ALG 07

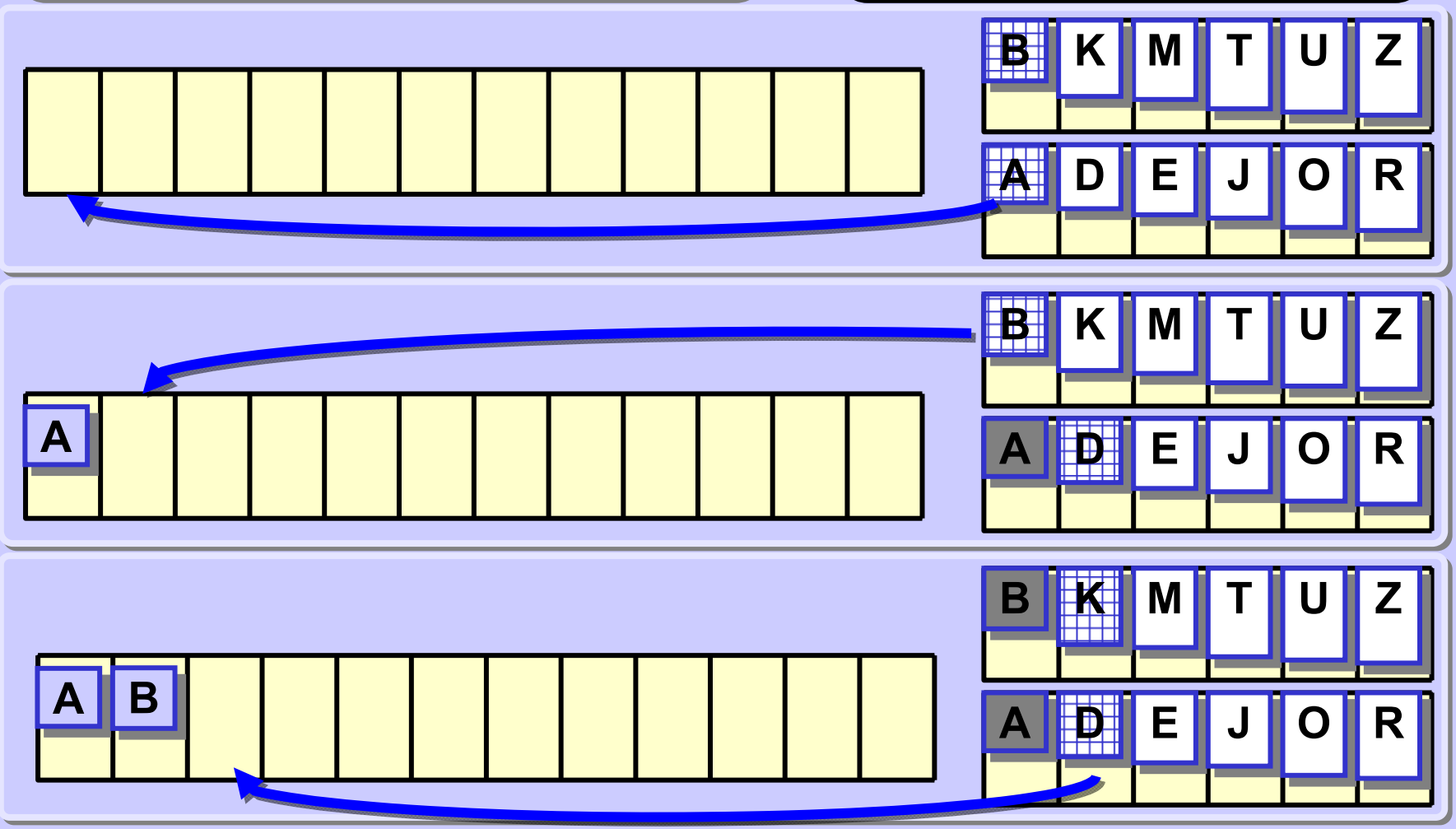
Merge sort (řazení sléváním)

Heap Sort (řazení haldou)

