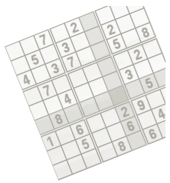

Constraint Programming

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Local search algorithms

Just to recall

Constraint Satisfaction Problem (CSP) consists of:

- a finite set of **variables**
- **domains** – finite sets of possible values for variables
- a finite set of **constraints**
  - constraint **arity** = the number of constrained variables

A feasible solution of a constraint satisfaction problem is a complete consistent assignment of values to variables.

- **complete** = each variable has assigned a value
- **consistent** = all constraints are satisfied

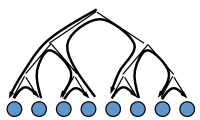
Looking for a solution

The goal: **find a complete and consistent instantiation of variables**

Two **core solving approaches**:

- **exploring complete but possibly inconsistent assignments** until a consistent assignment is found
  - generate and test, local search

- **extending a partial consistent** until a complete assignment is reached
  - backtracking and its extensions

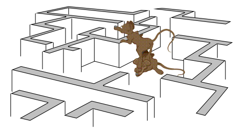


We can explore assignments in two ways:

- **systematically** (explore all possible assignments systematically)
  - a complete method, but could be too slow
- **non-systematically** (some assignments can be skipped)
  - an incomplete method, but can found solution much faster

Note:
We will use constraints in a *passive way*, just to verify whether the given assignment (even partial) satisfies the constraint.

Search techniques



Work plan:

- start simple (with a trivial algorithm)
- find weaknesses of the algorithm
- repair the problems to get better algorithms

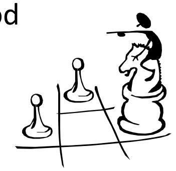
In particular:

- start with **generate and test** method
- **improve the generator**
  - local search methods (HC, RW, TS, GSAT, GENET, SA)
- **merge the generator with the tester**
  - backtracking methods
  - improvements of chronological backtracking
    - backjumping, dynamic backtracking, backmarking

Generate and test (GT)

Probably the most general problem solving method

1) generate a candidate for solution
2) test if the candidate is really a solution

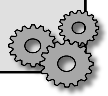


How to apply GT to CSP?

1) assign values to all variables
2) test whether all the constraints are satisfied

GT **explores complete but inconsistent assignments** until a (complete) consistent assignment is found.

```
procedure GT(X:variables, C:constraints)
    V ← construct a first complete assignment of X
    while V does not satisfy all the constraints C do
        V ← construct systematically a complete assignment next to V
    end while
    return V
```



Weaknesses and improvements of GT

The greatest weakness of GT is **exploring too many “visibly” wrong assignments**.

Example:

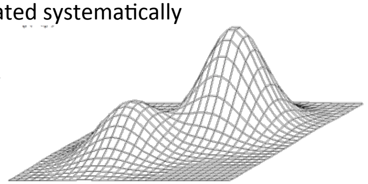
X::{1,2}, Y::{1,2}, Z::{1,2}   X = Y, X ≠ Z, Y > Z

X	**1**	**1**	**1**	**1**	**2**	**2**	**2**
Y	**1**	**1**	**2**	**2**	**1**	**1**	**2**
Z	**1**	**2**	**1**	**2**	**1**	**2**	**1**

- **How to improve GT?**
  - **smart generator**
    - the next assignment improves over the current assignment
    - the core idea of local search techniques
  - **merged generate and test stage** (earlier detection of clash)
    - constraints are tested as soon as the involved variables are instantiated
    - backtracking

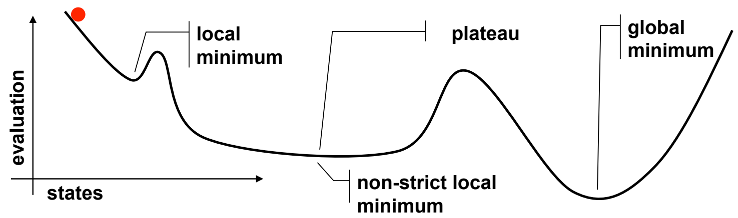
Local search

- ***Generate and test*** explores complete but inconsistent assignments until a complete consistent assignment is found.
- Weakness of GT – the generator does not exploit fully teh result of testing
- The next assignment can be constructed in such a way that constraint violation is smaller.
  - only “small” (**local**) **changes** of the assignment are allowed
  - the next assignment should be “better” than the current one
    - better = more constraints are satisfied
  - assignments are not necessarily generated systematically
    - we lost completeness, but
    - we (hopefully) get better efficiency



