Medical Imaging
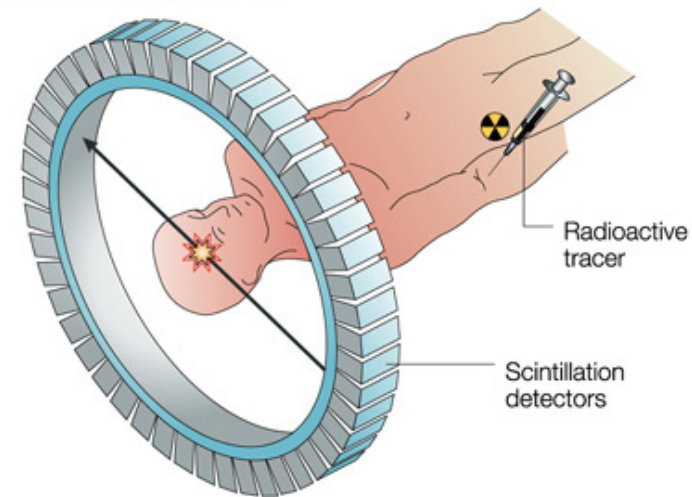# Positron Emission Tomography
(Outline of Lecture 6)

Nuclear Medicine

♦ Pathological conditions are initiated by a change in basic biochemistry of tissue.

♦ Detect these indicators by uptake of radioactive compounds (radio-pharmaceuticals).

♦ Abnormal tissue distribution of radio-pharmaceuticals $\Rightarrow$ strong indicator of disease.

## Positron Emission Tomography:

a Patient in a scanner

Radioactive tracer

Scintillation detectors

A. Basic Principles

◆ Radio-pharmaceuticals emit positrons, positron annihilates with an electron, two photons (511 KeV) travelling in opposite directions.

◆ Detect coincident pairs of photons, this defines the line along which the annihilation took place.

◆ Positron emitting isotopes ^{18}F, ^{11}C, ^{15}O, ^{13}N produced in a synchrotron. Half-times 2min – 110min.

◆ Rapid chemical synthesis of structural analogues of biologically active molecules: ^{15}O labelled water, fluorodeoxyglucose (FDG), $[^{18}$F]DOPA,...

B. Instrumentation

Summed signal
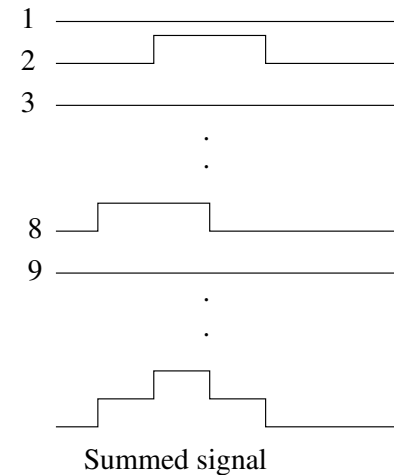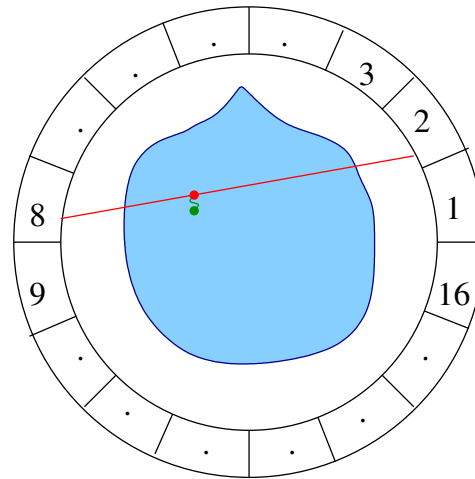
♦ Detectors:

- scintillation crystal: converts high energy photons into a shower of low energy photons (visible part of the spectrum), maximal count rates: 10^5 photons/second.

- photomultiplier tube: amplification up to 10^6 electrons per photo-electron.

♦ Planar ring of detectors

♦ Coincidence detection circuitry: measures coincident events by summing and thresholding logic pulses triggered by detector events.

C. Image reconstruction

We assume for the sake of simplicity:

♦ Travel distance of the positron before annihilation ≈ 0.

♦ All measured coincidences are correct.

♦ Consider only those emissions which are actually detected by some detector pair.