Merge sort

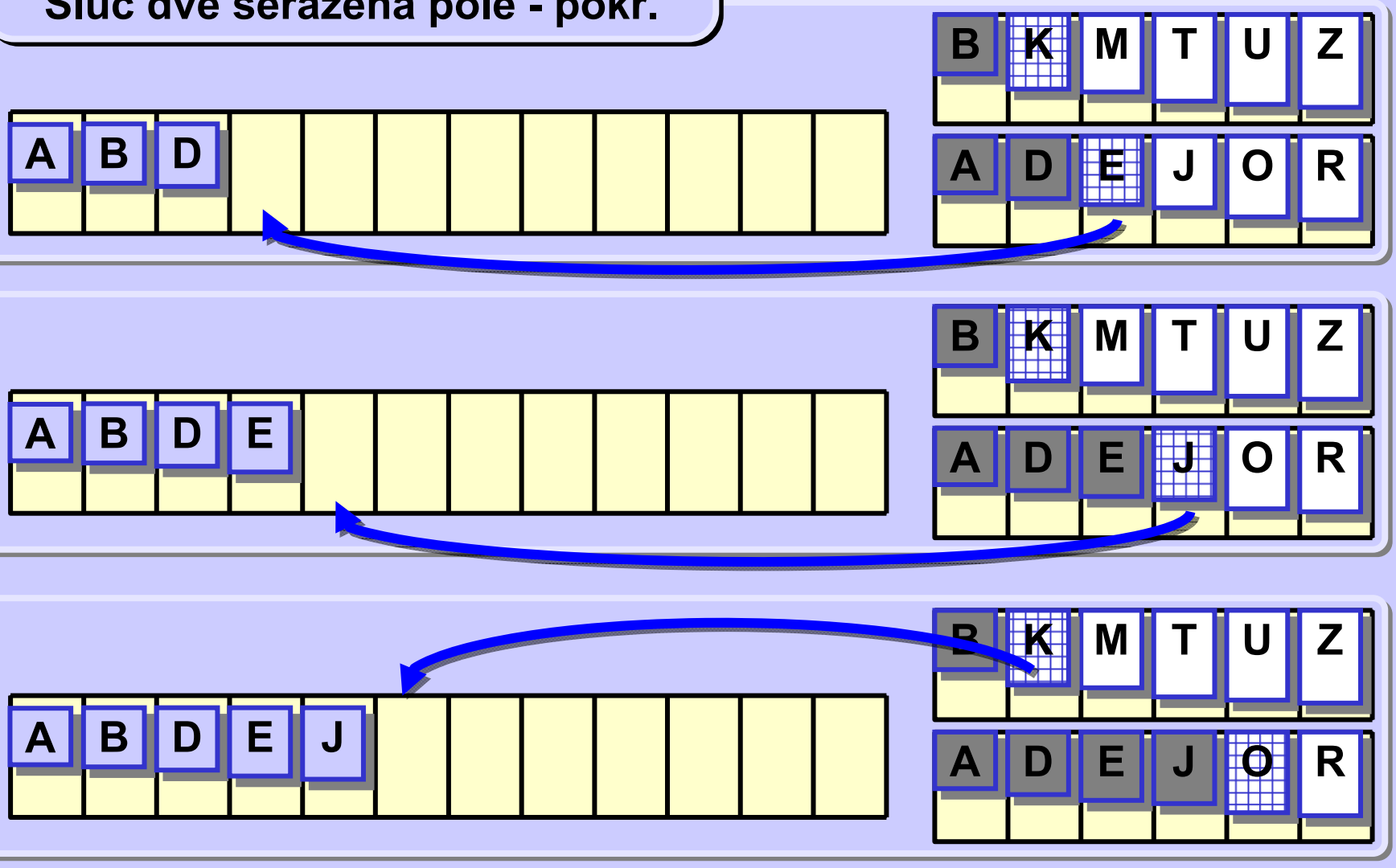
Sluč (slij?) dvě seřazená pole

Porovnávané prvky



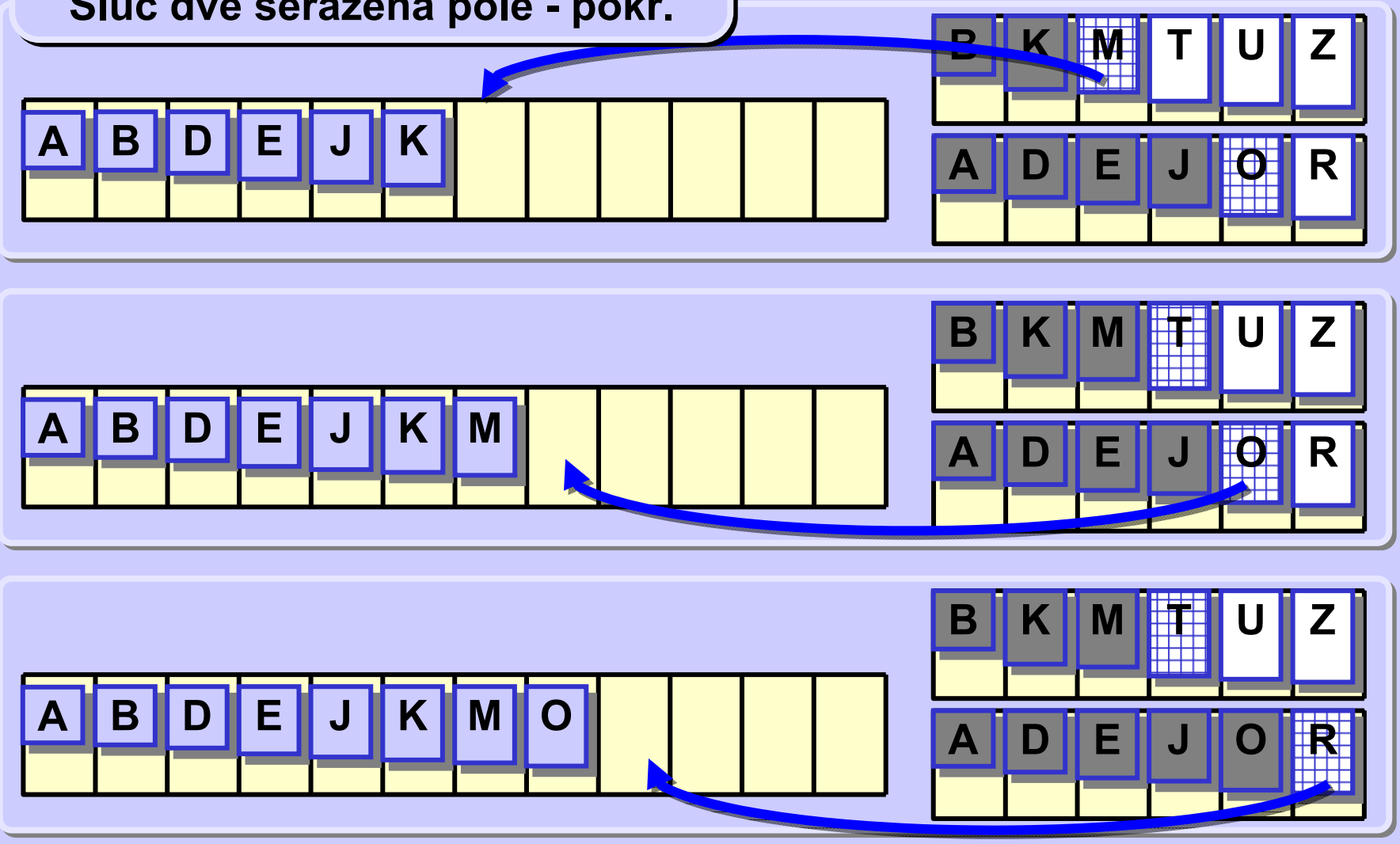
Merge sort

Sluč dvě seřazená pole - pokr.



Merge sort

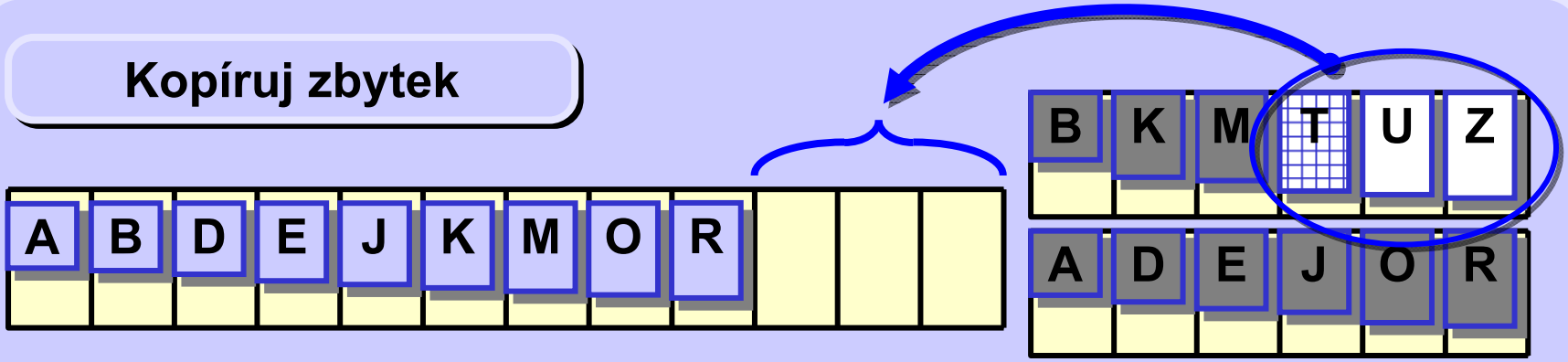
Sluč dvě seřazená pole - pokr.



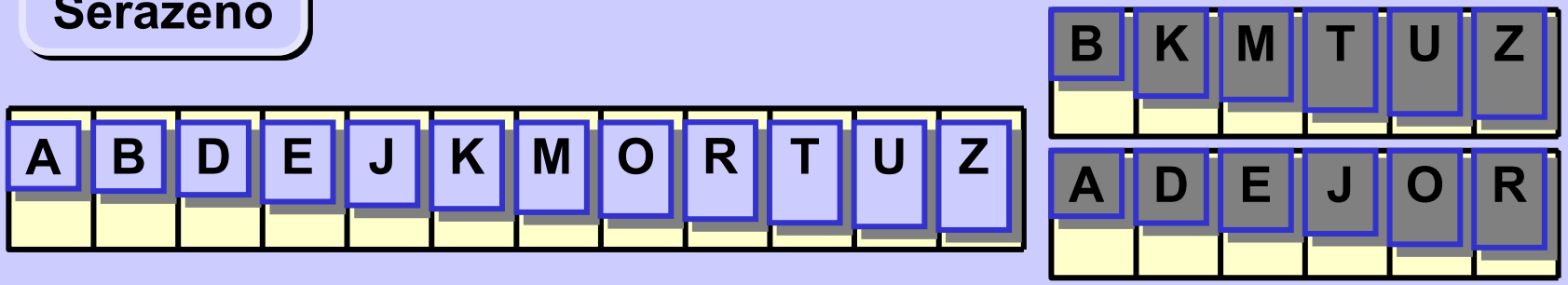
Merge sort

Sluč dvě seřazená pole - pokr.

