Properties of knowledge

(in the Kripke's semantics of possible worlds)

Can we „describe“ all formulas with modalities K_1, .., K_n, that are ever true?

Let us consider a Kripke strukture $M = (S, \pi, K_1, \ldots, K_n)$ and a formula ***A***. We define the following notions:

(i) ***A* *is* *valid* *in* *M*** (denoted as ***M* |= *A***), if ***A*** is true in all the states of ***M***, ie. in any state ***s*** of ***M*** holds (*M, s*) |= ***A*.**

(ii) ***A* *is* *satisfiable* *in* *M***, if there is a state ***t*** in ***M*** such that **(*M*, *t*) |=** ***A***.

(iii) ***A* *is* *valid*** (denoted as |= ***A***), if it is valid in all structures.

(iv) ***A* *is* *satisfiable***, if there is some structure ***M*** such that ***A*** is satisfiable in ***M***.

(v) a *formula* ***B*** *is a* ***logic consequence of* *A***, if ***B*** is valid in any structure ***M***, where ***A*** is valid (whenever***M* |= *A*** , then ***M* |= *B***).

Observation. A formula ***A*** is **valid** (is valid in *M*) <u>if and only if</u> (abr. iff) the formula ¬***A*** is not **satisfiable** (is not satisfiable in *M*).

There are many valid formulas (all propositional tautologies, ...)

*We search for some algorithm that would characterize all **valid formulas** and **logic consequences** using purely syntactic means (that apply transformations of formulas only)!*

Is there a FORMAL SYSTEM, that could do it?

Some examples of a FORMAL SYSTEM:

- *A set of axioms + derivation rules for propositional logic.*
- *Resolution rule for the 1st order logic.*

Let us identify some important valid formulas.

Our agents do know all the logical consequences of their knowledge:
Suppose the agent 1 knows both ***A*** and ***A*** implies ***B*** . This means that

- both formulas ***A*** and $A \rightarrow B$ are true in all the states the agent 1 considers possible,
- ***B*** must be true in all the states the agent 1 considers possible - this means that the agent 1 knows ***B***, too.

This can be written formally as: $\models (K_i A \wedge K_i(A \rightarrow B)) \rightarrow K_i B$

This formula is referred to as the **Distribution Axiom** or Kripke's axiom (denoted as **K**) because it allows to distribute the $\boldsymbol{K}_i$ operator over implication.

*Agents in the Kripke's structures are very strong and competent: Let us consider a structure **M** and a formula **A** valid in **M**. Each agent in **M** knows **A**.*

If ***A*** is true in all states of the structure ***M*** , then ***A*** must be true in all states of the structure ***M*** the agent considers possible. THUS:

*There holds for any structure **M** „if $M \models A$, then $M \models K_i A$"*

This observation confirms correctness of the **Knowledge Generalization** derivation rule *„If A is given, one can derive $K_i A$ "* for any ***i.***

This rule is sometimes depicted in the form

$$\frac{A}{K_i A}$$

Caution!

The Generalization Rule cannot be written in the form $A \rightarrow K_i A$

This formula claims „if A is true, then the agent i knows $\boldsymbol{A}$ “.
But this is NOT a **valid formula**!

An agent does not have to know all facts that are true in the considered state:

In the case of muddy children a child with muddy forehead does not know this fact first. This knowledge is acquired later!

Our agents know all the **formulas valid** in the considered structure, but nothing more! In other words they know only those formulas that are **true** *necessarily*.

They do not have to know formulas, that happen to be true in some of the worlds only (e.g. *by chance*).

Our agent does not have to know all facts that are true. But *if the agent knows something, then it holds*:

$$\models K_i A \rightarrow A$$

This property is often referred to as the ***Knowledge Axiom*** or the ***Truth Axiom*** (denoted as **T**).

Validity of this axiom is a consequence of reflexivity of the admissibility relation describing what the agent considers possible:

If $\boldsymbol{K_i A}$ is true in some world (M, s), $\boldsymbol{A}$ must hold in all states the agent $\boldsymbol{i}$ considers possible – this includes (M, s), since the considered admissibility relation is assumed to be reflexive.

{Philosophers use this axiom to highlight the difference between **knowledge** and **belief**.}

In the case we want to describe belief of an agent instead of its knowledge, it is necessary to replace the ***Truth Axiom***

$$\models K_i A \rightarrow A$$

by a weaker requirement that ensures consistency, namely

$$\neg K_i \mathit{false}$$

This is the ***Consistency Axiom*** , often refered to as **D**.

