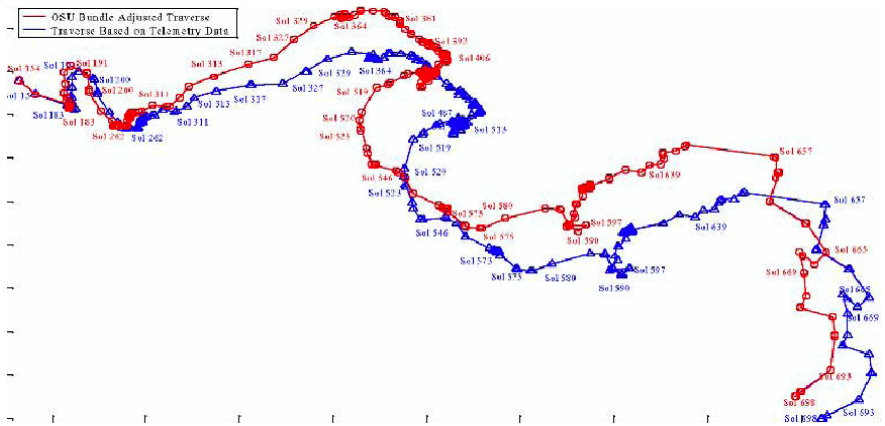
ROBOTIKA

MARS ROVER

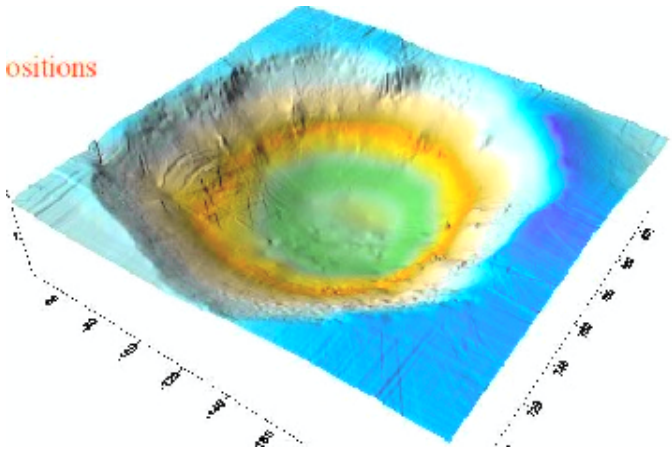


IMAGES





TRAJECTORY



MARS SURFACE

pajdla@cmp.felk.cvut.cz

Match-moving, Camera tracking, and Movies

pajdla@cmp.felk.cvut.cz

IMAGE SEQUENCE

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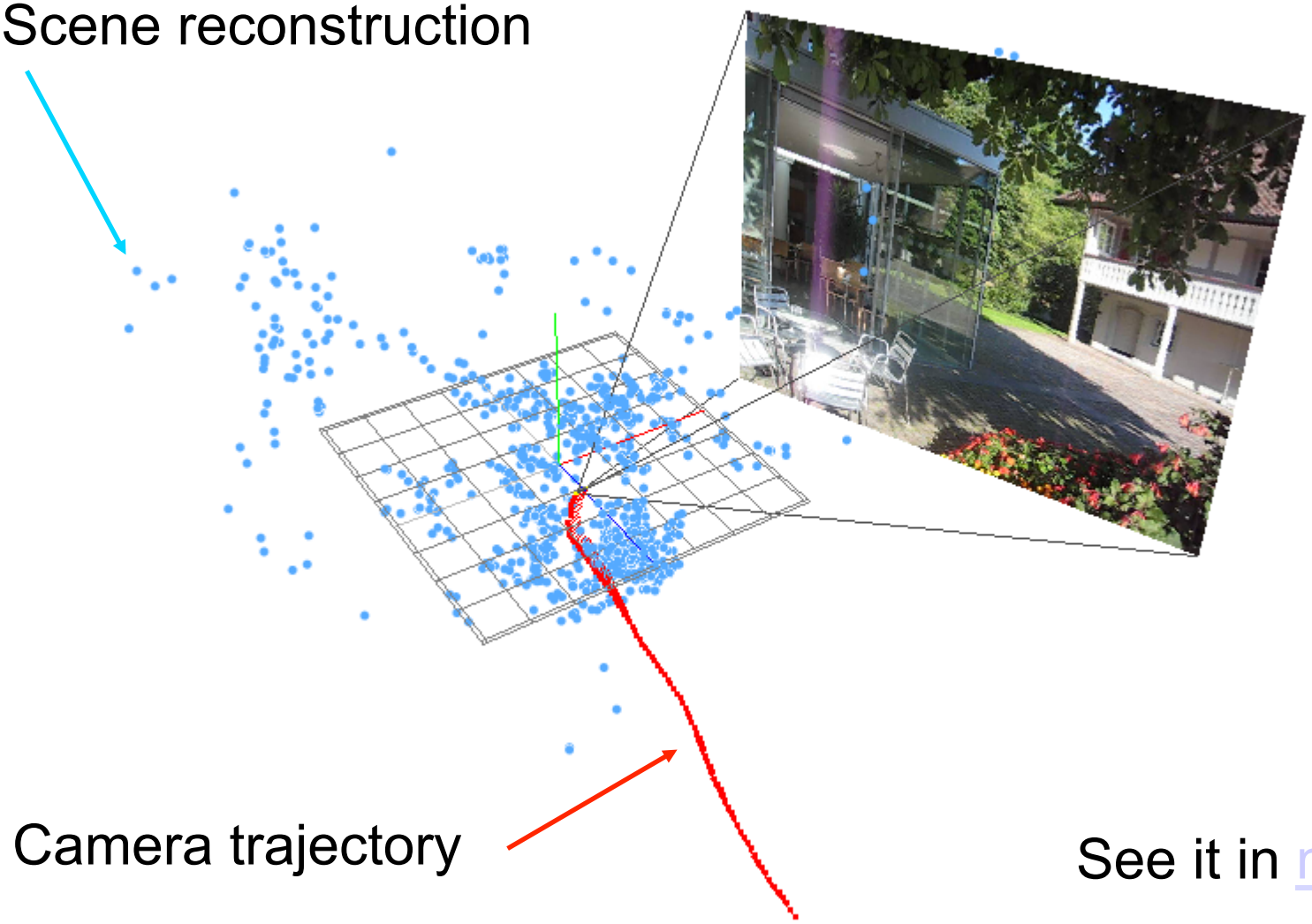
Scene reconstruction