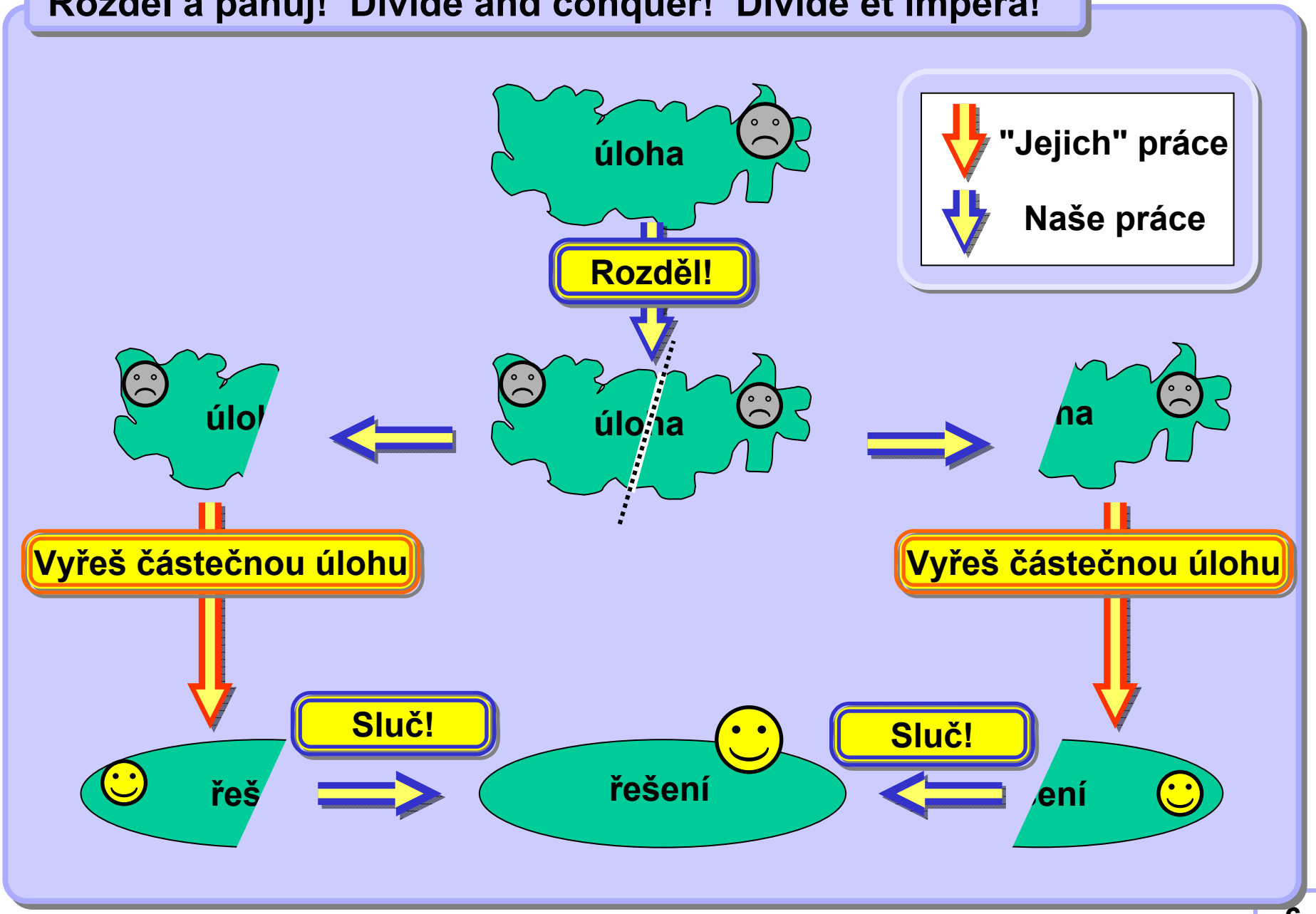
Kopíruj zbytek



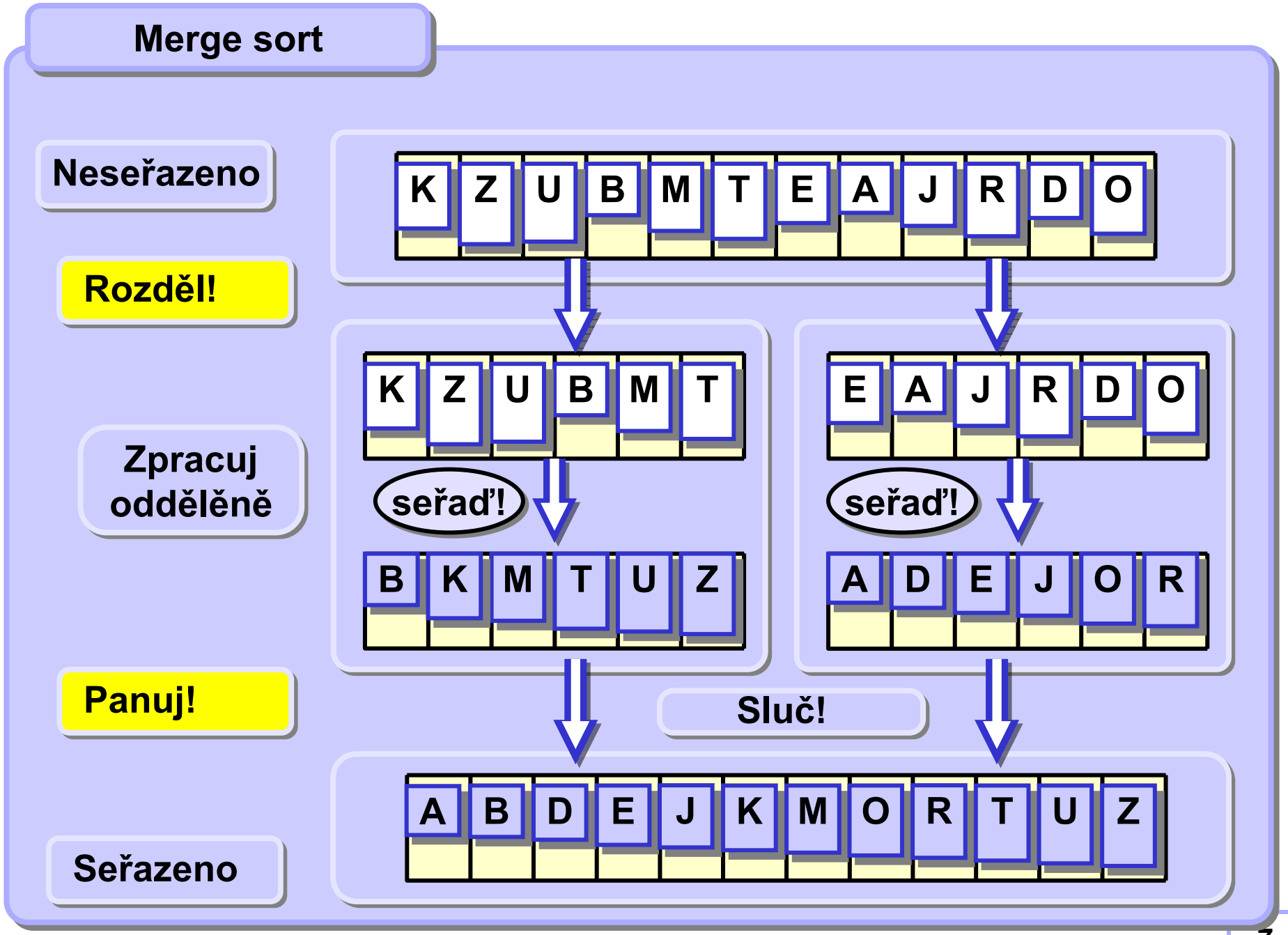
Seřazeno



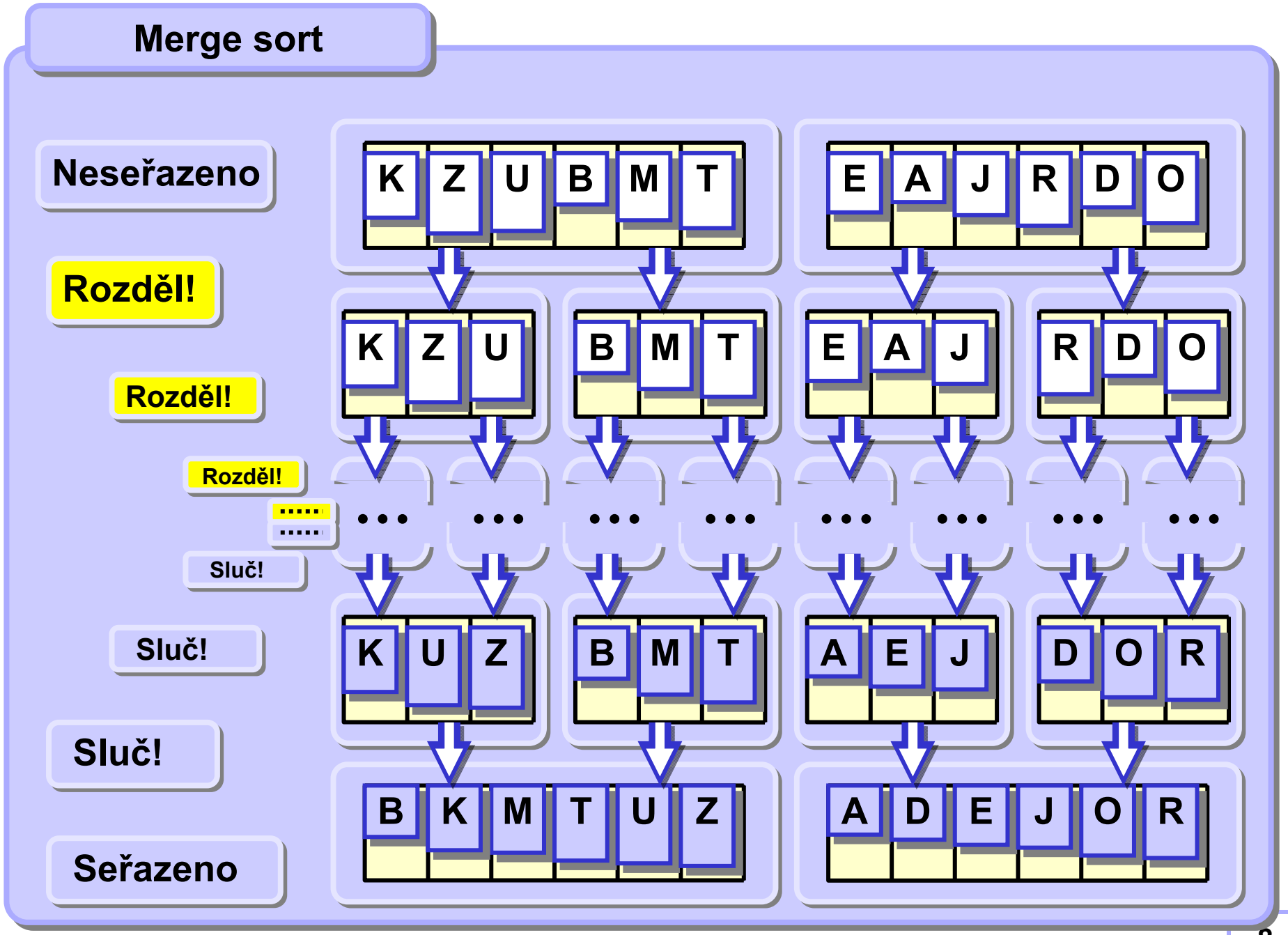
Rozděl a panuj! Divide and conquer! Divide et impera!