Denotations:

♦ $x(r)$ – density of the radio-pharmaceutical in the voxel $r \in V$

♦ $s(d)$ – number of coincidences detected in the tube $d \in D$

♦ Observation: $S(D) = \{s(d) \mid d \in D\}$
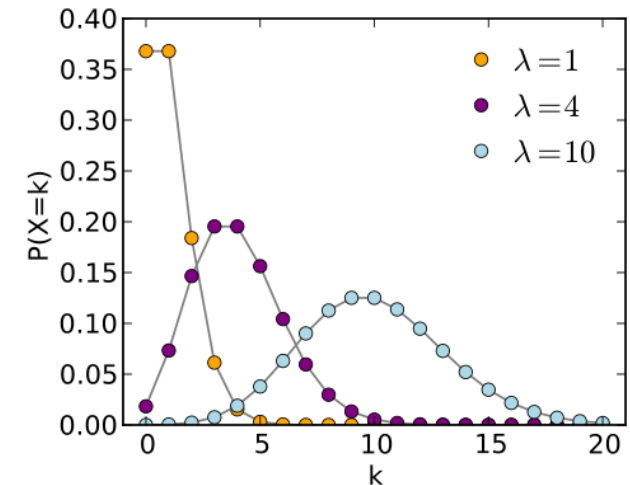
♦ Unknown: $X = \{x(r) \mid r \in V\}$
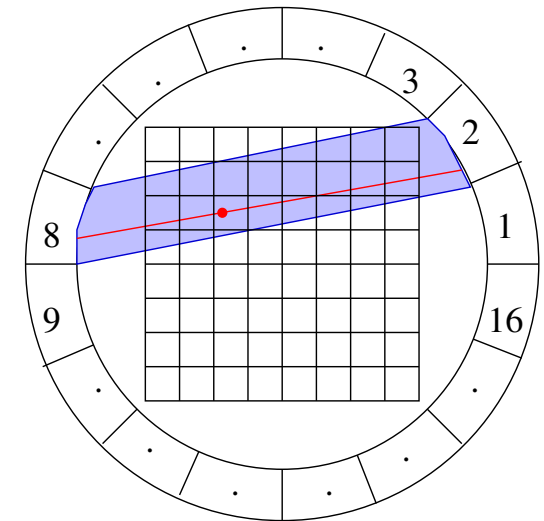
(1) Let λ denote the density of the r.p. in a voxel. How many emissions per unit of time we expect?
Poisson distribution

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}$$

(2) If an emission occurs in voxel $r \in V$, what is the probability that it will be detected by the tube $d \in D$?. Let us denote this probability by $\pi(d \mid r)$. These probabilities can be calculated by geometrical considerations. Clearly, $\sum_{d \in D} \pi(d \mid r) = 1$ holds.

(3) Let $s(d, r)$ denote the number of emissions in voxel $r \in V$ detected by the tube $d \in D$. This quantity is also Poisson distributed.

We have therefore

$$p(s; x) = \prod_{r \in V} \prod_{d \in D} e^{-\pi(d|r)x(r)} \frac{\left[\pi(d \mid r)x(r)\right]^{s(d,r)}}{s(d,r)!}$$

We don't know $s(d, r)$. But we know $S(d) = \{s(d) \mid d \in D\}$, where $s(d) = \sum_{r \in V} s(d, r)$.

Therefore solve the task

$$\sum_{s \in S(D)} p(s; x) = \sum_{s \in S(D)} \prod_{r \in V} \prod_{d \in D} e^{-\pi(d|r)x(r)} \frac{\left[\pi(d \mid r)x(r)\right]^{s(d,r)}}{s(d, r)!} \rightarrow \max_{x}$$

Expectation Maximisation Algorithm: Start with some initial estimate of x^0. Then maximise iteratively

E-step Given the current estimate of $x^{(t)}$, calculate an estimate for $s(d, r)$:

$$s(d, r) \approx \mathbb{E}\left[s(d, r) \mid s(D), x^{(t)}\right]$$

M-step Given the estimate for $s(d, r)$, re-estimate x:

$$x^{(t+1)} = \arg\max_{x} p(s; x)$$

Joining both steps gives the update formula

$$x^{new}(r) = x(r) \sum_{d \in D} \frac{s(d)\pi(d \mid r)}{\sum_{r' \in V} x(r)\pi(d \mid r)}$$

Positron Emission Tomography (PET)

D. Clinical applications

- ◆ Brain imaging

  - Measuring regional cerebral blood flow: use ^{15}O-labelled water

  - Measuring tissue metabolism: use FDG which passes BBB, enter cells, phosphorylates, is trapped in the cell

