Predicate logic, situation calculus

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http://cw.felk.cvut.cz/doku.php/courses/a4b33zui/start

Logic as a knowledge representation tool

- What convenient characteristics has propositional logic shown?
  - declarative nature
    - knowledge and reasoning naturally separated,
    - procedural languages – miss a general inference mechanism,
  - ability to represent uncertain information
    - with the aid of disjunction (and negation),
    - "Tomorrow afternoon I will go to the cinema or do sports."
    - cannot be taken for granted (see databases, data structures).
  - compositional and context-independent language
    - sentence semantics is a function of semantics of its parts,
    - semantics of $A \wedge B$ related to validity of A and B,
    - unlike natural language: "She saw it then."

Predicate logic

- Propositional logic with limited expressivity
  - problems: no mechanism for generalization and scaling (see wumpus world)
    - All the rooms that adjoining wumpus' room stench.
    - $W_{1,2} \Rightarrow (S_{1,1} \wedge S_{2,2} \wedge S_{1,3})$, $W_{2,1} \Rightarrow (S_{1,1} \wedge S_{2,2} \wedge S_{3,1})$ etc.
  - also relationships between objects, time, syllogisms,
- predicate logic introduces objects and relations among them
  - inspired by the strength of natural language, but preserves unambiguity.
- **1.order predicate logic (first-order logic, FOL)**
  - intentional restrictions in expressive power:
    - relations cannot be seen as objects,
    - general assertions about relations not allowed,
- higher-order logic systems
  - higher expressivity, more complex theory and typically lower efficiency.

FOL – syntax, language elements

- **constants** refer to particular objects
  - peter, paul, scotland, ai-foundations (Prolog notation),
- **predicates** represent object properties and relationhips among objects
  - man(peter), father(peter,paul), lives(peter,scotland),
- **functions** as indirect object references
  - father(peter), lives(peter) (functions can replace predicates and vice versa),
- **variables** refer to sets of objects,
  - X, Y, Z (Prolog notation: start with capitals),
- **quantifiers** determine variable interpretation
  - $\forall$ – for any substitution of object for variable the sentence holds,
  - $\exists$ – we can find an object for which the sentence holds,
  - $\forall$ X $\exists$ Y teaches(X,Y) – each course has at least one teacher,
- **logical operators** and parentheses – identical with propositional logic.

FOL – syntax

- Syntax – definition of correct/well-formed formulae
  - **term** is each constant, variable or function applied to them
    - refers to an object, nothing else cannot be term (i.e., predicates are not terms),
  - **well-formed formulas (WFFs)** are
    - **atomic formulas** $p(t_1, \ldots, t_n)$,
    - where p is a predicate symbol and t_i terms,
    - formulas $\neg p$ and $p \Rightarrow q$, provided that p and q are WFFs,
    - formula $\forall X p$, provided that X is a variable and p WFF.
- Syntax, notes
  - WFF may contain free variables (not quantified)
    - this may cause difficulties when evaluating WFFs,
    - **sentence** is a WFF if all the variables are bound (also closed formula),
  - the logical operators $\vee, \wedge, \Leftrightarrow$ and quantifier $\exists$ are derived.

FOL – semantics, interpretation

- How to determine the truth value of FOL formulas/sentences?
  - given by context, but ...
- can we check validity for all interpretations?
  - we work with large or infinite domains (e.g., real numbers),
  - as a consequence, an infinite number of models may exist.
- **Interpretation** defines:
  - objects, predicates and functions corresponding to symbols including their meaning
    - **domain** $\Delta = \{$Jiří Kléma, Michal Pěchouček, STM Y33ZUI, OI A4B33ZUI$\}$,
    - constant interpretation: $l_C : C \rightarrow \Delta$,
      jirka refers to Jiří Kléma, zui1 to course STM Y33ZUI, etc.
    - predicate interpretation (arity n): $l_P : P \rightarrow P(\Delta^n)$,
      teaches refers to teacher-course relationship,
      teaches/2 $= \{\{$jirka,zui1$\},\{$michal,zui2$\}\}$.

