Common knowledge

(and possible worlds semantics)

Common knowledge

$$E_G^1 A \Leftrightarrow \&_{i \in G} K_i A$$

$$C_G A \Leftrightarrow \&_{k>0} E_G^k A$$

$$E_G^{k+1} A \Leftrightarrow E_G E_G^k A$$

Let ***M*** *be any K. structure. There holds:*

- $\boldsymbol{M} \models C_G \boldsymbol{\alpha} \rightarrow E_G (\boldsymbol{\alpha} \,\&\, C_G \boldsymbol{\alpha})$
- If $\boldsymbol{M} \models \boldsymbol{\varphi} \rightarrow E_G (\boldsymbol{\psi} \,\&\, \boldsymbol{\varphi})$, then $\boldsymbol{M} \models \boldsymbol{\varphi} \rightarrow C_G \boldsymbol{\psi}$ **(Induction Rule)**

Following formulas are valid:

$(i) \quad (C_G A \wedge C_G (A \rightarrow B)) \rightarrow C_G B$

$(ii) \quad C_G A \rightarrow A$

$(iii) \quad C_G A \rightarrow C_G C_G A$

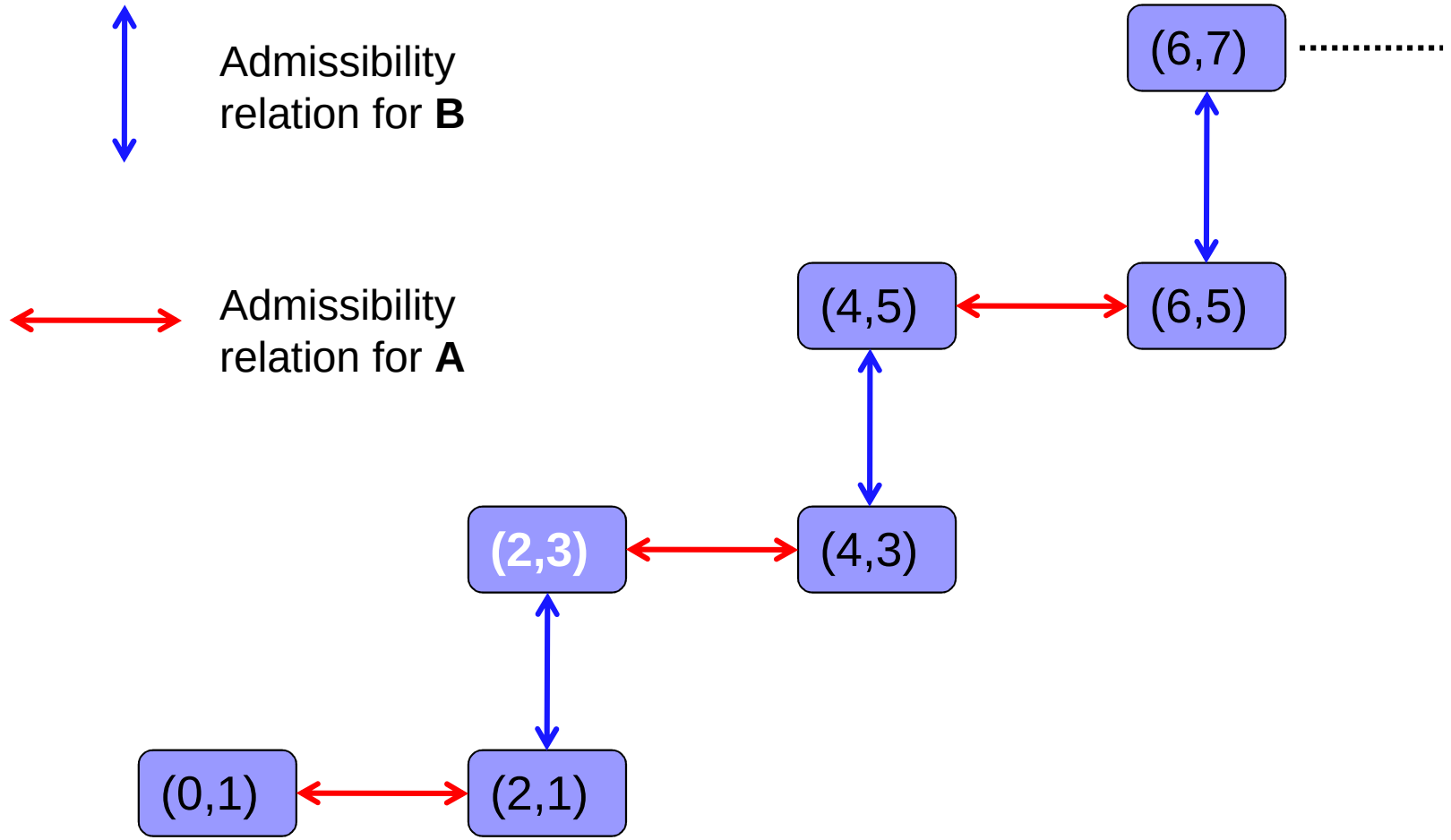
$(iv) \quad \neg C_G A \rightarrow C_G \neg C_G A$

The assumptions about the relations K_i are the same as in case of knowledge of individual agents.