Camera trajectory

See it in motion

pajdla@cmp.felk.cvut.cz

Doppelganger, www.skippingstone.net

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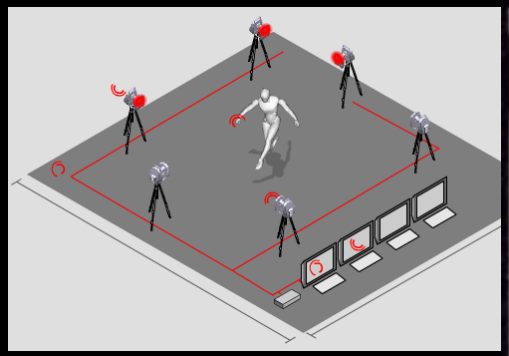
2d3 boujou 2

Showreel

pajdla@cmp.felk.cvut.cz

Motion tracking, Games and Movies



Showreel

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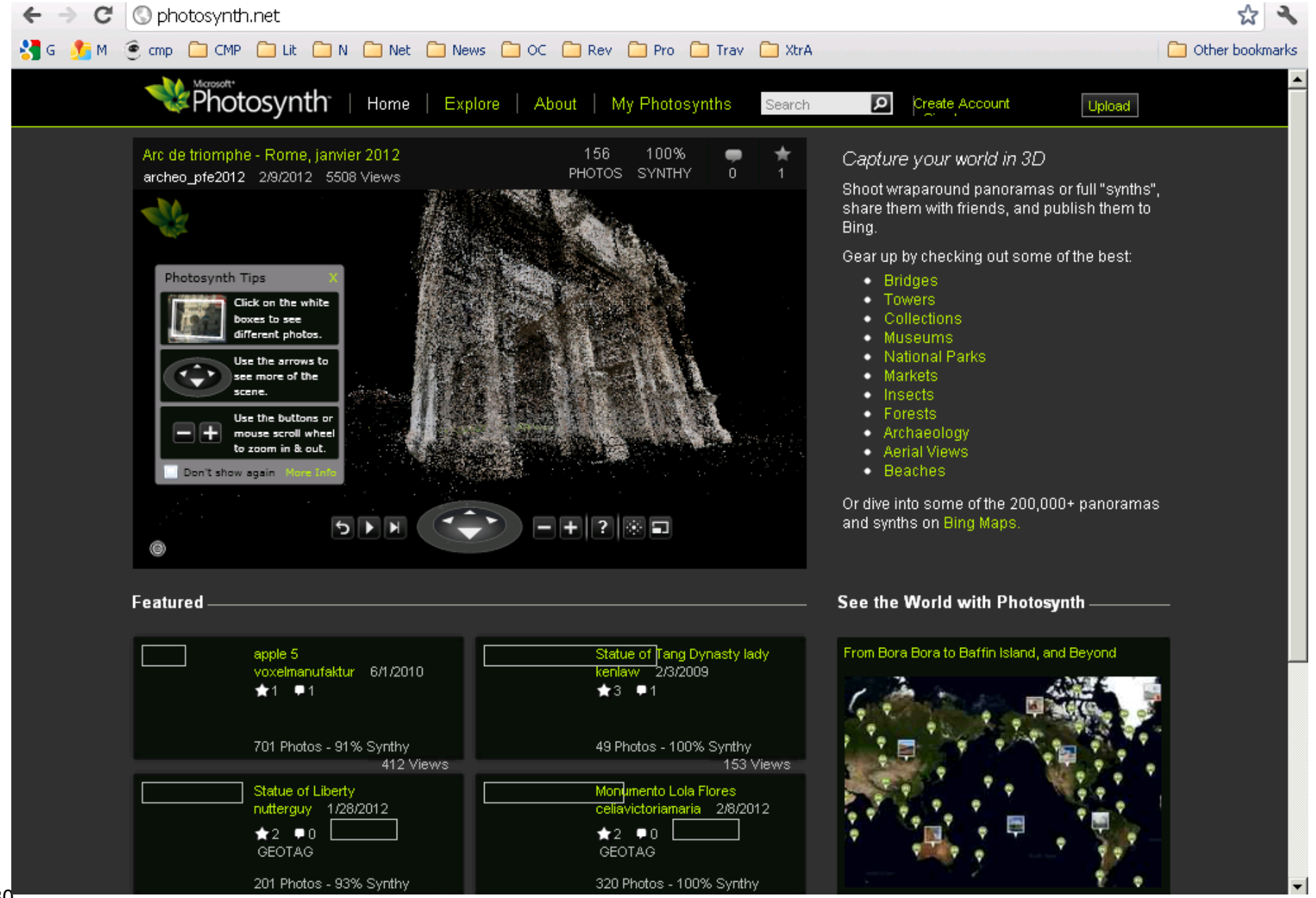
Microsoft Photosynth – Viewing Photos in 3D