Local search - Terminology

- **state** - a complete assignment of values to variables
- **evaluation** - a value of the objective function (# violated constraints)
- **neighbourhood** - a set of states locally different from the current state (the states differ from the current state in the value of one variable)
- **local optimum** - a state that is not optimal and there is no state with better evaluation in its neighbourhood
- **strict local optimum** - a state that is not optimal and there are only states with worse evaluation in its neighbourhood
- **non-strict local optimum** - local optimum that is not strict
- **plateau** - a set of neighbouring states with the same evaluation
- **global optimum** - the state with the best evaluation

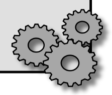


Hill Climbing

- Hill climbing is perhaps the most known technique of local search.
  - start at randomly generated state
  - look for the best state in the neighbourhood of the current state
    - **neighbourhood** = differs in the value of any variable
    - neighbourhood size = $\Sigma_{i=1..n}(D_i-1)$ (= n*(d-1))
  - "escape" from the local optimum via restart

Algorithm Hill Climbing

```
procedure hill-climbing(Max_Steps)
    restart: s ← random assignment of variables;
    for j:=1 to Max_Steps do              % restricted number of steps
        if eval(s)=0 then return s
        if s is a strict local minimum then
            go to restart
        else
            s ← neighbourhood with the smallest evaluation value
        end if
    end for
    go to restart
end hill-climbing
```



Min-Conflicts

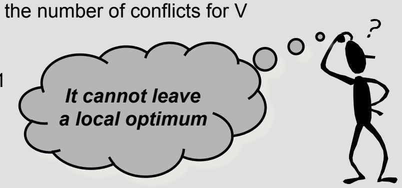
Minton, Johnston, Laird (1997)

Observation:
- the hill climbing neighbourhood is pretty large (n*(d-1))
- only change of a conflicting variable may improve the valuation

Min-conflicts method
- select randomly a variable in conflict and try to improve it
  - **neighbourhood** = different values for the selected variable *i*
  - neighbourhood size = (D_i-1) (= (d-1))

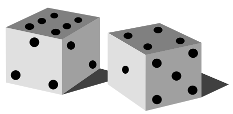
Algorithm Min-Conflicts

```
procedure MC(Max_Moves)
    s ← random assignment of variables
    nb_moves ← 0
    while eval(s)>0 and nb_moves<Max_Moves do
        choose randomly a variable V in conflict
        choose a value v' that minimises the number of conflicts for V
        if v' ≠ current value of V then
            assign v' to V
            nb_moves ← nb_moves+1
        end if
    end while
    return s
end MC
```



Random Walk

How to leave the local optimum without a restart (i.e. via a local step)?
- By adding some "noise" to the algorithm!



Random walk

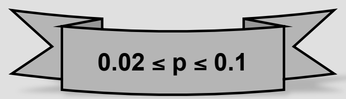
- **a state from the neighbourhood is selected randomly** (e.g., the value is chosen randomly)
- such technique can hardly find a solution
- so it needs some guide
  - Random walk can be combined with the heuristic guiding the search via probability distribution:
    - p - probability of using a random step
    - (1-p) - probability of using the heuristic guide

Min-Conflicts Random Walk

- MC guides the search (i.e. satisfaction of all the constraints) and RW allows us to leave the local optima.