FOL – semantics, interpretation

- Is formula $\forall X(p(X) \vee q(X)) \Rightarrow (\forall X p(X) \vee \forall X q(X))$ tautology?
  - to disprove we only need to find an interpretation for which is the formula false,
  - e.g., interpretation Δ={a,b}, $p_D = \{a\}$, $q_D = \{b\}$,
  - the left hand side of the implication holds:
    - X=a: $p(a) \vee q(a) = T \vee F = T$, X=b: $p(b) \vee q(b) = F \vee T = T$,
  - the right hand side does not hold:
    - X=b: $p(b) = F$, $\forall X p(X) = F$ similarly for X=a and $\forall X q(X) = F$.

Inference in FOL – unification

- How to **substitute** for free variables or terms when unifying terms?
  - KB1: Jirka knows Filip. Vaclav is known by everybody. Everybody knows his/hers mother.
    Monika knows someone.
  - question: Who knows Jirka?
  - return all the correct substitutions for the free variable considering KB,

  - KB2: $knows(jirka, filip)$, $\forall Y\, knows(Y, vaclav)$, $\forall Z knows(Z, mother(Z))$, $\exists W\, knows(monika, W)$,
  - KB3: $knows(jirka, filip)$, $knows(Y, vaclav)$, $knows(Z, mother(Z))$, $knows(monika, s1)$,
  - query: ?knows(jirka,X)
  - substitution: $\theta_1 = \{X|filip\}$, $\theta_2 = \{jirka|Y, X|vaclav\}$, $\theta_3 = \{jirka|Z, X|mother(jirka)\}$,
  - we search for **the most general** substitution to keep the terms unifiable.

Inference in FOL – unification

- Substitution and unification needed for any inference
  - see generalized modus ponens

$$\frac{A, A \Rightarrow B}{B} \rightarrow \frac{p'_1, \ldots, p'_n, (p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q)}{Subst(\theta, q)}$$

$$Subst(\theta, p'_i) = Subst(\theta, p_i)$$

  - When two persons know each other, they greet each other.
    - $\forall XY(knows(X, Y) \Rightarrow greets(X, Y))$.,
  - MP application on KB (KB $\vdash \alpha$)
    - $greets(jirka, filip)$, $greets(Y, vaclav)$,
    - $greets(U, mother(U))$, $greets(monika, s1)$.

Resolution in FOL – conversion into CNF (1)

- More difficult than in propositional logic – variables and quantifiers,
- E.g.: All Romans who know Marcus either hate Caesar or think that anyone who hates anyone is crazy.

$$\forall X \left[(roman(X) \wedge knows(X, markus)) \Rightarrow \\ (hates(X, caesar) \vee \forall Y \exists Z (hates(Y, Z) \Rightarrow thinkscrazy(X, Y))) \right]$$

- general steps of the conversion:

1. eliminate implications

$$\forall X \left[\neg(roman(X) \wedge knows(X, markus)) \vee (hates(X, caesar) \vee \\ \vee \forall Y \neg(\exists Z hates(Y, Z)) \vee thinkscrazy(X, Y))) \right]$$

2. move negations into atomic formulae, reduce their scope

$$\forall X \left[\neg roman(X) \vee \neg knows(X, markus) \vee hates(X, caesar) \vee \\ \vee \forall Y \forall Z (\neg hates(Y, Z) \vee thinkscrazy(X, Y)) \right]$$

Resolution in FOL – conversion into CNF (2)

4. skolemization – eliminate existential quantifiers
   - no occurrence, let us try: $\forall X \exists Y father(Y,X) \rightarrow \forall X father(s1(X),X)$
   - $s1$ is in the given context a Skolem function (assigns a father to each instantiation of X),
5. bind each quantifier to a unique variable

$$\forall X roman(X) \vee \forall X greek(X) \rightarrow \forall X roman(X) \vee \forall Y greek(Y)$$

6. move universal quantifiers to the left of formula – **prenex** form,

$$\forall XYZ \big[\neg roman(X) \vee \neg knows(X, markus) \vee \\ hates(X, caesar) \vee \neg hates(Y,Z) \vee thinkscrazy(X,Y)\big]$$

7. distribute disjunctions inwards over conjunctions
   - apply distributive laws $a \vee (b \wedge c) \rightarrow (a \vee b) \wedge (a \vee c)$
8. drop the prefix $=$ all universal quantifiers

$$\neg roman(X) \vee \neg knows(X, markus) \vee \\ hates(X, caesar) \vee \neg hates(Y,Z) \vee thinkscrazy(X,Y)$$

Motivation example 3 – Does Zuzana lay eggs?