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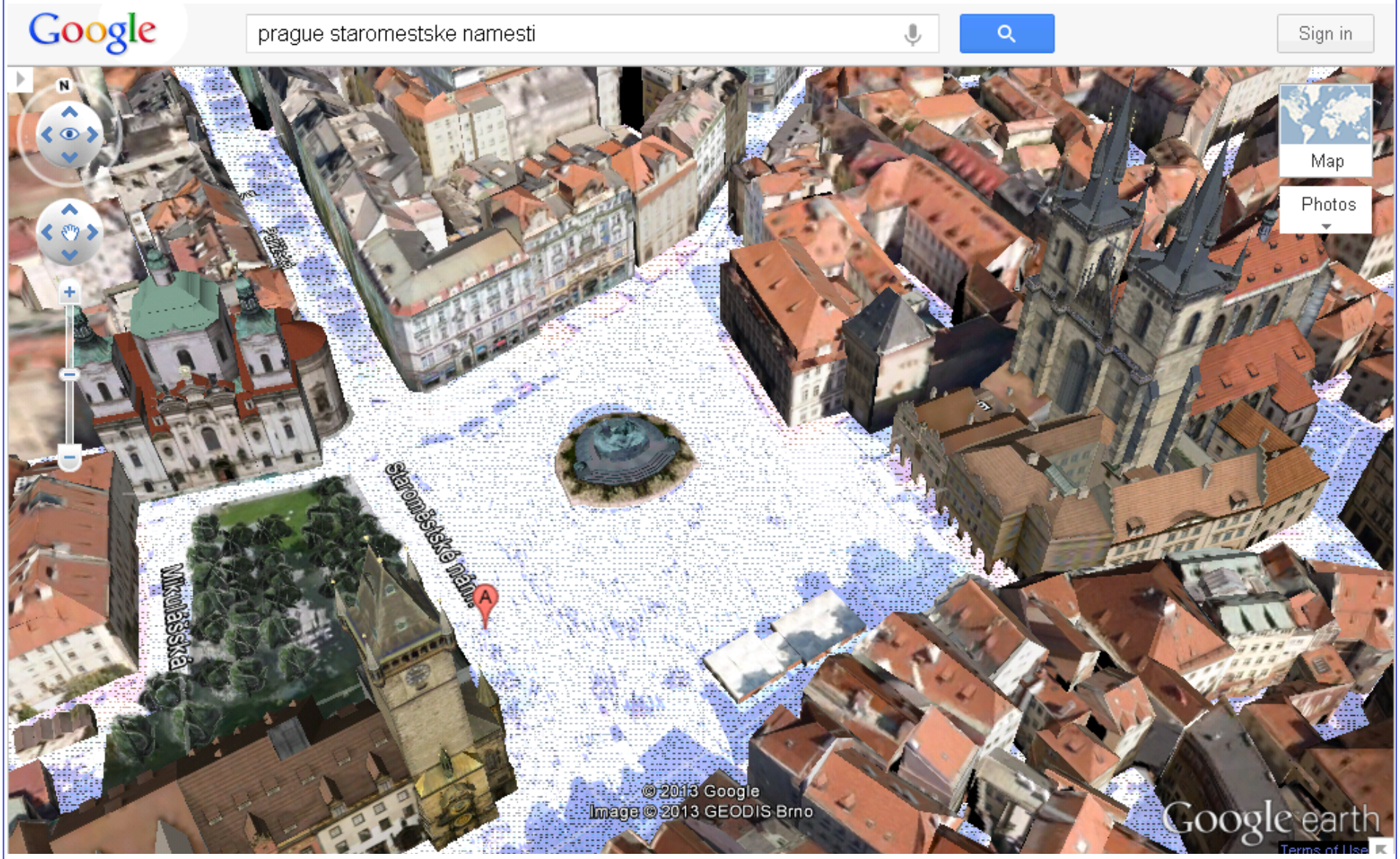
3D Google (Apple, …) Maps