Algorithm Min-Conflicts-Random-Walk

```
procedure MCRW(Max_Moves,p)
    s ← random assignment of variables
    nb_moves ← 0
    while eval(s)>0 and nb_moves<Max_Moves do
        if probability p verified then
            choose randomly a variable V in conflict
            choose randomly a value v' for V
        else
            choose randomly a variable V in conflict
            choose a value v' that minimises the number of conflicts for V
        end if
        if v' ≠ current value of V then
            assign v' to V
            nb_moves ← nb_moves+1
        end if
    end while
    return s
end MCRW
```

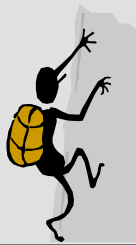


Steepest Descent Random Walk

- Random walk can be combined with the hill climbing heuristic too.
- Then, no restart is necessary.

Algorithm Steepest-Descent-Random-Walk

```
procedure SDRW(Max_Moves,p)
    s ← random assignment of variables
    nb_moves ← 0
    while eval(s)>0 and nb_moves<Max_Moves do
        if probability p verified then
            choose randomly a variable V in conflict
            choose randomly a value v' for V
        else
            choose a move <V,v'> with the best performance
        end if
        if v' ≠ current value of V then
            assign v' to V
            nb_moves ← nb_moves+1
        end if
    end while
    return s
end SDRW
```



Galinier, Hao (1997)

Tabu search

Observation:

Being trapped in a local optimum is a special case of cycling.

How to avoid cycles in general?

- **remember already visited states** and do not visit them again
  - memory consuming (too many states)
- it is possible to remember just few last states
  - prevents „short“ cycles

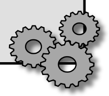
- **Tabu list** = a list of forbidden states
  - a state can be represented by a selected attribute
    - ⟨variable, value⟩ - describes the change of a state (a previous value)
  - the tabu list has a fix length k (tabu tenure)
    - „old“ states are removed from the list when a new state is added
  - a state included in the tabu list is forbidden (it is tabu)
- **Aspiration criterion** = re-enabling states that are tabu
  - i.e., it is possible to visit a state even if the state is tabu
  - **example:** the state is better than any state visited so far

Tabu search

- The tabu list prevents short cycles.
- It allows only the moves out of the tabu list or the moves satisfying the aspiration criterion.

Algorithm Tabu Search

```
procedure tabu-search(Max_Iter)
    s ← random assignment of variables
    nb_iter ← 0
    initialise randomly the tabu list
    while eval(s)>0 and nb_iter<Max_Iter do
        choose a move <V,v'> with the best performance among the non-tabu
            moves and the moves satisfying the aspiration criteria
        introduce <V,v> in the tabu list, where v is the current value of V
        remove the oldest move from the tabu list
        assign v' to V
        nb_iter ← nb_iter+1
    end while
    return s
end tabu-search
```

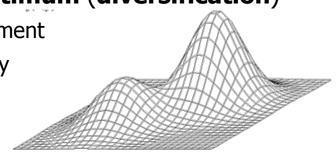


Local Search at Glance

- **LS methods explore complete but possible inconsistent assignments until a consistent assigned is found**
  - opposite to GT, they generate a new assignment based on the current assignment with the goal to increase the number of satisfied constraints

Local search algorithm is defined by:

- **neighbourhood** of the current assignment (state) and a method to **select the next assignment** from the neighbourhood (**intensification**)
  - HC heuristic – select the best assignment different at one variable from the current assignment
    - sometimes, the first better assignment from the neighbourhood is taken
  - MC heuristic – select the best assignment different at one selected conflict variable from the current assignment
- a method for **escaping from a local optimum** (**diversification**)
  - restart – start in a completely new assignment
  - RW – select the next assignment randomly
  - Tabu – forbid some assignments



Local Search for SAT

Many problems can be formulated as problems of Boolean SATisfiability
= **satisfying a logical formula** in conjunctive normal form (CNF)

- **CNF** = conjunction of clauses
- **clause** = disjunction of literals (constraint)
- **literal** = atomic variable or its negation

Example:

$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C \vee \neg A)$

- Similarly to a CSP, SAT is also an NP-complete problem so no fast (polynomial) solving algorithm can be expected.
- Local search can find a solution to pretty large formulas.

