Medical Imaging
Bloch equations, magnetic resonance imaging
(Outline of Lecture 2)

2. Bloch equations, Spin echo imaging

A. Bloch equations Torque on a dipole in an external field

$$\vec{\tau} = \vec{m}_{\mathrm{mg}} \times \vec{B}_0$$

where $\vec{m}_{\mathrm{mg}} = I\vec{A}$ for a planar circular current.

Motion equation for rigid bodies, rotational part

$$\dot{\vec{L}} = \vec{\tau} \quad \text{where } \vec{L} \text{ – angular momentum}$$

Here: charged particles moving on circular trajectories: $\vec{m}_{\mathrm{mg}} = \frac{q}{2m}\vec{L}$. Hence we obtain the motion equation

$$\dot{\vec{m}}_{\mathrm{mg}} = \gamma \vec{m}_{\mathrm{mg}} \times \vec{B}$$

Solution for a homogeneous and static external magnetic field $\vec{B}_0$:
Dipol precession with Larmor frequency $\vec{\omega} = \gamma \vec{B}_0$.

2. Bloch equations, Spin echo imaging

Macroscopic effect of many dipoles $\Rightarrow$ dipole density $\vec{M}(\vec{r})$

$$\dot{\vec{M}}(\vec{r},t) = \gamma \vec{M}(\vec{r},t) \times \vec{B}(\vec{r},t)$$

where $\vec{B}$ is the local magnetic field "seen" by dipoles at $\vec{r}$. $\vec{B} = \vec{B}_{\text{ext}} + \vec{B}_{\text{ind}}$

"Replace" unknown induced field $\vec{B}_{\text{ind}}$ by phenomenological terms accounting for relaxation

(1) $\vec{B}_{\text{ext}} = B_0 \vec{e}_z + (\vec{g} \cdot \vec{r})\vec{e}_z + \vec{B}_{\text{RF}}$

(2) Decay time T_1 for M_z relaxation (spin-lattice interaction). Protons loose energy to the surrounding "lattice". Different tissues have different values of T_1 (tissue contrast!).

(3) Decay time T_2 for $M_T = (M_x, M_y)$ relaxation (spin-spin interaction) involves the loss of "phase coherence" between protons precessing in the transverse plane. Again, different tissues have different values of T_2. For all tissues $T_2 \ll T_1$.

2. Bloch equations, Spin echo imaging

Bloch equations:

$$\dot{M}_x(\vec{r},t) = \gamma\Big[\vec{M}(\vec{r},t) \times \vec{B}_{\text{ext}}(\vec{r},t)\Big]_x - \frac{M_x(\vec{r},t)}{T_2}$$

$$\dot{M}_y(\vec{r},t) = \gamma\Big[\vec{M}(\vec{r},t) \times \vec{B}_{\text{ext}}(\vec{r},t)\Big]_y - \frac{M_y(\vec{r},t)}{T_2}$$

$$\dot{M}_z(\vec{r},t) = \gamma\Big[\vec{M}(\vec{r},t) \times \vec{B}_{\text{ext}}(\vec{r},t)\Big]_z - \frac{M_z(\vec{r},t) - M_0}{T_1}$$

B. Radiofrequency pulses

Applying a transversal rotating magnetic field with Larmor frequency i.e.

$$\vec{B}_{\text{ext}} = \vec{B}_0 + \vec{B}_{\text{RF}} = B_0\vec{e}_z + B_{\text{RF}}\cos(\omega t)\vec{e}_x + B_{\text{RF}}\sin(\omega t)\vec{e}_x.$$

As a result, magnetization $\vec{M}$ rotates around $\vec{B}_{\text{RF}}$ (rotating reference frame) by an angle α.

$$\alpha = \gamma\|B_{\text{RF}}\|\tau.$$

A $\pi/2$-pulse rotates the magnetization by $\pi/2$.

A π-pulse rotates the magnetization by π.

2. Bloch equations, Spin echo imaging

C. Measuring T_1 and T_2

If a α RF pulse is applied, then at a time t after this pulse

$$M_z(t) = M_0 \cos\alpha + M_0(1-\cos\alpha)(1-e^{-t/T_1})$$
$$M_{x,y}(t) = M_0 \sin\alpha \, e^{-t/T_2}$$

where M_0 denotes the equilibrium magnetization.

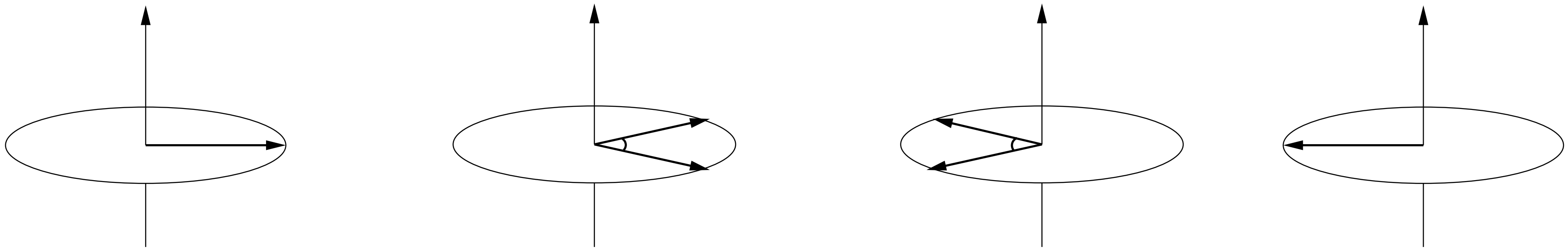
(1) Measuring T_1: apply a π-pulse, and a $\pi/2$-pulse after a variable delay τ. Measured signal $S(\tau)$