Merge sort


Neseřazeno
K Z U B M T E A J R D O
Rozděl!
K Z U B M T
E A J R D O
Zpracuj odděleně
seřaď!
seřaď!
B K M T U Z
A D E J O R
Panuj!
Sluč!
A B D E J K M O R T U Z
Seřazeno

Merge sort


Neseřazeno
Rozděl!
Rozděl!
Rozděl!
Sluč!
Sluč!
Sluč!
Seřazeno
K Z U B M T
E A J R D O
K Z U
B M T
E A J
R D O
K U Z
B M T
A E J
D O R
B K M T U Z
A D E J O R

Merge sort

```
void mergeSort (int a[], int aux[],
                                int low, int high)  {
  int half = (low+high)/2;
  int i;
  if (low >= high) return;          // too small!
                                // sort
  mergeSort(a, aux, low, half);    // left half
  mergeSort(a, aux, half+1, high); // right half
  merge(a, aux, low, high);        // merge halves

                                // put result back to a
  for (i = low; i <= high; i++)  a[i] = aux[i];

 // optimization idea:
  /* swapArray(a, aux) */ // better to swap
                                // references to a & aux!
}
```

Merge sort

```
void merge(int in[], int out[], int low, int high) {
  int half = (low+high)/2;
  int i1 = low;
  int i2 = half+1;
  int j = low;

                                  // compare and merge
  while ((i1 <= half) && (i2 <= high)) {
    if (in[i1] <= in[i2]) { out[j] = in[i1]; i1++; }
    else { out[j] = in[i2]; i2++; }
    j++;
  }
                                 // copy the rest
  while (i1 <= half) { out[j] = in[i1]; i1++; j++; }
  while (i2 <= high) { out[j] = in[i2]; i2++; j++; }
}
```

Merge sort

Asymptotická sožitost

Rozděl! ..... $\log_2(n)$ krát $\Rightarrow$

$\Rightarrow$ Sluč! ..... $\log_2(n)$ krát

Rozděl! ..... $\Theta(1)$ operací

Sluč! ..... $\Theta(n)$ operací

Celkem ..... $\Theta(n) \cdot \Theta(\log_2(n)) = \Theta(n \cdot \log_2(n))$ operací

Asymptotická složitost Merge sortu je $\Theta(n \cdot \log_2(n))$

Merge sort

Stabilita

Rozděl! ..... Nepohybuje prvky

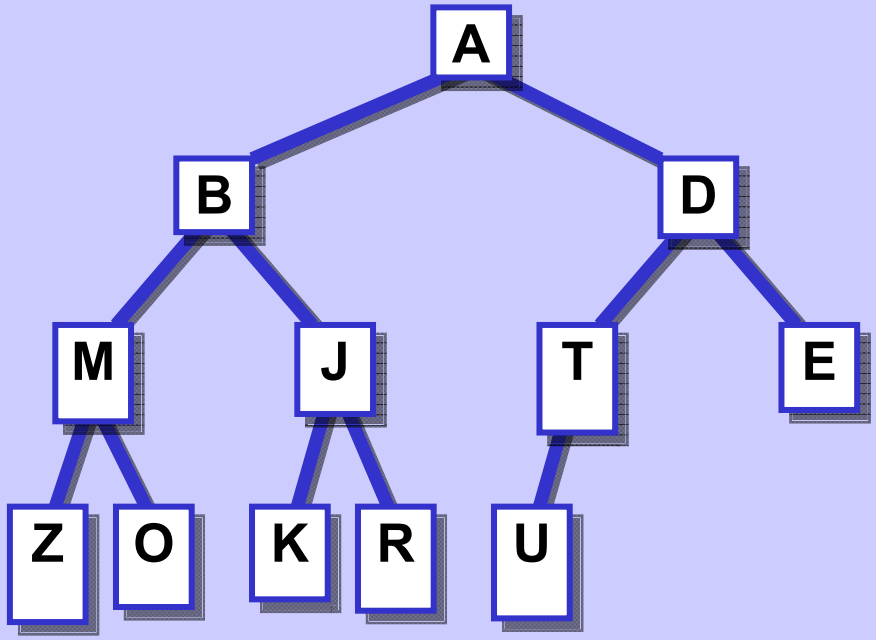
Sluč! ..... “ if (in[i1] <= in[i2]) { out[j] = in[i1]; …”

**Zařaď nejprve levý prvek,
když slučuješ stejné hodnoty**

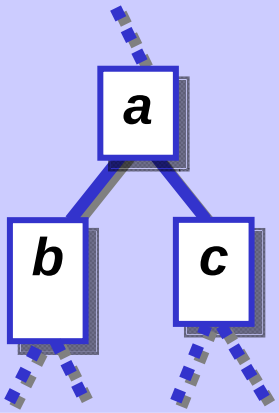
MergeSort je stabilní

Heap sort

Halda



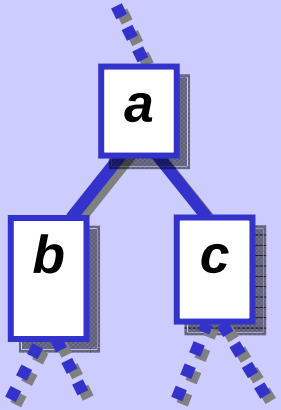
Pravidlo haldy



$$a \leq b \;\&\&\; a \leq c$$

Heap sort

Terminologie

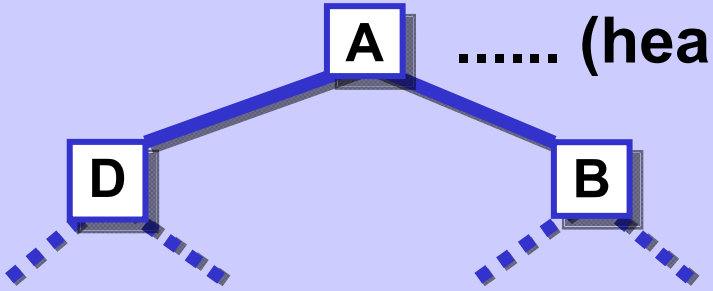


a ..... predecessor, parent of *b* *c*

..... předchůdce, rodič

b, *c* ..... successor, child of *a*

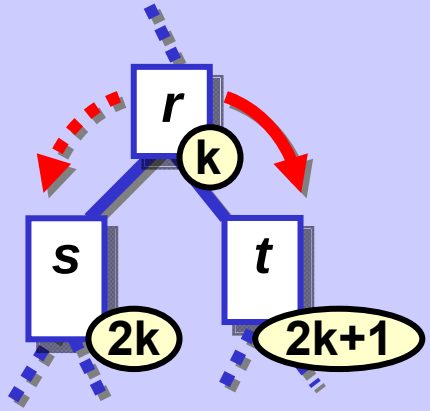
..... následník, potomek



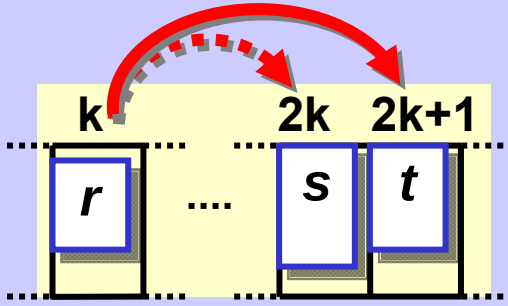
..... (heap) top ..... vrchol (haldy)