Notes:

- satisfaction formula in a disjunctive normal form can be decided fast
- SAT is a special case of a CSP and vice-versa, any CSP can be translated to SAT

Algorithm GSAT

- The GSAT method solves SAT problems by flipping the values of variables.
- The goal is to maximize teh (weighted) number of satisfied clauses.

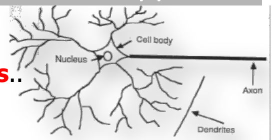
Algorithm GSAT

```
procedure GSAT(A,Max_Tries,Max_Moves)
    A: is a CNF formula
    for i:=1 to Max_Tries do
        S ← random assignment of variables
        for j:=1 to Max_Moves do
            if A satisfiable by S then return S
            V ← the variable whose flip yield the most important raise
                in the number of satisfied clauses
            S ← S with V flipped
        end for
    end for
    return the best assignment found
end GSAT
```

GSAT and heuristics

- GSAT can be combined with various heuristics improving its practical performance (especially for so called structured problems) :
- **Random-Walk**
  - can be used exactly as in MCRW
- **Clause weights**
  - Some clauses remain unsatisfied even after several iterations of the inner loop of GSAT – different clauses have different importance in formula satisfaction
  - satisfaction of "hard" clauses can be preferred by increasing their weights in the clause selection process
  - The algorithm can learn the weight itself
    - all clauses have identical weight at the beginning
    - After each iteration, the weights of unsatisfied clauses are increased
- **Solution averages**
  - in the GSAT algorithm each iteration starts from a random assignment of variables – hence the last reached assignment is "forgotten"
  - we can reuse the common parts of found assignments
    - the new assignment after restart is taken from the last assignments of previous two iterations by keeping the same parts and setting the remaining variables randomly

Connectionistic approach



- Based on idea of representing the problem as a **network of connected simple processors**..
  - processors have several states (usually only two – on/off).
  - The next state of the processor is derived from the states of connected processors (the connection strengths may be different).
- The goal is to find a stable state of the network, i.e., the processors are no more changing their states..
- This stable states represents a solution to the problem.

Features:

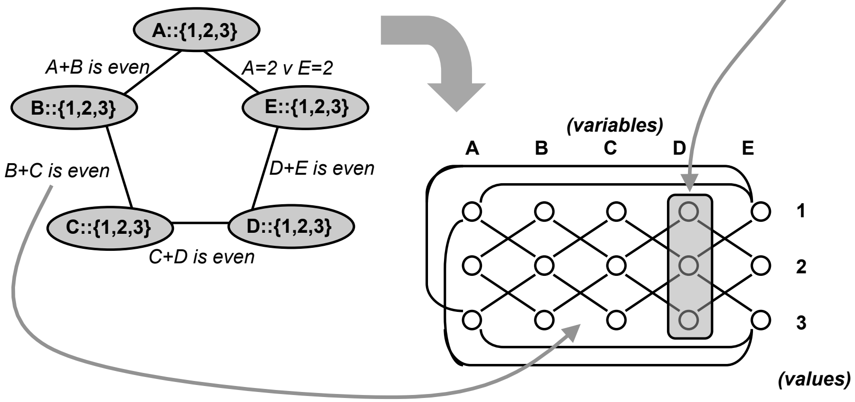
- massive parallelism (problems can be solved faster)
- Blackbox (not clear what is happening inside)

- Probably the most known representative is an artificial **neural network** (NN)
- A similar principle is used in **celular automata.**

GENET – Binary CSP as a NN

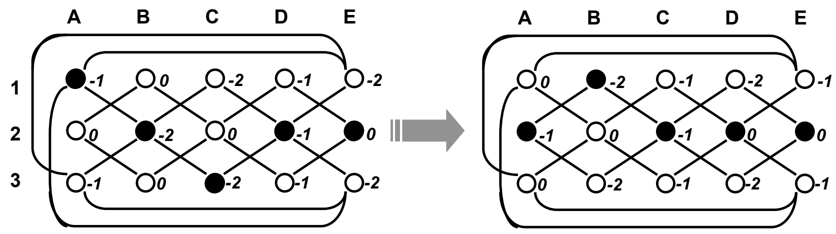
- Each variable is modelled as a cluster of „neurons" (each value models a single neuron)
- Two neurons are connected by the inhibition link with negative weight if teh corresponfing values are incompatible.