The next two properties describe what the agents know about their knowledge thanks to introspection. *Our agents know, what they know and what they do not know:*

$$\models K_i A \rightarrow K_i K_i A$$

$$\models \neg K_i A \rightarrow K_i \neg K_i A$$

The first property is called ***Positive Introspection Axiom*** (often denoted as **4**),

The second one is the ***Negative Introspection Axiom*** (often denoted as **5**).

Both are valid in the Kripke structures where admissibily relations are equivalences. ***Try to prove it!***

Formal (axiomatic) system K_n

Axioms: **A1.** All the propositional tautologies

A2. $(K_i \alpha \wedge K_i (\alpha \rightarrow \beta)) \rightarrow K_i (\beta)$

Derivation rules:

R1. From the formulas α and $\alpha \rightarrow \beta$ derive β (**Modus Ponens**)

R2. From the formula α derive $K_i \alpha$ (**Knowledge Generalization Rule**)

Proof of a formula φ in a formal system is a sequence of formulas δ_1, δ_2, **…** , δ_n such that δ_n is the formula φ and for any δ_i **(** $i < n+1$) holds

- either δ_i is an axiom of the considered formal system
- or there are numbers j and k smaller than i such that δ_i is the result of derivation rule application on δ_j or on δ_j are δ_k .

The formula φ is **provable in the formal system** (denoted as $\vdash$φ), if φ has a proof in this system.

Properties of the formal system K_n

Axioms:

A1. All the propositional tautologies

A2. $(K_i \alpha \wedge K_i(\alpha \rightarrow \beta)) \rightarrow K_i(\beta)$

Derivation rules:

R1. From the formulas α and $\alpha \rightarrow \beta$ derive β (**Modus Ponens**)

R2. From the formula α derive $K_i \alpha$ (**Knowledge Generalisation**)

What is the **relation between**

- **the formulas** that are **provable in the** system K_n and
- the formulas **valid** in all the Kripke structures with ***n*** agents ?

Formal system is **correct**, if any provable formula is also valid (ie. „For any formula A there holds that if $\vdash A$ than $\models A$“).

Formal system is **complete**, if all valid formulas can be proven (ie. „For any formula A there holds that if $\models A$ than $\vdash A$“)

$\mathbf{K_n} \vdash K_i(\alpha \wedge \beta) \rightarrow K_i\alpha$:

Formal proof – sequence of formulas**:** [Each formula in the sequence ***must provide a reference to one of*** $\mathbf{K_n}$ ***axioms*** or a ***precise description of the derivation rule*** *as it is applied* to the formulas appearing earlier in the proof].

1. $(\alpha \wedge \beta) \rightarrow \alpha$ [Prop.tautology **]**

2. $K_i((\alpha \wedge \beta) \rightarrow \alpha)$ [**KG:** 1, ie. "**KG** is applied to the formula from the row 1 **]**

3. $(K_i(\alpha \wedge \beta) \wedge K_i((\alpha \wedge \beta) \rightarrow \alpha)) \rightarrow K_i\alpha$ [$\mathbf{K_n}$ distribution axiom]

4. $((K_i(\alpha \wedge \beta) \wedge K_i((\alpha \wedge \beta) \rightarrow \alpha)) \rightarrow K_i\alpha)$

$\rightarrow (K_i((\alpha \wedge \beta) \rightarrow \alpha) \rightarrow (K_i(\alpha \wedge \beta) \rightarrow K_i\alpha))$

[Prop. tautology $((p \wedge q) \rightarrow r) \rightarrow (q \rightarrow (p \rightarrow r))$]

5. $K_i((\alpha \wedge \beta) \rightarrow \alpha) \rightarrow (K_i(\alpha \wedge \beta) \rightarrow K_i\alpha)$ [**MP:** 3,4**]**

6. $K_i(\alpha \wedge \beta) \rightarrow K_i\alpha$ [**MP:** 2,5**]**

$\mathbf{K}_n \vdash K_i(\alpha \wedge \beta) \rightarrow (K_i\alpha \wedge K_i\beta)$