pajdla@cmp.felk.cvut.cz

Image and its coordinate system

pajdla@cmp.felk.cvut.cz

Camera

Digital cameras

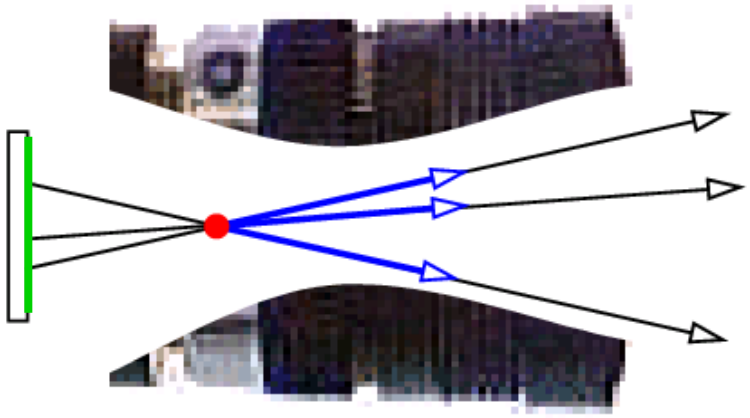


Image projection model

1. Light extends along straight rays
2. Projection center
3. Projection plane

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Image

pixel

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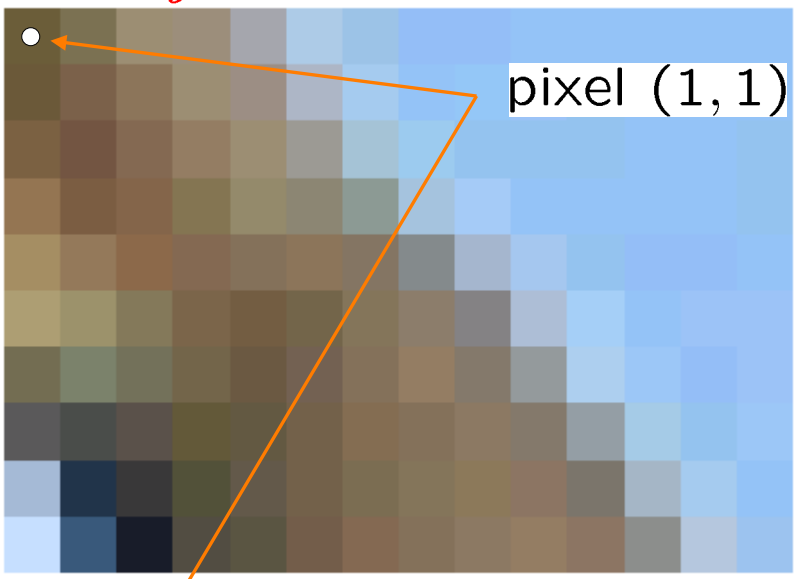
Image is a *m x n x 3* matrix in Matlab

$(0,0)^\top$




```
>>im = imread('karluv-most.jpg');
>>imagesc(im(1:10,1:14,:));
>>axis image
```

Indexing

```
>>im(3,4,1)
ans = 144
```

i j R

RGB

1 ... R

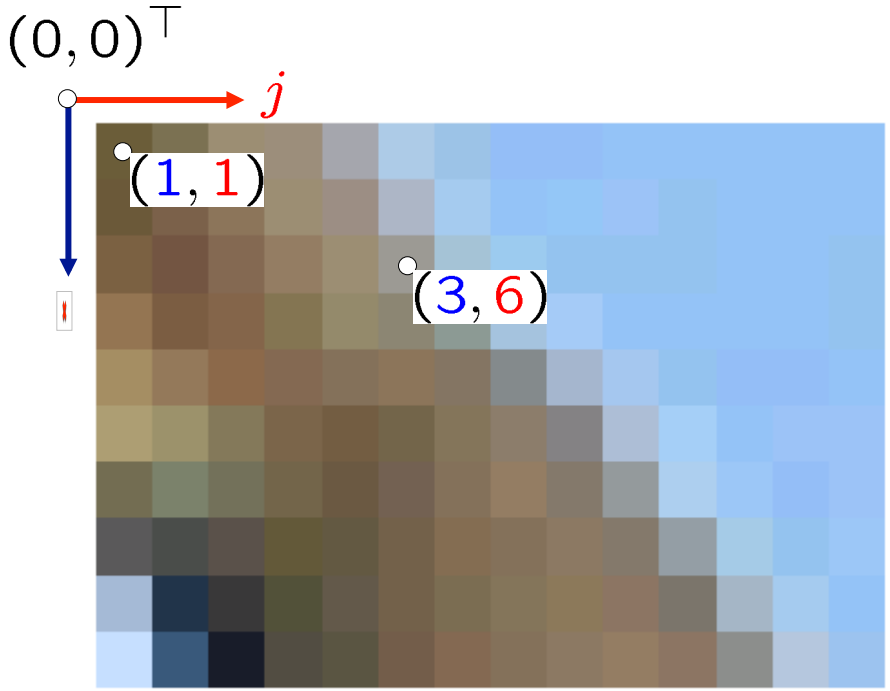
2 ... G

3 ... B

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	107	126	155	154	161	172	155	145	149	151	150	150	150	149
2	107	121	139	156	154	174	166	146	149	152	150	148	149	151
3	125	116	130	144	159	157	167	159	149	150	148	150	151	151
4	148	125	128	132	147	140	140	167	160	146	148	151	150	150
5	164	149	136	128	131	137	130	133	160	164	150	151	149	151
6	168	154	134	122	114	118	134	137	131	173	165	148	152	153
7	112	127	116	112	106	116	130	144	129	145	170	152	147	152
8	89	76	91	98	102	115	128	131	139	130	148	167	151	152
9	164	37	62	85	98	115	127	131	139	139	125	165	164	149
10	194	62	27	84	95	112	128	131	137	150	139	142	176	158