:: If I tell you that:

- (S1) Platypus and echidna are the only mammals that lay eggs.
- (S2) Only birds and mammals are warm-blooded.
- (S3) Zuzana, my armadillo, is warm-blooded and has no feathers.
- (S4) Every bird has feathers.

:: and ask: (D) Does Zuzana lay eggs?

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:: Inference in natural language:

- Zuzana has no feathers and thus it is not a bird.
- Zuzana is warm-blooded and it is not a bird, it must be a mammal.
- Zuzana is mammal and armadillo, not platypus/echidna, it cannot lay eggs.

:: How to implement automatically? Strings difficult for inference ...

Does Zuzana lay eggs? – resolution proof (1)

- (S1) $\forall X(mammal(X) \wedge eggs(X) \Rightarrow echidna(X) \vee platypus(X))$
- (S2) $\forall X(warm - blooded(X) \Rightarrow bird(X) \vee mammal(X))$
- (S3) $armadillo(zuzana) \wedge warm-blooded(zuzana) \wedge \neg feathers(zuzana)$
- (S4) $\forall X(bird(X) \Rightarrow feathers(X))$
- (F) $\forall X(armadillo(X) \Rightarrow \neg(platypus(X) \vee echidna(X)))$
- (D) $eggs(zuzana)$

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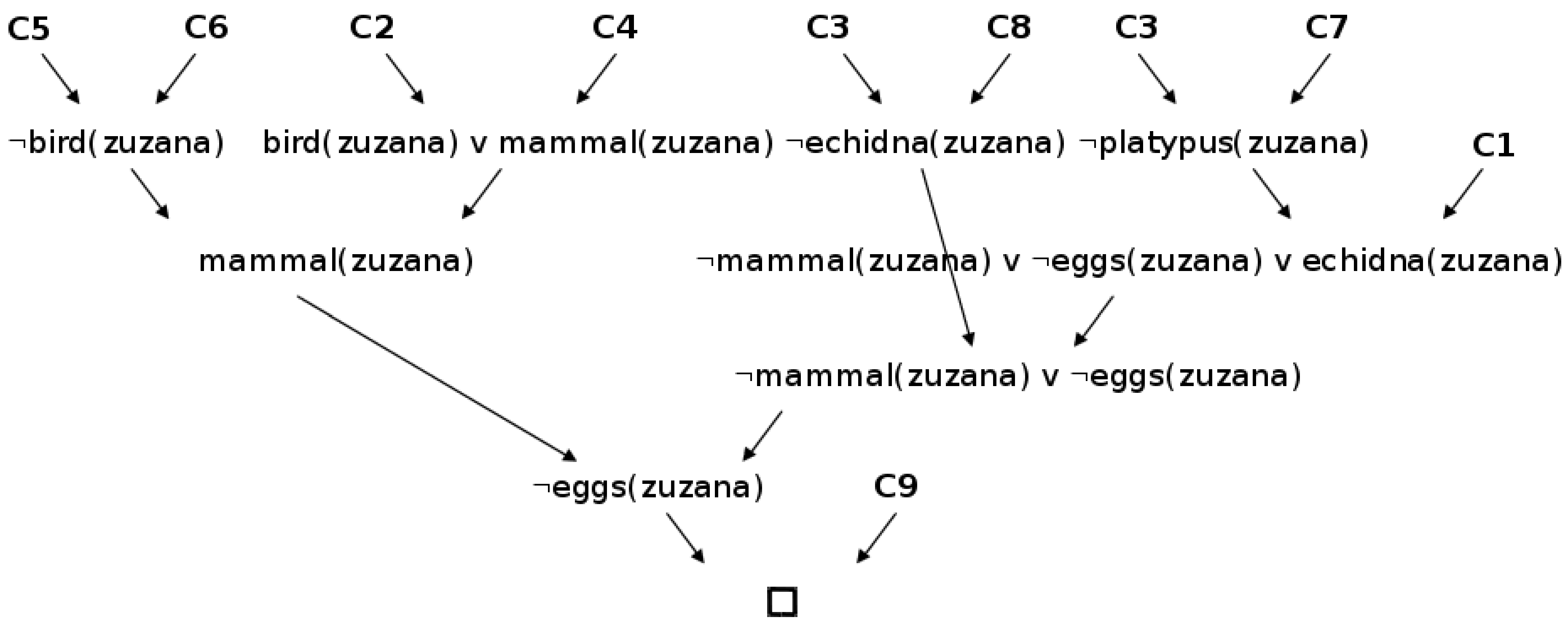
- (F) $\forall X(armadillo(X) \Rightarrow \neg(platypus(X) \vee echidna(X)))$
- (D) $eggs(zuzana)$