1. $\mathbf{K}_n \vdash K_i(\alpha \wedge \beta) \rightarrow K_i\alpha$ [see the former page]
2. $\mathbf{K}_n \vdash K_i(\alpha \wedge \beta) \rightarrow K_i\beta$ [This proof is a minor modification of that of the formula on the line 1]
3. $(K_i(\alpha \wedge \beta) \rightarrow K_i\alpha) \rightarrow ((K_i(\alpha \wedge \beta) \rightarrow K_i\beta) \rightarrow (K_i(\alpha \wedge \beta) \rightarrow (K_i\alpha \wedge K_i\beta)))$
   $[(\rho \rightarrow \varphi) \rightarrow ((\rho \rightarrow \psi) \rightarrow (\rho \rightarrow (\varphi \wedge \psi)))$ [propositional tautology]
4. $(K_i(\alpha \wedge \beta) \rightarrow K_i\beta) \rightarrow (K_i(\alpha \wedge \beta) \rightarrow (K_i\alpha \wedge K_i\beta))$ [**MP**: 1,3]
5. $K_i(\alpha \wedge \beta) \rightarrow (K_i\alpha \wedge K_i\beta)$ [**MP**: 2,4]

Claim 1: $\mathbf{K}_n \vdash K_i(\alpha \wedge \beta) \equiv K_i\alpha \wedge K_i\beta$

Proof:

- The implication $\rightarrow$ has been proven above.
- The inverse implication is on the next page.

$\mathbf{K}_n \vdash K_i\,\alpha \wedge K_i\,\beta \rightarrow K_i(\alpha \wedge \beta)$

6. $\alpha \rightarrow (\beta \rightarrow (\alpha \wedge \beta))$ [prop.tautology]

7. $K_i(\alpha \rightarrow (\beta \rightarrow (\alpha \wedge \beta)))$ [**KG**:6]

8. $K_i\alpha \rightarrow (K_i(\alpha \rightarrow (\beta \rightarrow (\alpha \wedge \beta))) \rightarrow K_i(\beta \rightarrow (\alpha \wedge \beta)))$ [distribution axiom A2]

9. $K_i(\alpha \rightarrow (\beta \rightarrow (\alpha \wedge \beta))) \rightarrow (K_i\alpha \rightarrow K_i(\beta \rightarrow (\alpha \wedge \beta)))$ [„prop.modification of" 8]

10. $(K_i\alpha \rightarrow K_i(\beta \rightarrow (\alpha \wedge \beta)))$ [**MP**: 9,7]

11. $(K_i\alpha \rightarrow K_i(\beta \rightarrow (\alpha \wedge \beta))) \rightarrow (K_i\beta \rightarrow (K_i\alpha \rightarrow K_i(\beta \rightarrow (\alpha \wedge \beta))))$ [Prop.tautology]

12. $K_i\beta \rightarrow (K_i\alpha \rightarrow K_i(\beta \rightarrow (\alpha \wedge \beta)))$ [**MP** 11,10]

13. $(K_i\,\alpha \wedge K_i\beta) \rightarrow K_i(\beta \rightarrow (\alpha \wedge \beta))$ [„prop.modification of" 12]

14. $(K_i\,\alpha \wedge K_i\beta) \rightarrow (K_i\beta \wedge K_i(\beta \rightarrow (\alpha \wedge \beta)))$ [„prop.modification of"13, see *]

15. $K_i\beta \wedge K_i(\beta \rightarrow (\alpha \wedge \beta)) \rightarrow K_i(\alpha \wedge \beta)$ [A2]

16. $((K_i\,\alpha \wedge K_i\beta) \rightarrow (K_i\beta \wedge K_i(\beta \rightarrow (\alpha \wedge \beta)) \rightarrow K_i(\alpha \wedge \beta)))$
$\rightarrow (((K_i\,\alpha \wedge K_i\beta) \rightarrow K_i\beta \wedge K_i(\beta \rightarrow (\alpha \wedge \beta))) \rightarrow ((K_i\,\alpha \wedge K_i\beta) \rightarrow K_i(\alpha \wedge \beta)))$ [Prop.taut. * *]

17. $(K_i\beta \wedge K_i(\beta \rightarrow (\alpha \wedge \beta)) \rightarrow K_i(\alpha \wedge \beta)) \rightarrow ((K_i\,\alpha \wedge K_i\beta) \rightarrow (K_i\beta \wedge K_i(\beta \rightarrow (\alpha \wedge \beta)) \rightarrow K_i(\alpha \wedge \beta)))$ [P.t. * * *]