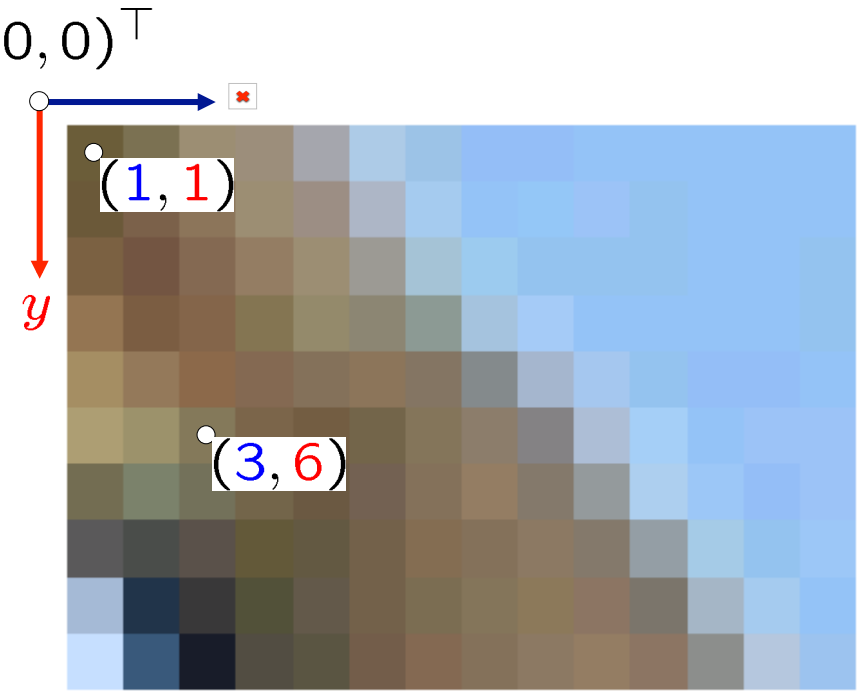
pajdla@cmp.felk.cvut.cz

Image coordinate systems in Matlab



Indexing

```
>>im(3,6,1)
```



Plotting

```
>>axis image
>>plot(3,6,'.');
```

```
im(i,j,:) ↔ plot(j,i,'.');
```


pajdla@cmp.felk.cvut.cz

A few more notes ...

pajdla@cmp.felk.cvut.cz

How to study difficult technical subjects?

Studying a difficult and unknown subject is like walking a swamp:

1. always probe the terrain around you (inspect, visualize and understand results of all actions)
2. always keep one foot on the safe ground (never change more than one thing at a time)
3. avoid stepping on a path unless you have tested it (test correctness on simple synthetic data)
4. keep track of your journey (doccument and comment)
5. be able to go back (be always able to re-run all your experiments)
6. make a map for your followers (write an informative report)

The fastest way is the longest way!.

pajdla@cmp.felk.cvut.cz

How to program homework in Matlab?