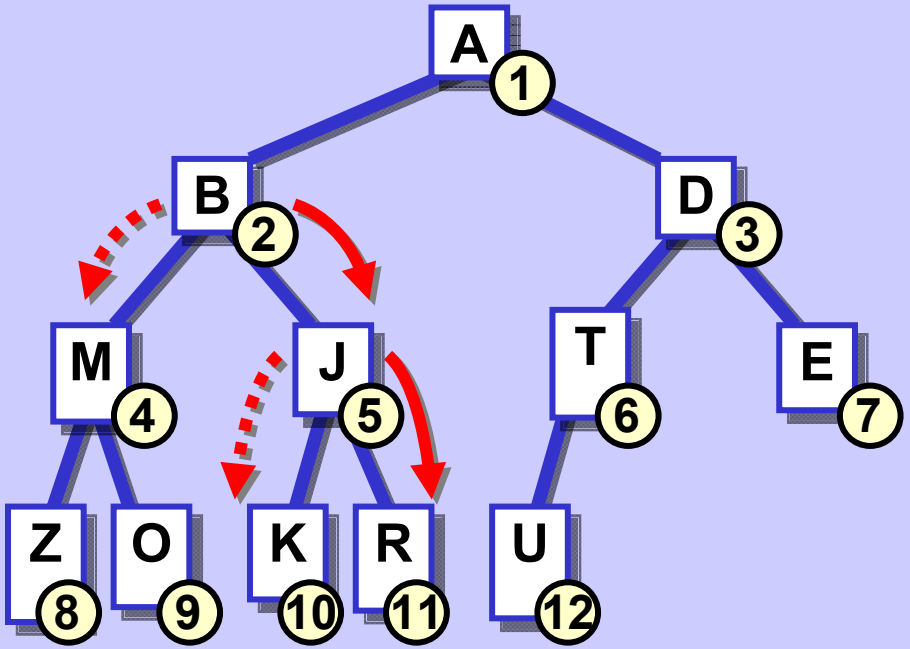
Heap sort

Halda uložená v poli

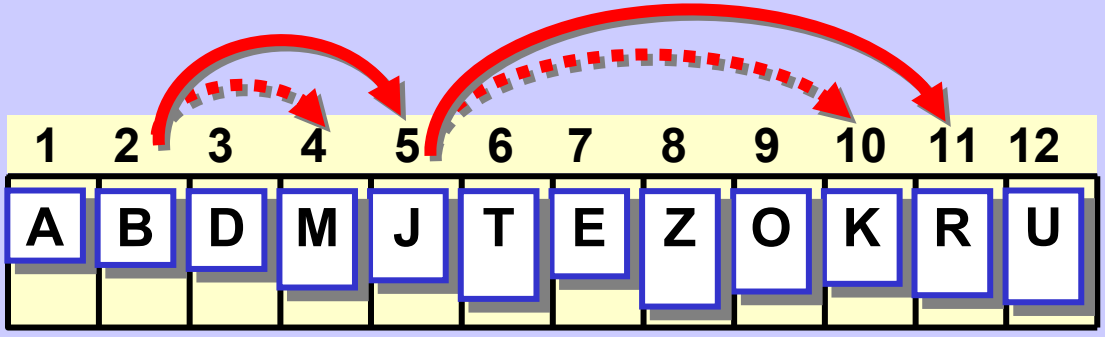


následníci





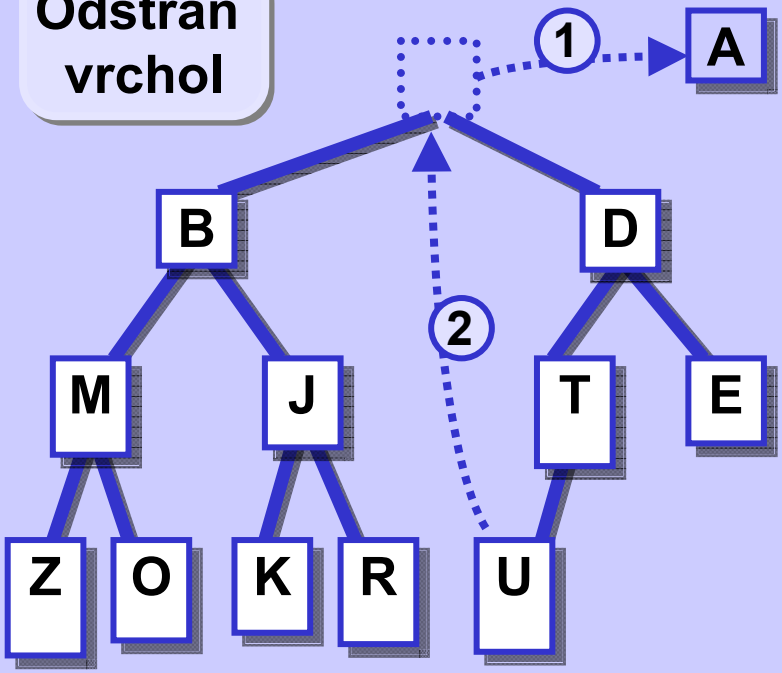
následníci



Oprava haldy

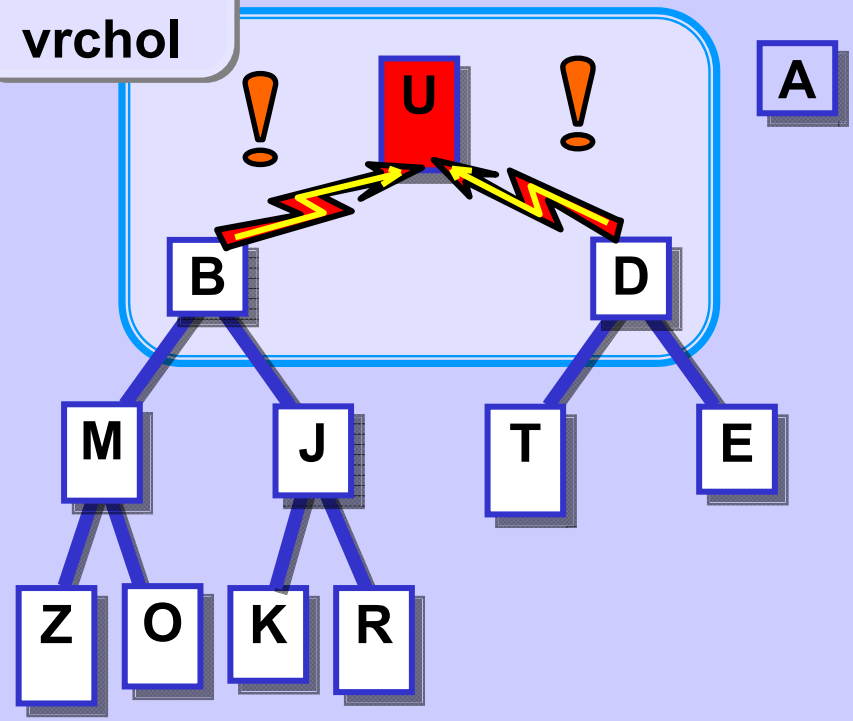
Vrchol odstraněn (1)

① Odstraň vrchol



② poslední ⟶ první

③ Vlož na vrchol



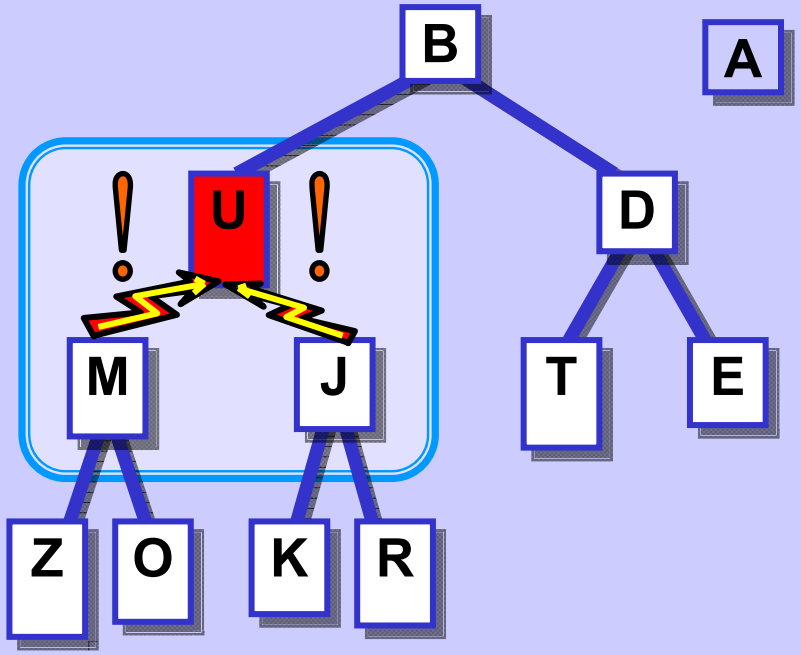
U > B, U > D, B < D

⇒ prohoď B ⟷ U

Oprava haldy

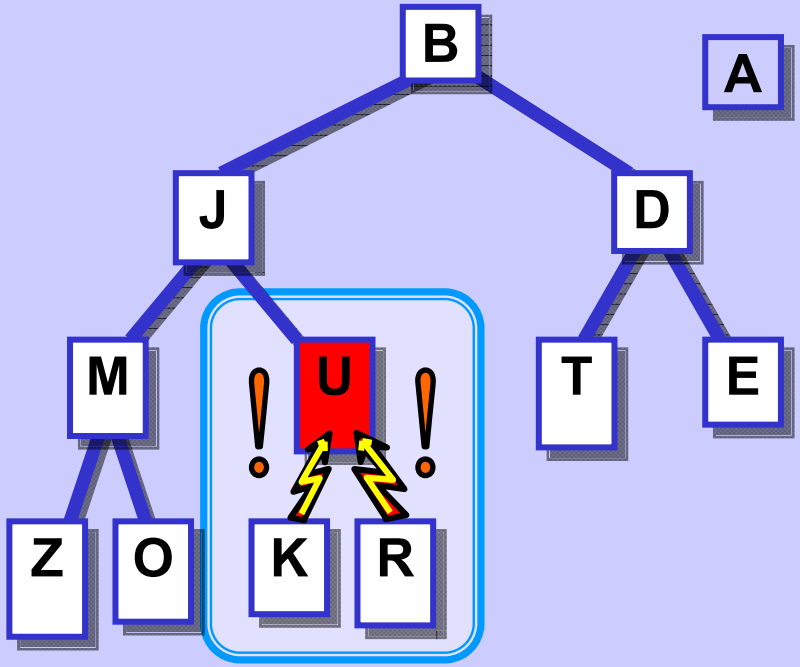
Vrchol odstraněn (2)