Anna a Bob

Ann and **B**ob take part in a quizz. First, the organizer selects from an urn a natural number ***n***, that he writes on the forehead of one of the players and continues by writing the neighboring number (either ***n*+1** or ***n*-1**) on the forhead of the second player. Neither **A**nn nor **B**ob knows her/his number – each sees only the other´s forehead. They can take turns in announcing nothing but „*I do not know my number*.“ or „*I know my number*.“
Suppose A has on her forehead 3 and B has 4.

- Draw the corresponding Kripke structure.
- Do they have some common knowledge, e.g. that their numbers are smaller than 100?


Admissibility relation for **B**
Admissibility relation for **A**
(6,7)
(4,5)
(6,5)
(2,3)
(4,3)
(0,1)
(2,1)

Valid property

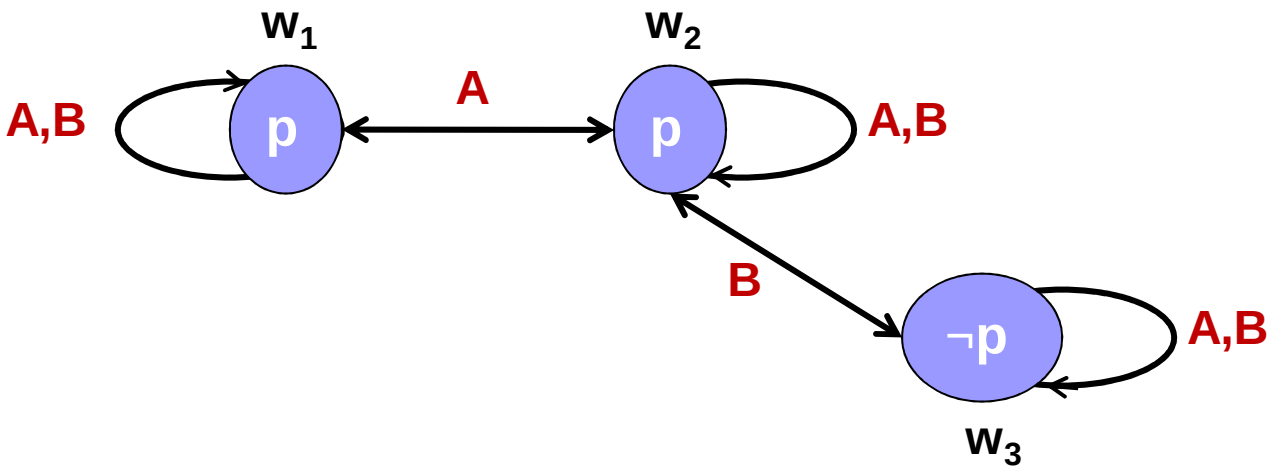
(Lemma on the slide 27 of Lecture Mol1)

It can be proven for any Kripke structure ***M***, its state ***s***, a formula **φ,** group of agents ***G*** and ***k*>0** that

- $(M,s) \models E_G^k \varphi$ iff $(M,v) \models \varphi$ for any state **v,** that is ***G*** reachable from the state ***s*** in ***k*** steps.
- $(M,s) \models C_G \varphi$ iff $(M,v) \models \varphi$ for any state **v,** that is ***G*** reachable fro, the state ***s***.

Consequently, Ann and Bob cannot have a common knowledge on the upper bound of their numbers (the corresponding Kripke structure must contain numbers of arbitrarily high values).

Is there a difference between common knowledge and a fact everyone knows?



Compare truth evaluation of formula ***E p*** and ***C p*** in the state w_1 !

Mr. Product and **Mr.S**um

The organizer of a quizz selected from an urn 2 natural numbers ***x*** and ***y*** from the domain {2,3,4, ..,14}. He did not inform anyone about these numbers but he provided a hint to Mr. ***S*** by saying him the value of their sum, ie. ***x***+***y*** and a different hint to Mr.***P*** who obtained info about their product, ie. ***x*******y***.

Both men ***P*** and ***S*** started to reason:

1. Mr. ***P*** said „I do not know the conisdered numbers ***x,y***.“
2. Mr. ***S*** answered „I knew, you do not know.“
3. Mr. ***P*** continued „Now, I know both numbers“

- Verify that selection of numbers 5 and 6 would result in this dialog.
- Suggest the next sentence of Mr.***S***.

Provide your solutions via the upload system by 18.5. 9:00 am!