18. $((K_i\,\alpha \wedge K_i\beta) \rightarrow (K_i\beta \wedge K_i(\beta \rightarrow (\alpha \wedge \beta)) \rightarrow K_i(\alpha \wedge \beta)))$ [**MP**: 15,17]

19. $(((K_i\,\alpha \wedge K_i\beta) \rightarrow K_i\beta \wedge K_i(\beta \rightarrow (\alpha \wedge \beta))) \rightarrow ((K_i\,\alpha \wedge K_i\beta) \rightarrow K_i(\alpha \wedge \beta)))$ [**MP**: 16, 18]

20. $(K_i\,\alpha \wedge K_i\beta) \rightarrow K_i(\alpha \wedge \beta)$ [**MP**: 19, 14]

* If |-(A&B)→C then |- (A&B)→(B & C))

* * (α→ (β→γ)) → ((α → β) → (α → γ))

* * * (α→ (β→ α))

Theorem (see the lab work presentation).

For all structures $\boldsymbol{M}$ with n agents where the admissibility relations are interpreted by relations that are equivalences, there holds for any formulas A , B :

$$(i)\ \ M \models (K_i A \wedge K_i(A \to B)) \to K_i B$$

$$(ii)\ \ if \quad M \models A \quad than \quad M \models K_i A$$

$$(iii)\ \ M \models K_i A \to A$$ reflexivity

$$(iv)\ \ M \models K_i A \to K_i K_i A$$ transitivity

$$(v)\ \ M \models \neg K_i A \to K_i \neg K_i A$$ symmetric+transitive r.

Axioms of propositional modal logics

1. **Propositional tautologies**
2. **Distribution Axiom** (denoted as **K**) $(K_i A \wedge K_i(A \rightarrow B)) \rightarrow K_i B$
3. **Knowledge Axiom** (denoted as **T**) $K_i A \rightarrow A$ r
4. **Positive Introspection Axiom** (den.as **4**) $K_i A \rightarrow K_i K_i A$ t
5. **Negative Introspection Axiom** (den.as **5**) $\neg K_i A \rightarrow K_i \neg K_i A$ s+t
6. **Consistency Axiom** (den.as **D**) $\neg K_i \mathit{false}$

Derivation rules:

R1. From the formulas α and $\alpha \rightarrow \beta$ derive β (**Modus Ponens**)

R2. From the formula α derive $K_i\, \alpha$ (**Knowledge Generalization**)

Proof of a formula φ in the formal system **under assumption** α is a sequence of formulas $\delta_1, \delta_2, \dots, \delta_n$ such that δ_n is the formula φ and for any δ_i ($i < n+1$) holds

- either δ_i is an axiom of the considered formal system or the assumption α
- or there are numbers j and k smaller than i such that δ_i is the result of derivation rule application on δ_j or on δ_j are δ_k .

The formula φ is **provable** in the formal system **under assumption** α (denoted as $\alpha \vdash \varphi$), if φ has a proof **under assumption** α .

Claim 1 $\mathbf{K}_n$, $(\varphi \to \psi) \vdash K_i\, \varphi \to K_i\, \psi$

(The formula $K_i\, \varphi \to K_i\, \psi$ is a consequence of the assumption $(\varphi \to \psi)$ in the formal system $\mathbf{K}_n$) : $(\varphi \to \psi)$ [assumption]

1. $K_i(\varphi \to \psi)$ [KG „assumption“]
2. $K_i\varphi \to (K_i(\varphi \to \psi) \to K_i\psi)$ [distribution axiom **K**]
3. $(K_i\varphi \to (K_i(\varphi \to \psi) \to K_i\psi)\,) \to (K_i(\varphi \to \psi) \to (K_i\varphi \to K_i\psi)\,)$ [Prop-T1: $(\varphi \to (\psi \to \tau\,)) \to (\psi \to (\varphi \to \tau\,))$]
4. $K_i(\varphi \to \psi) \to (K_i\varphi \to K_i\psi)$ [MP 2,3]
5. $(K_i\varphi \to K_i\psi)$ [MP 1,4]

Claim 2: Let the formulas φ , ψ be equivalent (ie. the formula $(\varphi \to \psi) \wedge (\psi \to \varphi)$ is a tautology, denoted as $\varphi \equiv \psi$). There holds $\mathbf{K}_n$, $\varphi \equiv \psi \mid\!- K_i\, \varphi \equiv K_i\, \psi$.