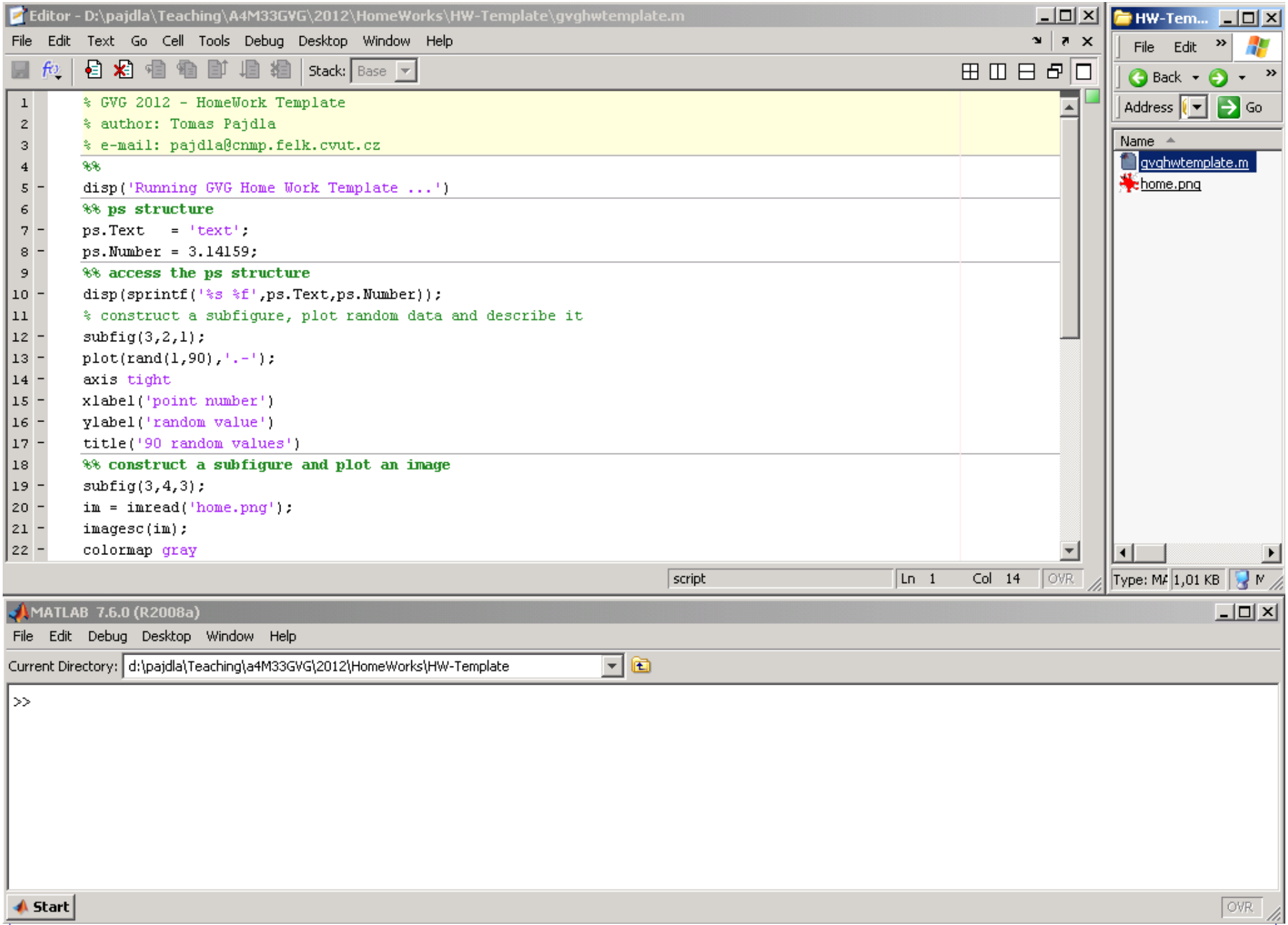
1. use Matlab functions in

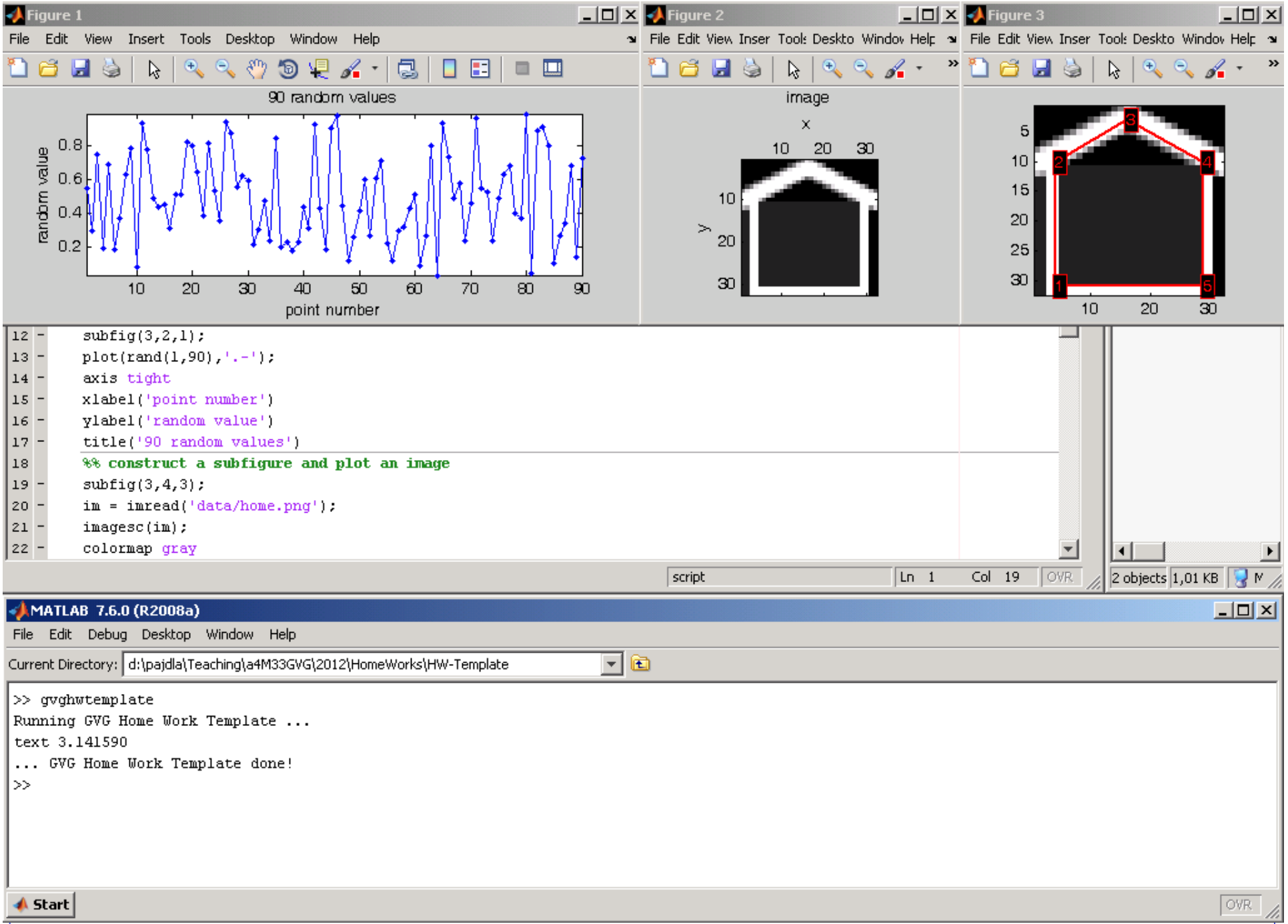
   `http://cw.felk.cvut.cz/courses/GVG/2012/HomeWorks/Tools`