- (C1) $\neg mammal(X) \vee \neg eggs(X) \vee echidna(X) \vee platypus(X)$
- (C2) $\neg warm-blooded(X) \vee bird(X) \vee mammal(X)$
- (C3,4,5) $armadillo(zuzana) \wedge warm-blooded(zuzana) \wedge \neg feathers(zuzana)$
- (C6) $\neg bird(X) \vee feathers(X)$
- (C7,8) $(\neg armadillo(X) \vee \neg platypus(X)) \wedge (\neg armadillo(X) \vee \neg echidna(X))$
- (C9) $eggs(zuzana)$

Does Zuzana lay eggs? – resolution proof (2)

:: Resolution proof tree:



:: The assertion that Zuzana lays eggs contradicts the theory.

:: **Zuzana does not lay eggs.**

FOL in wumpus world

- FOL is expressive enough to describe wumpus world and for inference in it,
- environment description
  - adjacency of cave rooms

$$\forall XYAB\left[adjacent([X,Y],[A,B]) \Leftrightarrow [A,B] \in \{[X+1,Y],[X-1,Y],[X,Y+1],[X,Y-1]\}\right]$$

  - diagnostic rules
    - infer cause from effect – for breeze:

$$\forall S\,(breeze(S) \Rightarrow \exists R\,(adjacent(R,S) \wedge pit(R))),$$
$$\forall S\,(\neg breeze(S) \Rightarrow \neg\exists R\,(adjacent(R,S) \wedge pit(R))),$$

  - causal rules
    - infer effect from cause –for cave rooms:

$$\forall R\,(pit(R) \Rightarrow \forall S\,(adjacent(R,S) \Rightarrow breeze(S))),$$
$$\forall S\,[\forall R\; adjacent(R,S) \Rightarrow \neg pit(R)] \Rightarrow \neg breeze(S),$$

  - the above written diagnostic and causal rules equivalent to formula:

$$\forall S\,(breeze(S) \Leftrightarrow \exists R\,(adjacent(R,S) \wedge pit(R)))\,.$$