  - Measure the rate of dopamine synthesis: use [^{18}F]DOPA
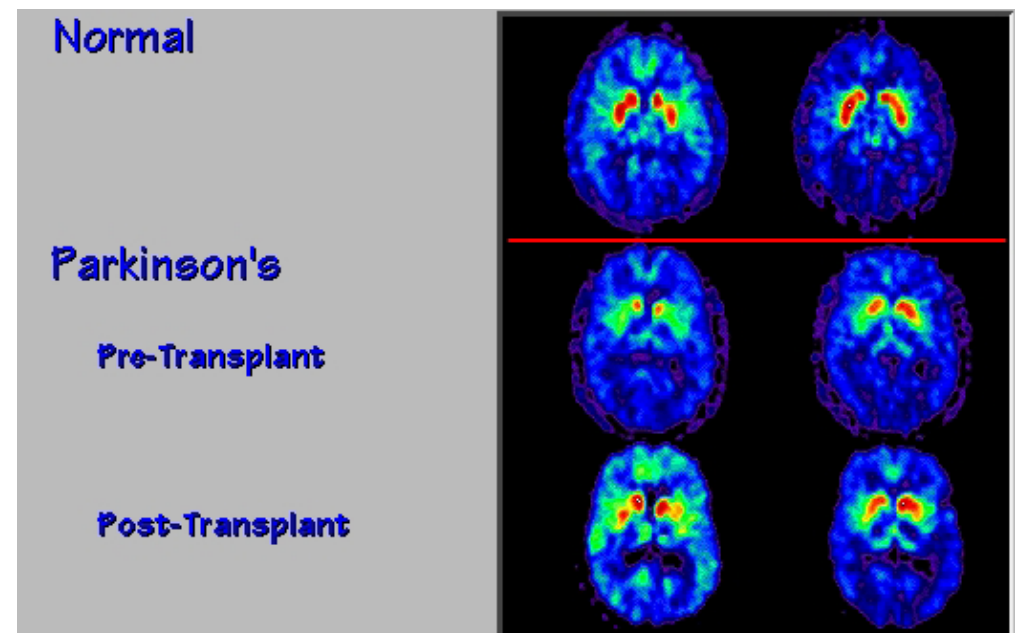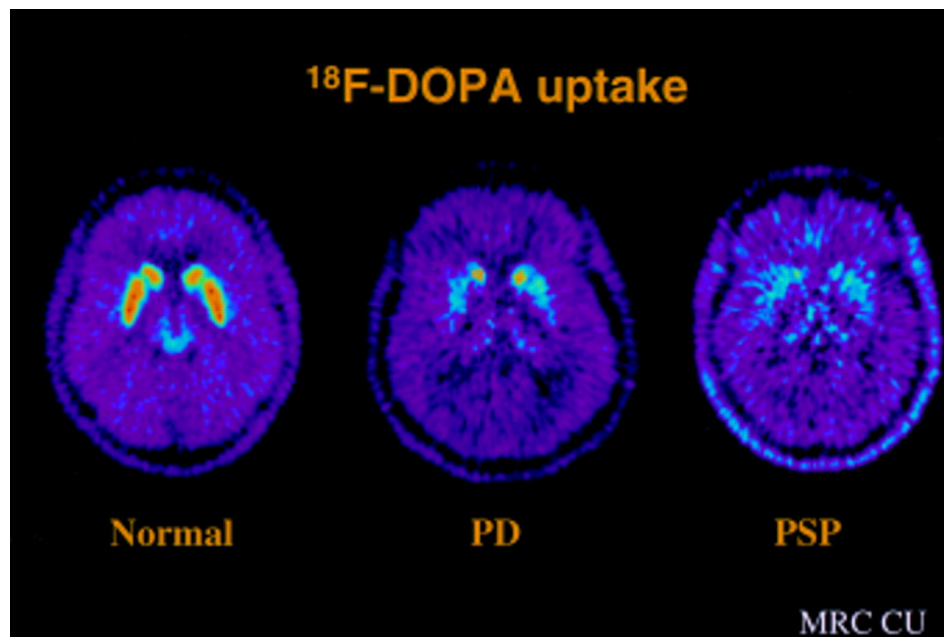
◆ Cardiac studies

- Measure blood flow: use $^{13}NH_3$.

- Measure metabolism: use FDG

◆ Tumor and metastatic cancer diagnosis

- Measure metabolism (e.g. whole body scan): use FDG.