$$S(\tau) = M_0(1-2e^{-\tau/T_1})$$

(2) Measuring T_2: apply a $\pi/2$-pulse followed by a delay τ, then apply a π-pulse followed by the same delay τ, then measure. Measured signal $S(\tau)$

$$S(\tau) = M_0 e^{-2\tau/T_2}$$

2. Bloch equations, Spin echo imaging



$\pi/2$-pulse | defocussing after time τ | π-pulse | refocusing after τ

2. Bloch equations, Spin echo imaging

D. Slice selective RF pulses

Apply:

- Gradient field $(\vec{g} \cdot \vec{r})\vec{e}_z = g_0 z \vec{e}_z$
- Rotating excitation field $\vec{B}_{\text{RF}}$ with constant frequency spectrum in some interval $\omega = \gamma(B_0 + g_0 \bar{z}) \pm \Delta$ i.e. sinc-function!

for the time $T = \pi/2\gamma\|\vec{B}_{\text{RF}}\|$ (i.e. $\pi/2$-pulse).

Result:

- Spins in the slice around $\bar{z}$ precess in the x-y plane
- They dephase (T_2) due to spin-spin interactions and local inhomogeneities.

Applying a π-pulse $\Rightarrow$ refocusing.

2. Bloch equations, Spin echo imaging

E. x-y Gradients

Application of x-y gradient field after RF pulses, $B_{\text{ext}} = \big(B_0 + \vec{g}(t)\cdot\vec{r}\big)\vec{e}_z$.

Simplification: omit decay terms. Denote $M_T = M_x + iM_y$. Bloch equation for transversal magnetization becomes

$$\dot{M}_T = -i\gamma\big[B_0 + \vec{g}(t)\cdot\vec{r}\big]M_T$$

Integration by separation of variables gives

$$M_T(\vec{r},t) = M_T(\vec{r},0)\exp\Big[-i\gamma\Big(B_0 t + \vec{r}\cdot\int_0^t \vec{g}(t')dt'\Big)\Big]$$

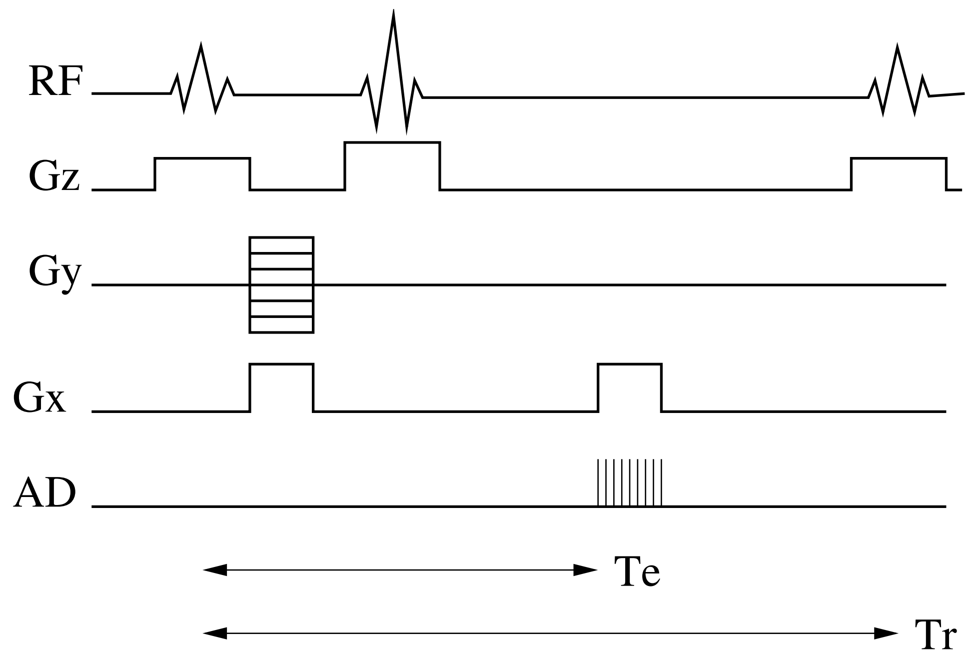
Total transverse magnetization is

$$M_T(t) = \iint_{\text{slice}} dxdy\, M_T(x,y,t) =$$
$$e^{-i\gamma B_0 t}\iint_{\text{xy}} dxdy\, M_T(x,y,0)\exp\Big[-i\big(k_x(t)x + k_y(t)y\big)\Big]$$

The gradient $\vec{g}(t)$ has to be arranged so as to measure the Fourier transformed of $M_T(x,y,0)$ for all k_x, k_y.

2. Bloch equations, Spin echo imaging

E. Spin-Echo Imaging Sequence



The intensity of an axial acquired image is

$$I(x,y) = \rho(\vec{x,y})\left[1 - \exp^{-T_R/T_1(x,y)}\right]\exp^{-T_E/T_2(x,y)}$$

Gives images with high contrast and SNR, but takes several minutes to acquire. Other sequences are used if fast acquisition is necessary.