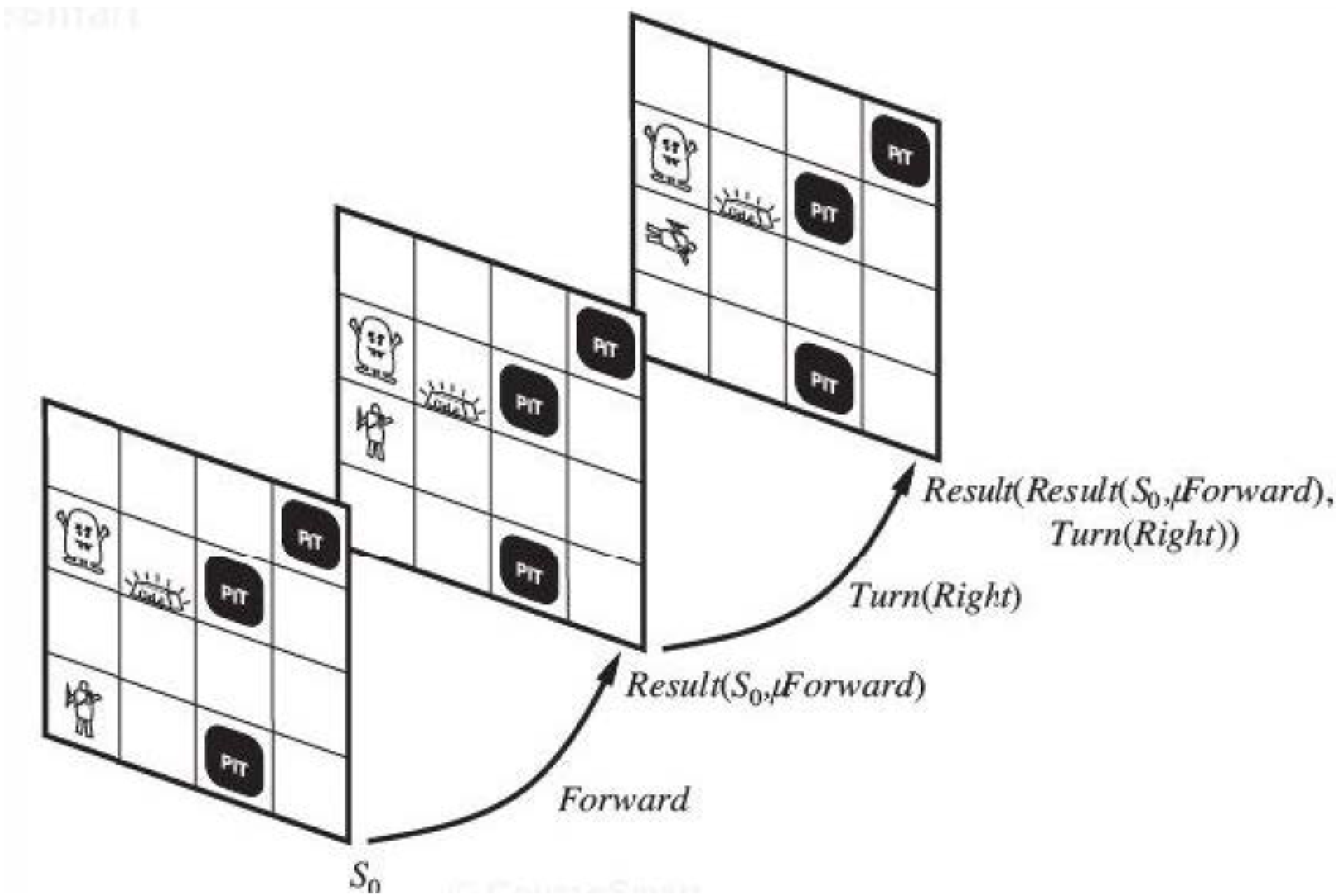
FOL in wumpus world

- it turns out more difficult to represent perception, reflexes and environment modifications
  - perception (T denotes time, resp. the number of step)
    $\forall TSB\,(sensors([S, B, glitter], T) \Rightarrow glitter(T))$
  - reflex
    $\forall T\,(glitter(T)) \Rightarrow best\_action(grab, T))$

Keeping track of change

- facts hold in particular **situations**, rather than eternally,
- **situation calculus** is one way to represent change in FOL
  - predicates either rigid (eternal) or fluent (changing)
    - with or without possibility to change during time,
  - adds a situation argument to each fluent predicate
    - e.g. $holds(gold, now)$, term now denotes a situation,
  - rigid predicates e.g. $pit(R)$, $adjacent(R, S)$,
  - situations connected by the $result$ function,
    - s is a situation, $result(a, s)$ is a situation too,
    - $result(a, s)$ reached by doing action a in situation s.

Keeping track of change



Russell, Norvig: AIMA, 3rd edition, 2010.

Description and application of actions

- "effect" axiom – defines changes corresponding to the outcome of action
  - $\forall s \; withGold(s) \Rightarrow holds(golds, result(grab, s))$
- "frame" axiom – defines all that **remains the same**
  - $\forall s \; hasArrow(s) \Rightarrow hasArrow(result(grab, s))$
- **frame problem**: how to cope with the unchanged facts smartly

  (a) representational

  avoid the frame axioms in the local world,

  F fluent predicates, A actions $\rightarrow \mathcal{O}(FA)$ frame axioms,

  (b) inferential

  avoid copying of unchanged to keep the state information complete,

Description and application of actions

- **qualification problem**
  - precise description of real actions requires infinite care,
  - what if gold is slippery or nailed to the ground or . . . ?
- **ramification problem**
  - real actions have many secondary/hidden consequences,
  - agent moves with all that he holds,
  - when the gold is dusty, the dust moves with it,
  - gloves needed to grab get worn out.

Description and application of actions

- “successor-state” axioms diminish the representational frame problem,
- each axiom attached to one predicate (instead of an action)

  P holds after execution of action $\Leftarrow$ [action caused P
  $\vee$ P held before and action did not touch P]

- for gold handling

$$\forall a, s\ holds(gold, Result(a, s)) \Leftarrow$$
$$[(a = grab \wedge withGold(s)) \vee (holds(gold, s) \wedge a \neq release)]$$

- we obtain F axioms
  - the total number of literals is $\mathcal{O}(AE)$
    (E is the number of effects per action),
- alternative notation for $holds$ with frame and effect axioms?