Example:



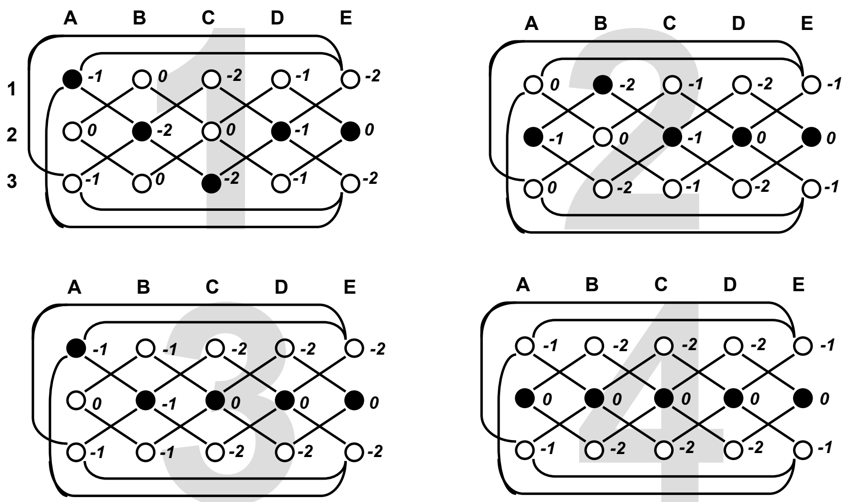
GENET computation

- At the beginning one active neuron is selected in each cluster.
- Neurons change state in a **synchronous way** (all together)
  - based on the inputs (Σ w*s – weighed sum of states of connected neurons)
  - For each cluster, the neuron with the largest input is activated
- The computation stops is a **stable state**.



● = „active" neuron; the numbers indicate inputs to neurons

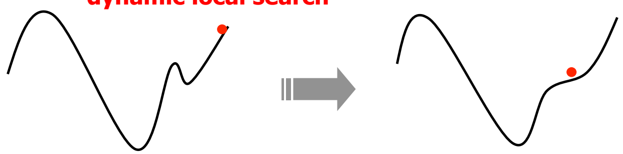
GENET computation cont'd



● = „active" neuron; the numbers indicate inputs to neurons

Escape from local optimum

- What of we reach a stable state that is not a solution?
  - So far we used either restart or "noise".
  - We can try to modify the **space of state evaluations**.
    - How? By modifying the evaluation function!
      - **dynamic local search**

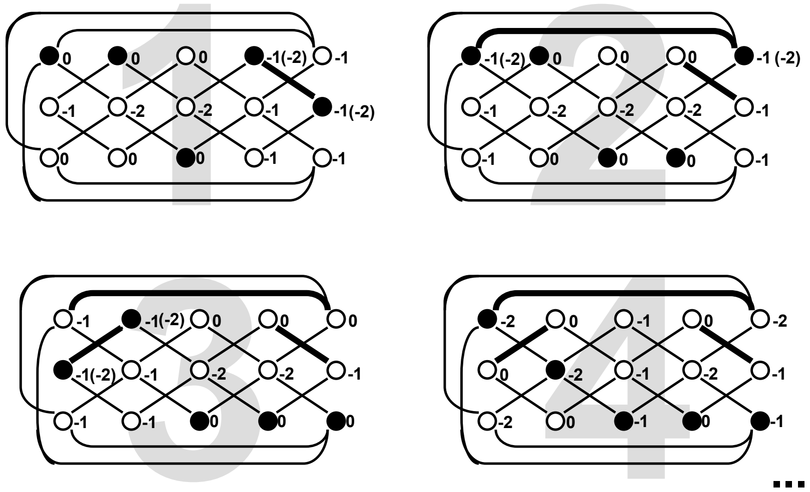


This can be done by modifying the **weight of connections** in GENET!