③ Vlož na vrchol - pokračování



$U > M,\ U > J,\ \underline{J < M}$

$\Rightarrow$ prohoď $J \leftrightarrow U$



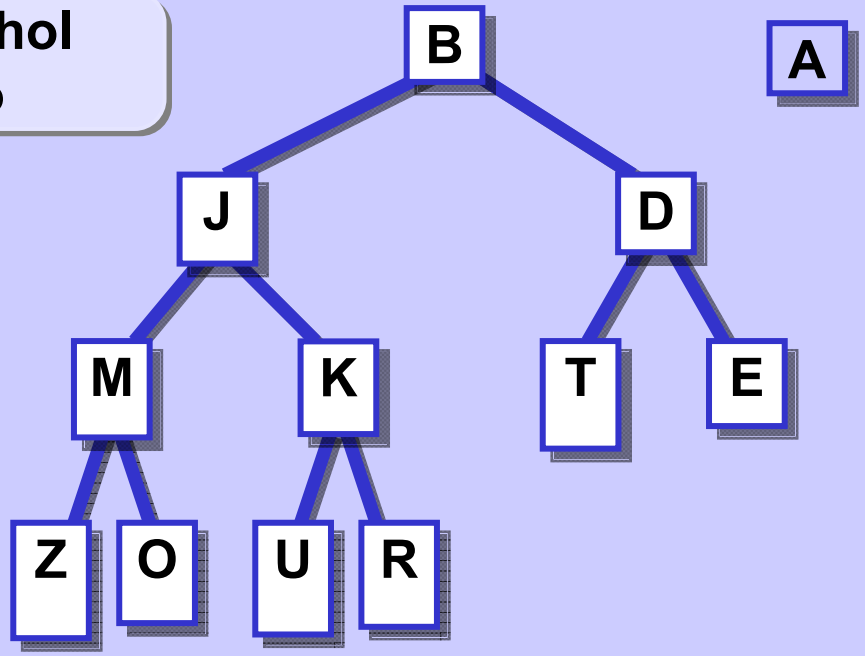
$U > K,\ U > R,\ \underline{K < R}$

$\Rightarrow$ prohoď $K \leftrightarrow U$

Oprava haldy

Vrchol odstraněn (3)

③ Vlož na vrchol - hotovo

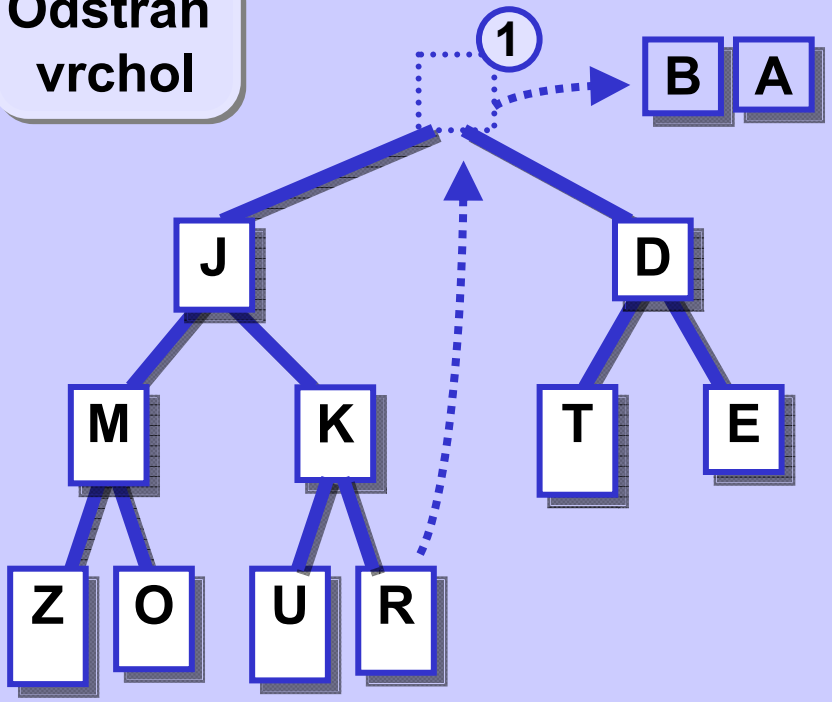


Nová halda

Oprava haldy

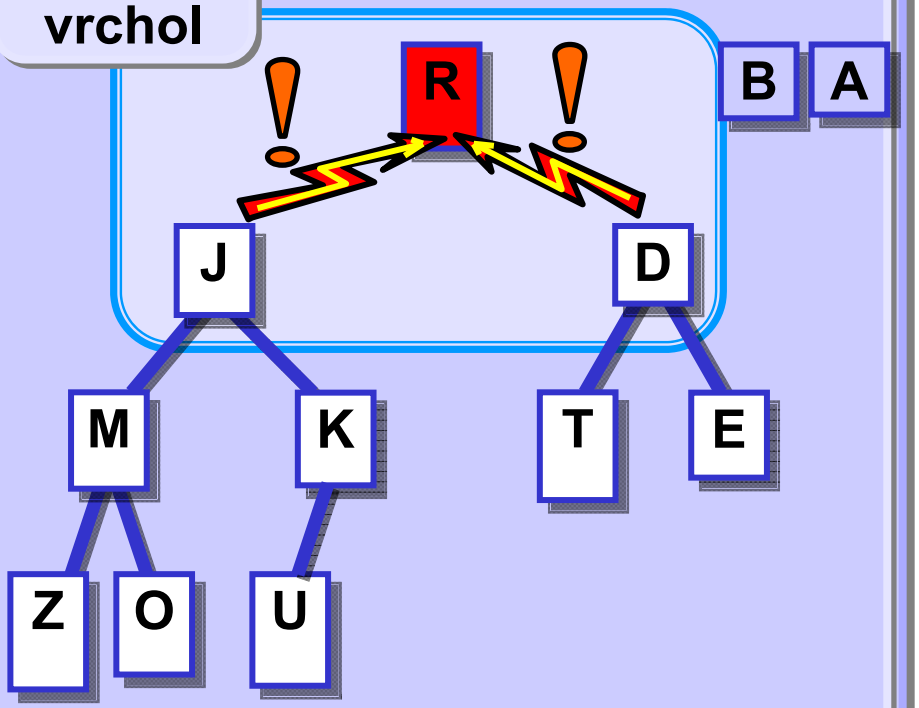
Vrchol odstraněn II (1)

① Odstraň vrchol



② poslední ⟶ první

③ Vlož na vrchol



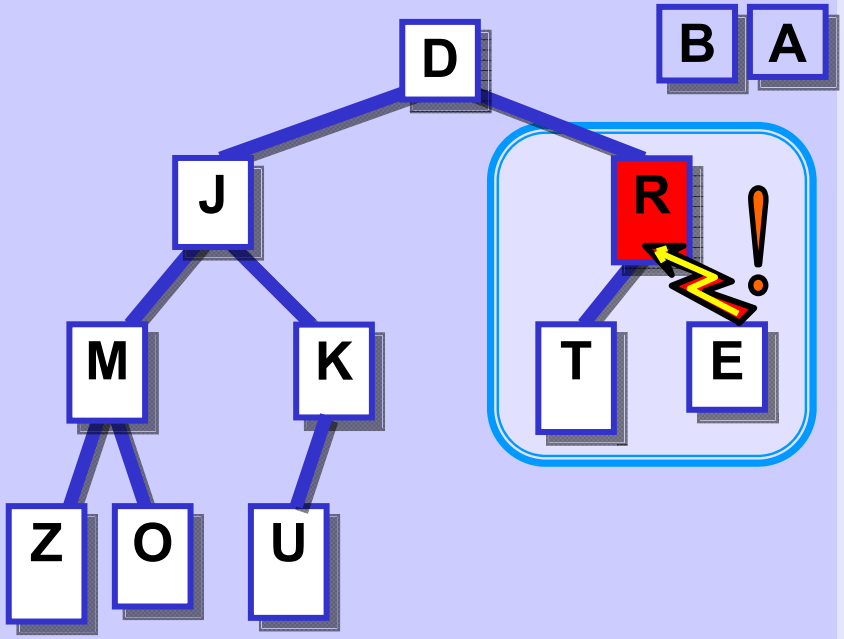
R > J, R > D, D < J

⇒ prohoď D ⟷ R

Oprava haldy

Vrchol odstraněn II (2)