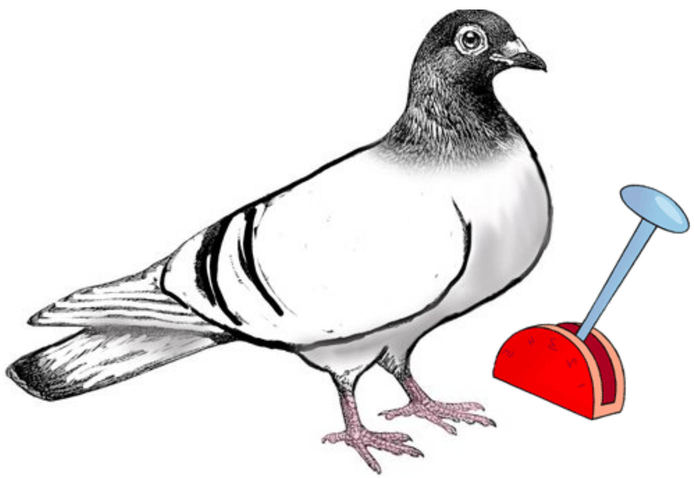
Frame problem and human brain

- Sherlock Holmes and the dog that did not bark

```
Is there any point to which you would wish to draw my attention?
To the curious incident of the dog in the night-time.
The dog did nothing in the night-time.
That was the curious incident, remarked Sherlock Holmes.
```

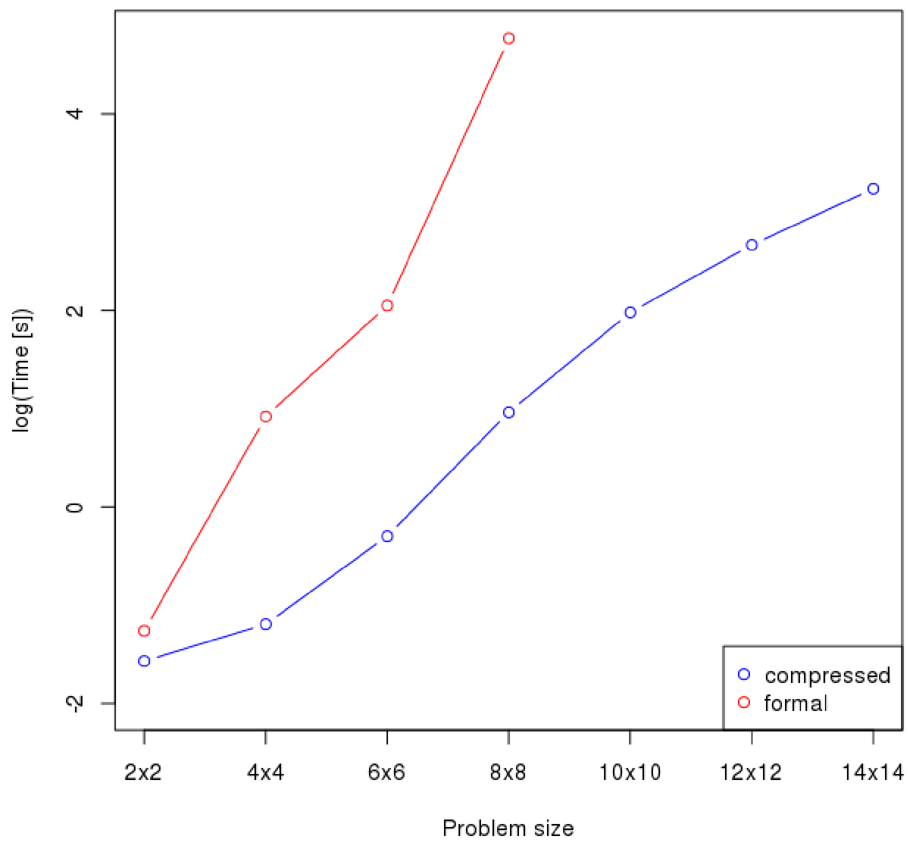
- What can human brain and memory do?
  - even human memory does not store everything,
  - but often it pretends so,
  - people do not realize the simplification tricks,
  - focus on action and its results,
  - frame gets reconstructed,
  - that is why people have difficulties to notice the events, that *did not happen*.