2. use `gvghwtemplate.m` to start

3. use `subfig(m,n,i);` to make non-overlapping figures

pajdla@cmp.felk.cvut.cz


Editor - D:\pajdla\Teaching\A4M33GVG\2012\HomeWorks\HW-Template\gvghwtemplate.m
File Edit Text Go Cell Tools Debug Desktop Window Help
Stack: Base
% GVG 2012 - HomeWork Template
% author: Tomas Pajdla
% e-mail: pajdla@cmp.felk.cvut.cz
%%
disp('Running GVG Home Work Template ...')
%% ps structure
ps.Text = 'text';
ps.Number = 3.14159;
%% access the ps structure
disp(sprintf('%s %f',ps.Text,ps.Number));
% construct a subfigure, plot random data and describe it
subfig(3,2,1);
plot(rand(1,90),'.-');
axis tight
xlabel('point number')
ylabel('random value')
title('90 random values')
%% construct a subfigure and plot an image
subfig(3,4,3);
im = imread('home.png');
imagesc(im);
colormap gray
script Ln 1 Col 14 OVR
HW-Tem...
File Edit
Back
Address Go
Name
gvghwtemplate.m
home.png
Type: MA 1,01 KB
MATLAB 7.6.0 (R2008a)
File Edit Debug Desktop Window Help
Current Directory: d:\pajdla\Teaching\a4M33GVG\2012\HomeWorks\HW-Template
>>
Start
OVR



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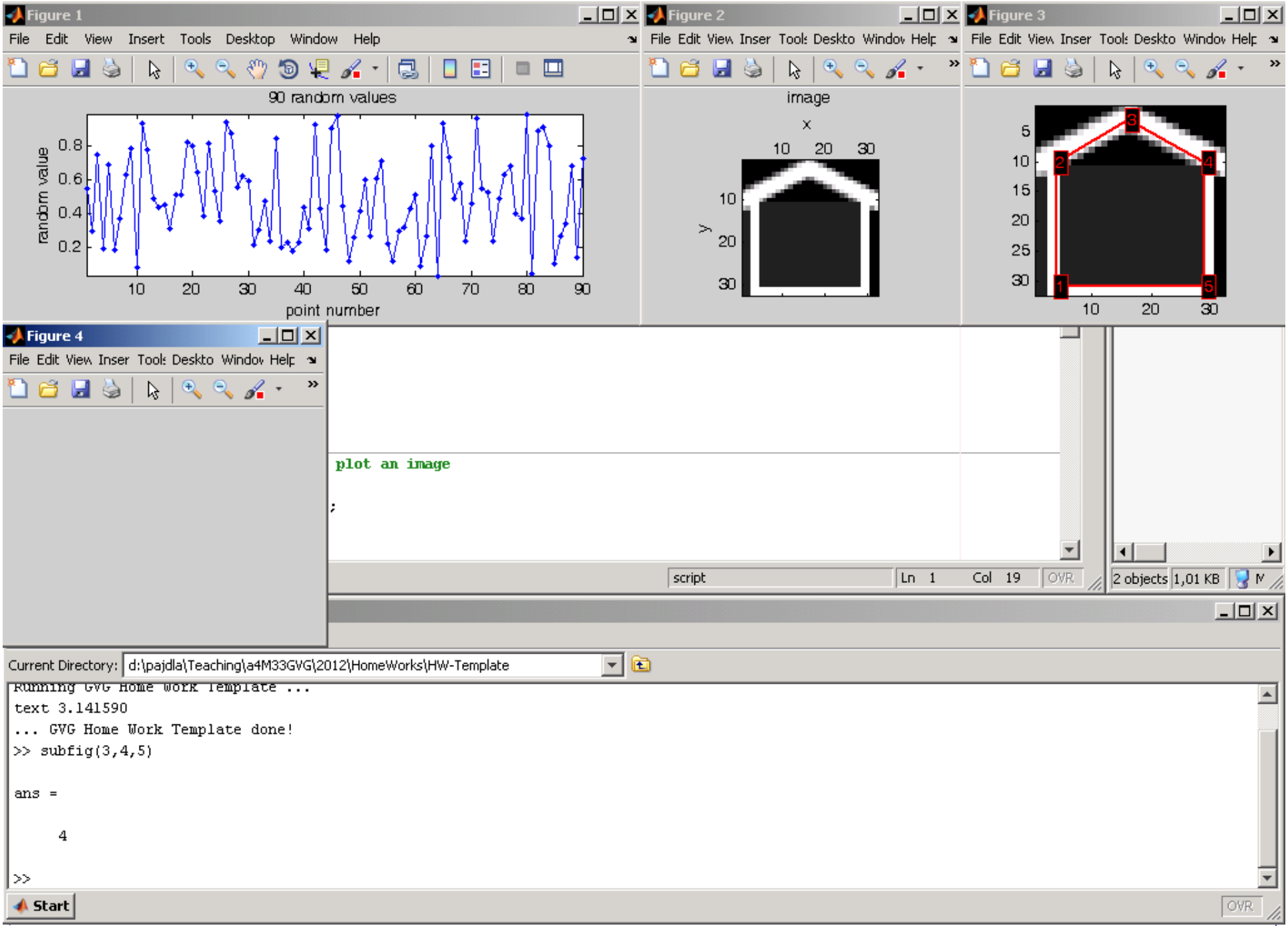
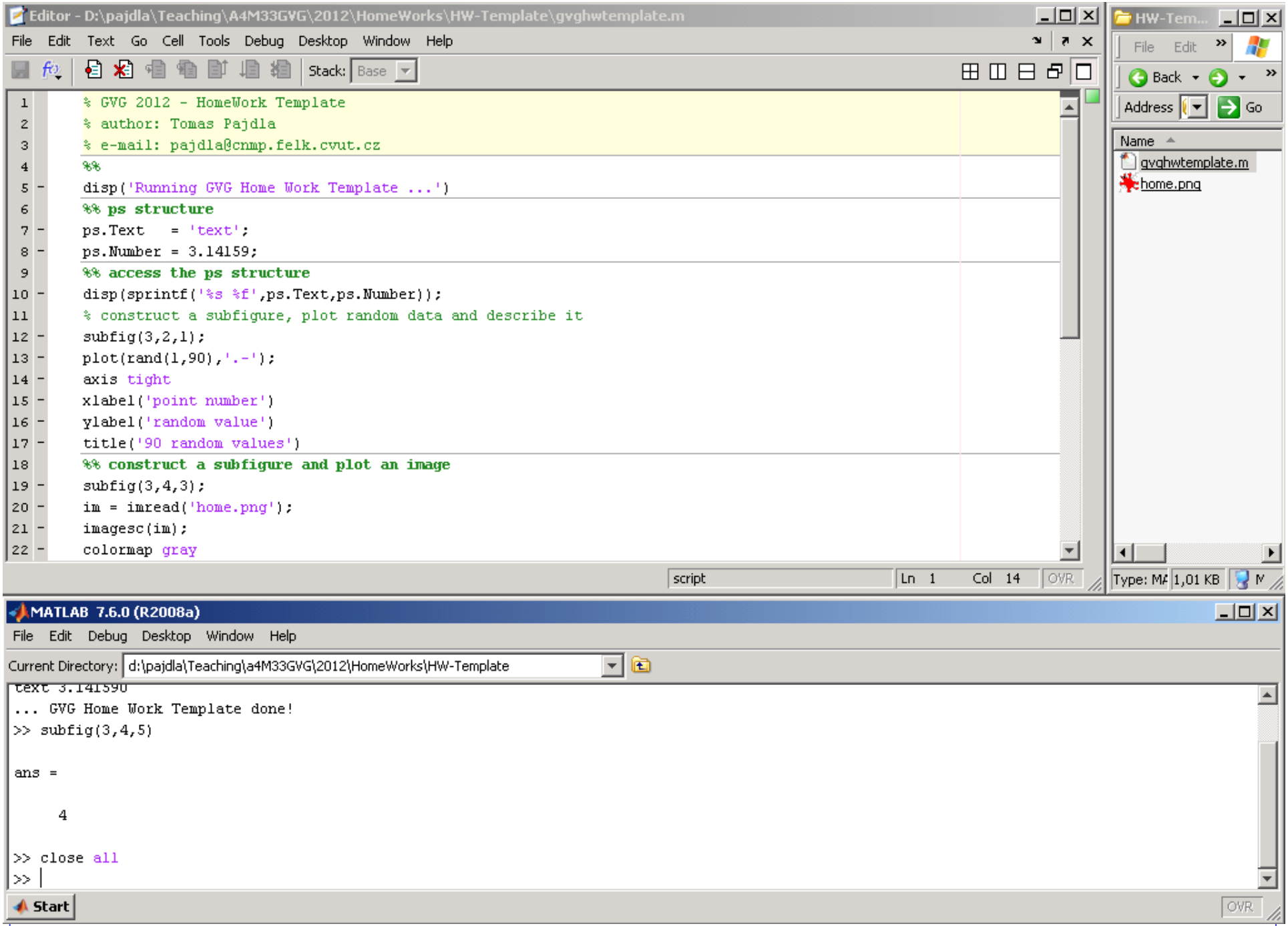