③ Vlož na vrchol - pokračování

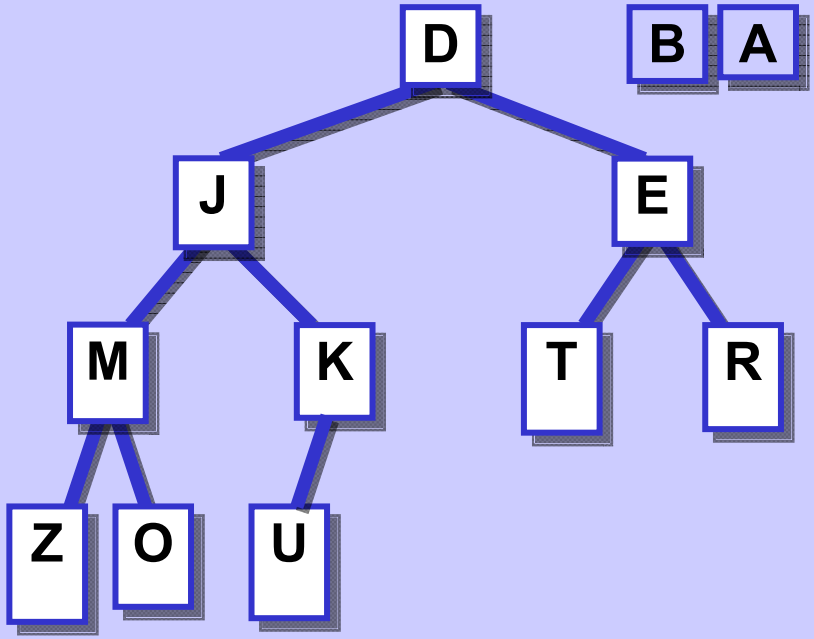


R < T, R > E

⇒ prohoď E ↔ R

Vrchol odstraněn II (3)

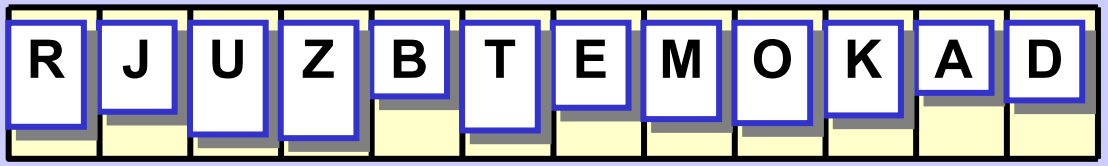
③ Vlož na vrchol - hotovo



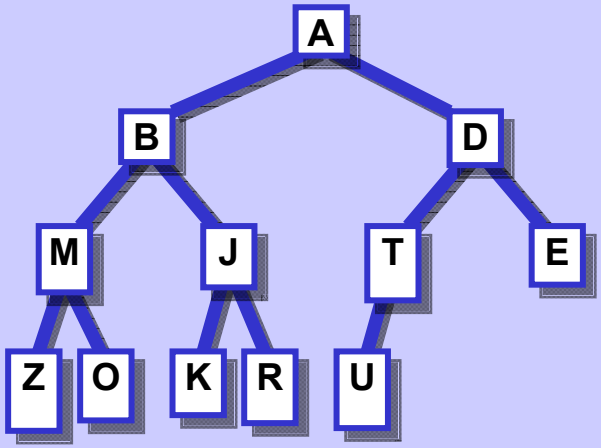
Nová halda

Heap sort

I Neseřazeno

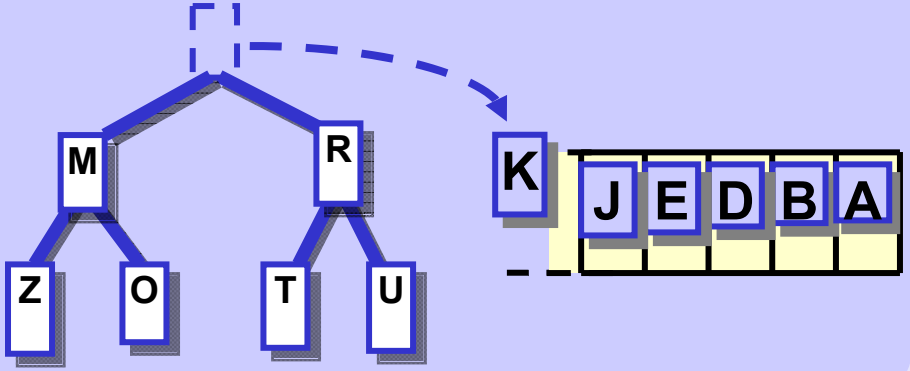


II Vytvoř haldu

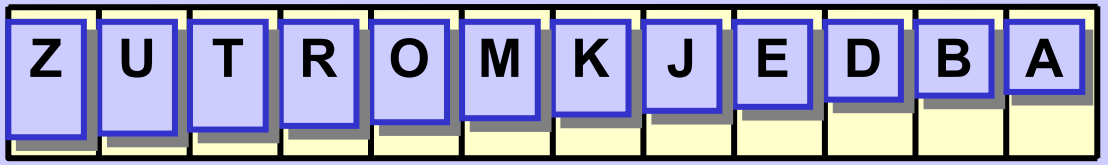


III

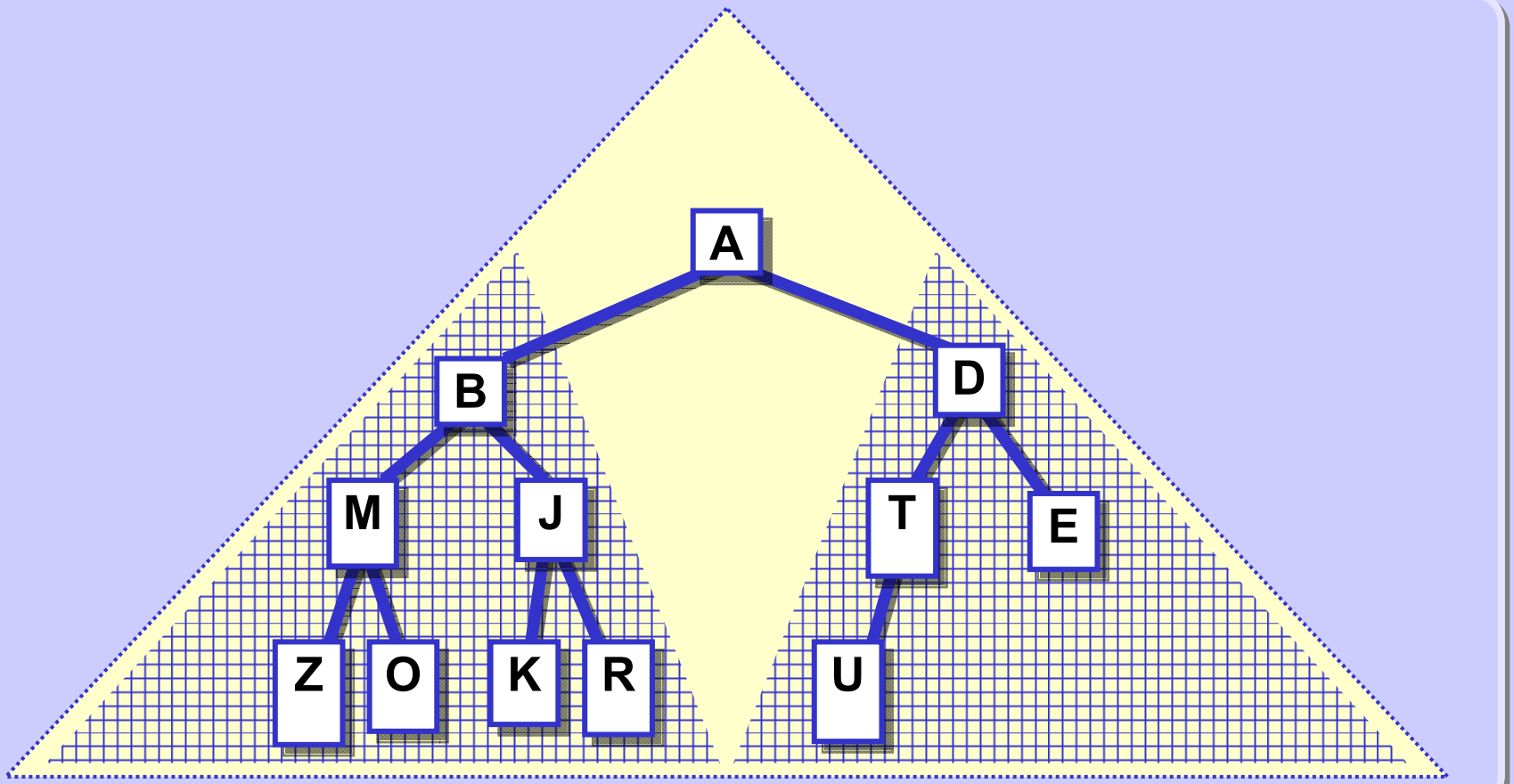
```
for (i = 0; i < n; i++)
    a[i] = “odstraň vrchol”;
```