Frame problem and human brain



Wumpus: formal and efficient solution, comparison

- formal solution
  - modular knowledge,
  - extendable,
  - easy to maintain,
- efficient solution
  - minimizes the number of predicates,
  - the ultimate case is 1,
  - implicit search space pruning,
- demo.



Making plans

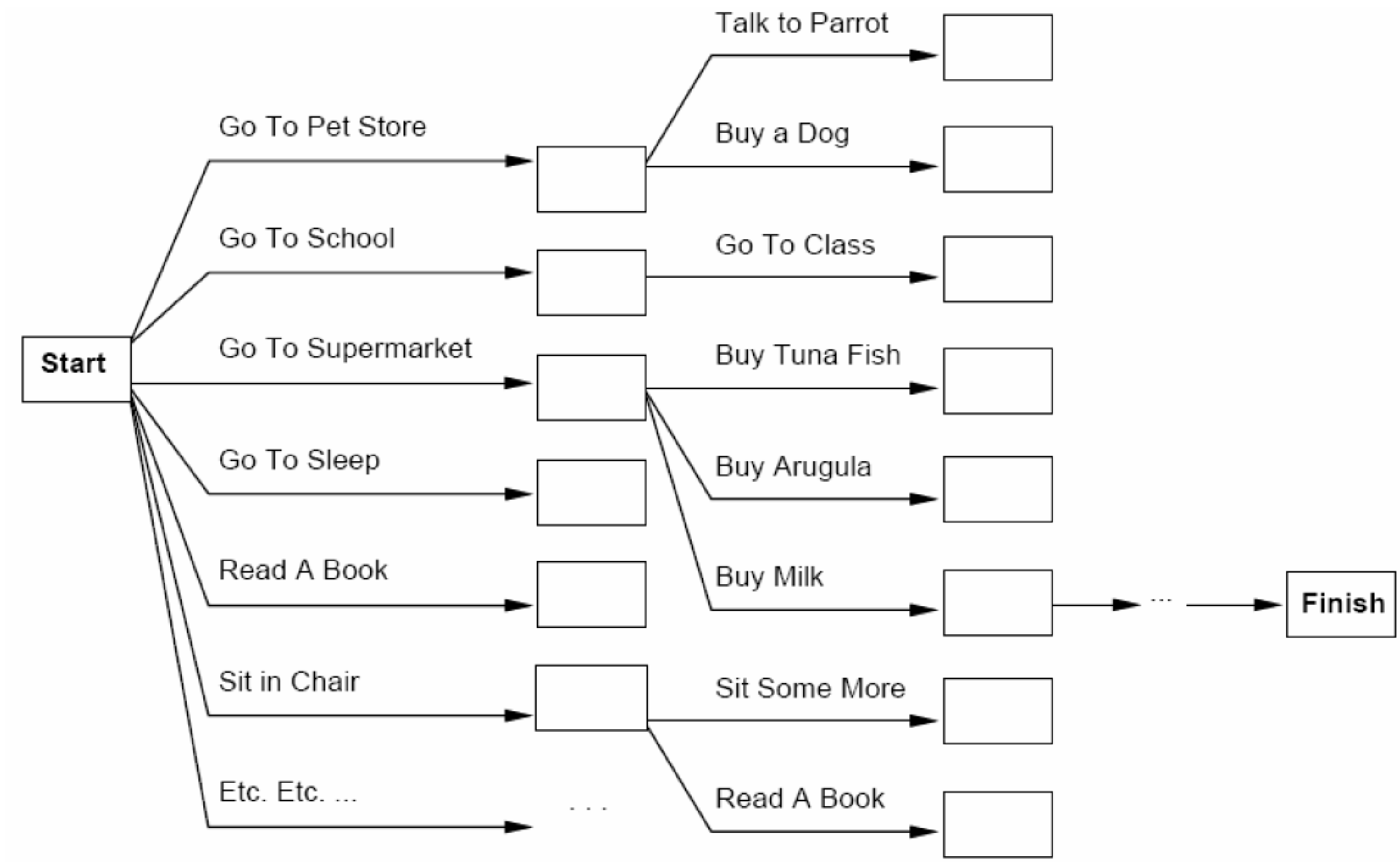
- initial facts in the knowledge base
  - $position(agent, [1,1], s_0)$, $position(gold, [1,2], s_0)$,
- query:
  - $Ask(KB, \exists s\ holds(gold, s))$,
  - i.e., in which situation will the agent hold gold?
- answer:
  - $\{s/result(grab, result(go\_forward, s_0))\}$,
  - i.e., go forward and then grab the gold,
- assumptions:
  - agent is interested only in plans starting in s_0,
  - s_0 is the only situation described in KB.