Figure 1
File Edit View Insert Tools Desktop Window Help
90 random values
random value
point number
Figure 2
image
Figure 3
Figure 4
plot an image
script
Ln 1 Col 19
2 objects 1,01 KB
Current Directory: d:\pajdla\Teaching\a4M33GVG\2012\HomeWorks\HW-Template
Running GVG Home Work Template ...
text 3.141590
... GVG Home Work Template done!
>> subfig(3,4,5)
ans =
4
>>
Start


Editor - D:\pajdla\Teaching\A4M33GVG\2012\HomeWorks\HW-Template\gvghwtemplate.m
File Edit Text Go Cell Tools Debug Desktop Window Help
Stack: Base
% GVG 2012 - HomeWork Template
% author: Tomas Pajdla
% e-mail: pajdla@cmp.felk.cvut.cz
%%
disp('Running GVG Home Work Template ...')
%% ps structure
ps.Text = 'text';
ps.Number = 3.14159;
%% access the ps structure
disp(sprintf('%s %f',ps.Text,ps.Number));
% construct a subfigure, plot random data and describe it
subfig(3,2,1);
plot(rand(1,90),'.-');
axis tight
xlabel('point number')
ylabel('random value')
title('90 random values')
%% construct a subfigure and plot an image
subfig(3,4,3);
im = imread('home.png');
imagesc(im);
colormap gray
script Ln 1 Col 14 OVR
HW-Tem...
File Edit
Back
Address Go
Name
gvghwtemplate.m
home.png
Type: MA 1,01 KB
MATLAB 7.6.0 (R2008a)
File Edit Debug Desktop Window Help
Current Directory: d:\pajdla\Teaching\a4M33GVG\2012\HomeWorks\HW-Template
text 3.141590
... GVG Home Work Template done!
>> subfig(3,4,5)
ans =
4
>> close all
>>
Start
OVR