This is a direct consequence of the above statement .

Let us denote by $\mathcal{M}_n(\Phi)$ the set of all Kripke structures over the set Φ of primitive propositions and a set of n agents. Denote that no requirements are set on the relations K_i in this case.

Let $\mathcal{M}_n^{rst}(\Phi)$ be the subset of $\mathcal{M}_n(\Phi)$ consisting of all the Kripke structures from where all the admissibility relations have the identified properties ***rst***, namely they are:

reflexive

symetric

transitive.

(ie. The considered admissibility relationsare **equivalences**).

Theorem 1: The system $\mathbf{K}_n$ represents correct and complete syntactic description of all formulas that are valid in the set $\mathcal{M}_n(\Phi)$ of all Kripke struktures ($\mathbf{K}_n$ is an axiomatization w.r.t. $\mathcal{M}_n(\Phi)$).

Theorem 2:

Let T be the axiom $K_i A \rightarrow A$. The system $\mathbf{T}_n = (\mathbf{K}_n +$ axiom T) is the axiomatization w.r.t. $\mathcal{M}_n^{r}(\Phi)$.

Let 4 be the axiom $K_i A \rightarrow K_i K_i A$. The system $\mathbf{S}4_n = (\mathbf{T}_n +$ axiom 4)) is the axiomatization w.r.t. $\mathcal{M}_n^{rt}(\Phi)$.

Let 5 be the axiom $\neg K_i A \rightarrow K_i \neg K_i A$. The systém $\mathbf{S}5_n = (\mathbf{S}4_n +$ axiom 5) is the axiomatization w.r.t. k $\mathcal{M}_n^{rts}(\Phi)$.

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Some more valid statements:

c1) $\mathbf{K_n}$, T(Axiom 3: $K_i\boldsymbol{\alpha} \rightarrow \boldsymbol{\alpha}$) $\vdash \neg K_i \textit{\textbf{false}}$

1. $K_i \textit{\textbf{false}} \rightarrow \textit{\textbf{false}}$ [A3]
2. $\neg \textit{\textbf{false}} \rightarrow (\neg K_i \textit{\textbf{false}})$ [prop.modification of 1]
3. $\neg \textit{\textbf{false}}$ [prop.tautology]
4. $\neg K_i \textit{\textbf{false}}$ [MP: 3,2]

c2) $\mathbf{K_n}$, T $\vdash \neg K_i \boldsymbol{\alpha} \vee \neg K_i \neg K_i \boldsymbol{\alpha}$

c3) $\mathbf{K_n}$, T $\vdash \neg K_i (\boldsymbol{\alpha} \wedge \neg K_i \boldsymbol{\alpha})$

1. $K_i \neg K_i \boldsymbol{\alpha} \rightarrow \neg K_i \boldsymbol{\alpha}$ [A3, Truth Axiom]
2. $\neg K_i \neg K_i \boldsymbol{\alpha} \vee \neg K_i \boldsymbol{\alpha}$ [prop.modification of $\rightarrow$ in 1], *viz* a1
3. $\neg (K_i \neg K_i \boldsymbol{\alpha} \wedge K_i \boldsymbol{\alpha})$ [prop.modification of $\vee$ in 2]
4. $\neg K_i (\neg K_i \boldsymbol{\alpha} \wedge \boldsymbol{\alpha})$ [transitivity of K_i in the formula 3], *viz* a2

Some more relations that can be proven:

a) $(\mathrm{K}_n + \mathrm{A6}) \vdash \neg (K_i \boldsymbol{\alpha} \wedge K_i \neg \boldsymbol{\alpha})$

b) $(\mathrm{K}_n + \mathrm{A3}) \vdash \mathrm{A6}$

c) $\mathrm{K}_n \vdash K_i \neg (\boldsymbol{p} \rightarrow K_i \boldsymbol{p}) \equiv K_i (\boldsymbol{p} \wedge \neg K_i \boldsymbol{p}) \equiv (K_i \boldsymbol{p} \wedge K_i (\neg K_i \boldsymbol{p}))$

d) It is not possible to prove $K_i \neg (\boldsymbol{p} \rightarrow K_i \boldsymbol{p})$ in $(\mathrm{K}_n + \mathrm{A3})$.