Making plans: a better way

- make plans in the form of sequence of actions $[a_1, a_2, \ldots, a_n]$,
- $planResult(p, s)$ is a result of execution of plan p in state s
  - projection task – situation after application of sequence of actions?
    $position(agent, [1,1], s_0) \wedge position(gold, [1,2], s_0) \wedge \neg holds(O, s_0)$
    $position(gold, [1,1], result([go([1,1],[1,2]), grab(zlato), go([1,2],[1,1])], s_0))$
  - planning task – which sequence of actions leads to the desired situations?
- query $Ask(KB, \exists p\ holds(gold, planResult(p, s_0)))$
  - has the solution $\{p/[go\_forward, grab]\}$,
- definition of $planResult$ in terms of $result$
  - $\forall s\ planResult([\,], s) = s$,
  - $\forall a, p, s\ planResult([a|p], s) = planResult(p, result(a, s))$,
- motivation for **planning** as a discipline
  - specialized planners more efficient than general reasoning/search.

Why specialized planners

- Find a suitable sequence of actions (a path) in graph below



Russell, Norvig: Artificial Intelligence: A Modern Approach.

Situation calculus: example

- Let us have a logical KB defining distances between Czech cities:
  - $dist(praha, brno, 209)$, $dist(brno, zlin, 96)$,
- agent state determined by its position and the amount of gasoline in the tank
  - $position(City, State)$, $tank(Gas, State)$,
  - let fuel consumption be 6l/100km,
- define the predicate journeyLength(P,D)
  - e.g. $journeyLength([praha, brno, zlin], 305)$,
- define a situation axiom which specifies consequences of action $takeJourney(P)$
  - consider the change of position and the amount of gasoline in the tank,
- do we need any frame axioms?

Situation calculus: example

- recursive definition of journeyLength(P,D):

$$\forall X\ journeyLength([X], 0)$$

$$\forall X, Y, L, D_1, D_2\ \ dist(X, Y, D_1) \wedge journeyLength([Y|L], D_2)\ \Rightarrow$$
$$journeyLength([X, Y|L], D_1 + D_2)$$

- the effects of action $takeJourney(P)$ arrive only when its assumptions are met

$$\forall X, Y, B, D, P, S\ position(X, S) \wedge journeyLength(P, D) \wedge$$
$$tank(B, S) \wedge (B > D * 6/100) \wedge first(P, X) \wedge last(P, Y) \Rightarrow$$
$$position(Y, Result(takeJourney(P), S)) \wedge$$
$$tank(B - D * 6/100, Result(takeJourney(P), S))$$

- action $takeJourney(P)$ changes both the fluent predicates
  - frame axioms not needed, needed e.g. for a hypothetical predicate: $roadworthyCar(Roadworthy, State)$.

Summary

- In a dynamic world it is more difficult to derive and maintain knowledge
  - general bottlenecks – frame and ramification problems,
  - even human memory reconstructs the frame only roughly,
  - representation of fluent worlds in FOL $\rightarrow$ situation calculus,
- another KR systems
  - production systems,
- practical utilization
  - e.g. semantic web,
- where you can learn more?
  - A4M33RZN – Advanced methods of knowledge representation,
  - A4M33AU – Automated reasoning.

Recommended reading, lecture resources

:: Reading

- Russel, Norvig: **AI: A Modern Approach**, Logical Agents, chapter 7
  - representation for intelligent agents,
  - available in pdf – `http://aima.cs.berkeley.edu/newchap07.pdf`.
- Mařík a kol. **Umělá inteligence 1**
  - kapitola Reprezentace znalostí
    - základní formáty, logika, sémantické sítě, rámce,
  - kapitola Řešení úloh a dokazování vět
    - predikátová logika a důkazní prostředky,
- Mařík a kol. **Umělá inteligence 2**
  - kapitola Znalostní inženýrství
    - praktická, znalostní systémy v konkrétních aplikacích.