- If there is a connection between two active neurons (= constraint violation), increase the weight of the connection.
  - new_weight$_{,y}$ = old_weight$_{x,y}$ – s_x* s_y
- This also changes the evaluation function (**Guided Local Search**).

Example of changing connection weights

In local optimum we **strengthen weights** of violated connections (which makes the state instable).



Algorithm GENET

```
procedure GENET(connectionist network)
   one arbitrary node per cluster is switched on;
   repeat
      repeat  % network convergence
         modified ← false;
         for each cluster C do in parallel
            on_node ←  currently switched on node in cluster C;
            label_set ← the set of nodes in C which input are maximum;
            if on_node is not in label_set then
               switch off on_node;
               modified ← true;
               switch on arbitrary node in label_set;
            end if
         end for
      until not modified
      if sum of input to all switched-on nodes < 0 then
         for each connection c connecting nodes x & y do in parallel
            if both x and y are switched on then
               decrease the weight of c by 1;
         end for
      end if
   until input to all switched-on nodes are 0
end GENET
```

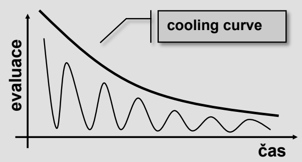
Simulated annealing

- Base on the idea of **simulating the process of metal cooling**.
  - Higher temperature means faster movement of atoms so the probability of changing position is higher.
  - By cooling down, the atoms "try" to find the "best" position – the position with the smallest energy.
- A very similar process can be modelled in optimisation algorithms:
  - so called **simulated annealing**:
    - start with a random state
    - a local change is accepted if:
      - improves the current state
      - makes the state worse, but such a state is accepted only with some probability dependent on "temperature"
    - „temperature" is continuously decreased so the probability of accepting a worsening step is also decreasing – a **cooling scheme** is used to define how the temperature decreases

Algorithm SA

```
procedure SA(InitT, MinT, MaxMoves)
   s ← random assignment of variables
   best ← s
   T ← InitT
   while MinT<T do
      num_errors ← 0
      while num_error<MaxMoves do
         next_s ← a random local change of s
         if eval(next_s) < eval(s) then
               s ← next_s
               if eval(s) < eval(best_s) then best ← s
         else
               p ← random number in [0,1)
               if p < e^((eval(s)-eval(next_s))/T) then
                  s ← next_s
               else
                  num_errors ← num_errors+1
      end while
      T ← 0.8 x T
   end while
   return best
end SA
```

Metropolis heuristic



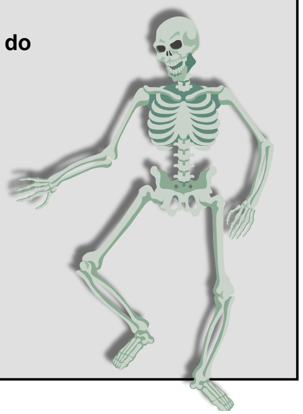
Localizer

Michel, Van Hentenryck (1997)

- The local search algorithms have a similar structure that can be encoded in the common skeleton. This skeleton is filled by procedures implementing a particular technique.

Local Search Skeleton

```
procedure local-search(Max_Tries,Max_Moves)
    s ← random assignment of variables
    for i:=1 to Max_Tries while Gcondition do
        for j:=1 to Max_Moves while Lcondition do
            if eval(s)=0 then
                    return s
            end if
            select n in neighbourhood(s)
            if acceptable(n) then
                    s ← n
            end if
        end for
        s ← restartState(s)
    end for
    return best s
end local-search
```



Local Search - Summary

- **Local search techniques start from some state and by moving to neighbouring states they try to reach a goal state.**
- Each algorithm is specified by:
  - state neighbourhood and allowed states in the neighborhood
  - heuristic to select the next state from the neighbourhood (**intensification**)
  - meta-heuristic to escape local optima (**diversification**)

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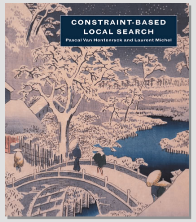

Lokalizer was the base of the **Comet** system (MaxOS X, Linux, Win), that allows description of local search algorithms in a declarative way.

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