$\boldsymbol{E_G} \quad \boldsymbol{C_G} \quad \boldsymbol{D_G}$

Let $\boldsymbol{G}$ be a subset of {1, 2, . . . , n}, $\boldsymbol{E_G A}$ holds iff every agent from $\boldsymbol{G}$ knows $\boldsymbol{A}$. Thus

Axiom C1. $$E_G A \Leftrightarrow \underset{i \in G}{\&} K_i A$$

Intuitively, **common knowledge** specifies something „*what is crystal clear to everyone*“. It should be no surprise that common knowledge has the properties that have been described in the **Distribution Axiom**, in the **Knowledge Axiom**, and in the **Positive** and **Negative Introspection Axioms**, see the next page.

Common knowledge of two groups of agents:

$$\textit{If} \quad Q \subseteq G \quad \textit{then} \quad C_G A \rightarrow C_Q A$$

It can be verified that the following formulas are valid (they are true in all Kripke structures):

$(i) \quad (C_G A \,\&\, C_G(A \rightarrow B)) \rightarrow C_G B$

$(ii) \quad C_G A \rightarrow A$

$(iii) \quad C_G A \rightarrow C_G C_G A$

$(iv) \quad \neg C_G A \rightarrow C_G \neg C_G A$

The assumptions on properties of the underlying admissibility relations for all $\boldsymbol{K_i}$ are the same as in the case of reasoning about knowledge.

Distributed knowledge

charakterize knowledge the agents can acquire when „*all of them share all their individual knowledge*“.

Even this modal operator has simmilar properties (axioms) as knowledge of a single agent. Let us point to some specific cases:

- *Distributed knowledge in the group with a single agent is that of the agent*, namely $\models D_{\{i\}} A \leftrightarrow K_i A$
- *The bigger the considered group the bigger their distributed knowledge :*

$$If \quad Q \subseteq G \; then \; \models D_Q A \rightarrow D_G A$$

Task7 Could the modality be defined as a boolean function?(2 points)

Let us consider for simplicity only Kripke structures with a single agent whose knowledge is described by the modal operator ***K***. We know that in all the corresponding Kripke structures where ***K*** is interpreted by equivalence there holds for any formula α

a) the formula ***K*** $\alpha \rightarrow \alpha$ (Knowledge Axiom) is valid ,

b) but the formulas $\alpha \rightarrow$ ***K*** α and $\neg$ ***K*** α are not valid.

Utilize these facts to show that such a behaviour of the modal operator ***K*** cannot be encoded by any boolean function (ie. Truth values defined by a table).

Hint: Suppose the truth value of ***K*** α can be calculated from the truth value of α using a truth table for ***K*** (in the same way as $\neg\alpha$ is calculated form α). Consider all possible truth tables for ***K*** and show that none of them grants the properties a) and b) mentioned above.

Task 8 Math puzzle (1 point)

X and ***Y*** are two different whole numbers greater than 1. Their sum is not greater than 100, and ***Y*** is greater than ***X***.

S and **P** are two mathematicians (and consequently perfect logicians); **S** knows the sum ***X* + *Y*** and **P** knows the product ***X* * *Y***. Both **S** and **P** know nothing about ***X*** and ***Y*** but the facts listed in this paragraph. The following conversation occurs:

- **S** says "**P** does not know ***X*** and ***Y***."
- **P** says "Now I know ***X*** and ***Y***."
- **S** says "Now I also know ***X*** and ***Y***."

Can the input of ***X***=4 and ***Y***=13 result in this conversation?

Task 9 Ann and Bob (2 points)

Ann and **B**ob take part in a quizz. First, the organizer selects from an urn a natural number ***n*** < **10**, that he writes on the forehead of one of the players and continues by writing the neighboring number (either ***n*+1** or ***n*-1**) on the forhead of the second player. Neither **A**nn nor **B**ob knows her/his number – each sees only the other´s forehead. They can take turns in announcing nothing but „*I do not know my number*." or „*I know my number*." Suppose Ann starts and she can see the symbol **5**.

- Who will be the first to identify her/his number?
- Demonstrate your conclusion about the winner using the corresponding Kripke structure and its modification during information exchange between **A** and **B**.