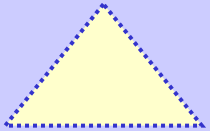


IV Seřazeno

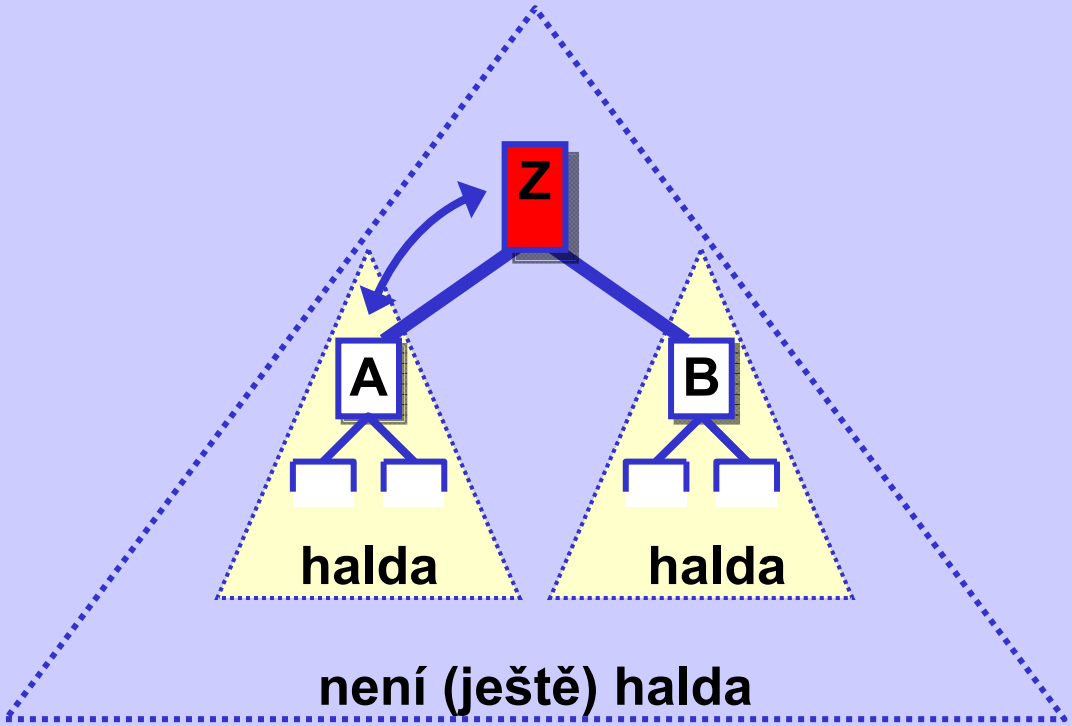


Rekurzivní vlastnost "býti haldou"



 je halda $\Rightarrow$  je halda a  je halda

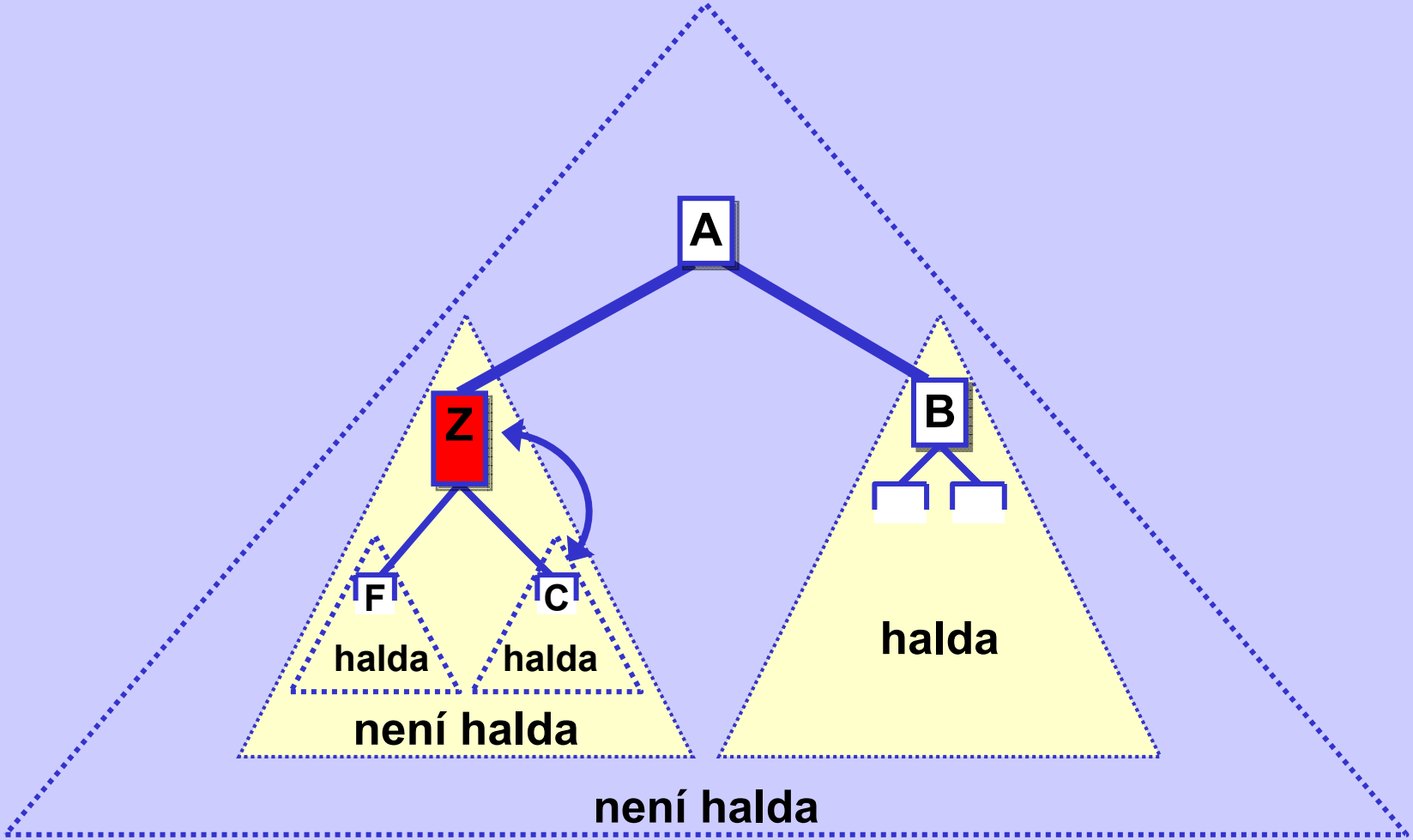
Vytvoř jednu haldu ze dvou menších



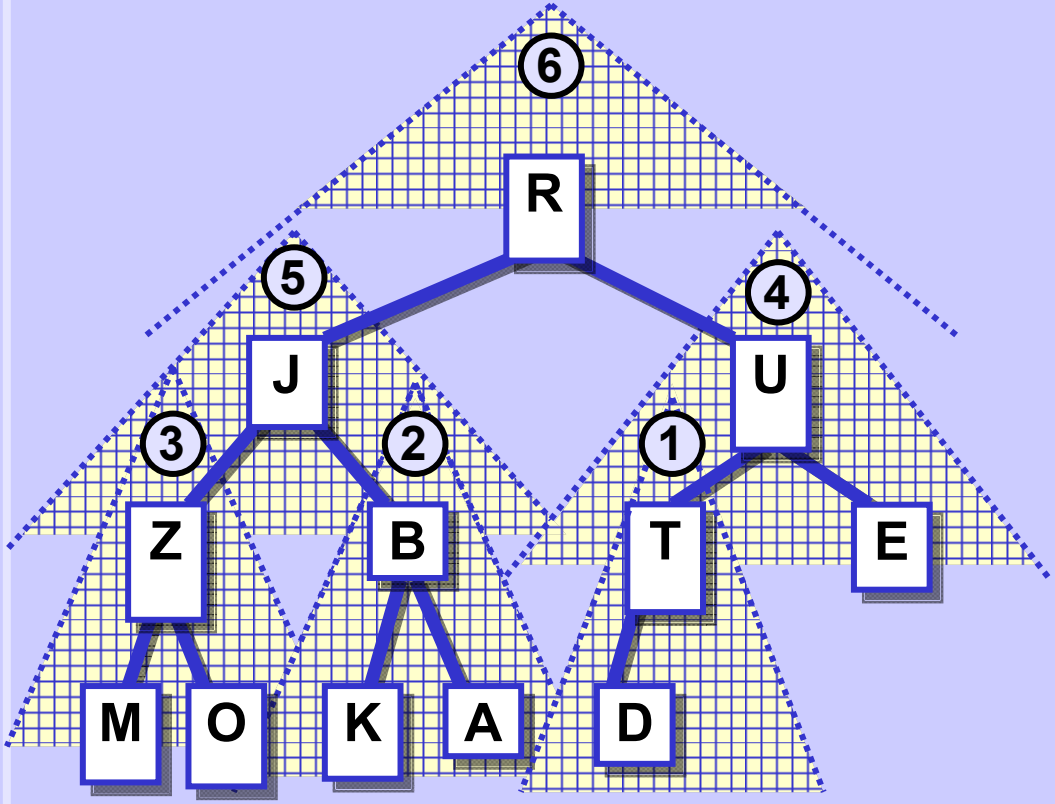
$Z > A$ nebo $Z > B$

$\Rightarrow$ prohoď: $Z \leftrightarrow \min(A,B)$

Vytvoř jednu haldu ze dvou menších



Vytvoř haldu



Vytvoř haldu v ① ...

... a vytvoř haldu v ② ...

... a vytvoř haldu v ③ ...

... a vytvoř haldu v ④ ...

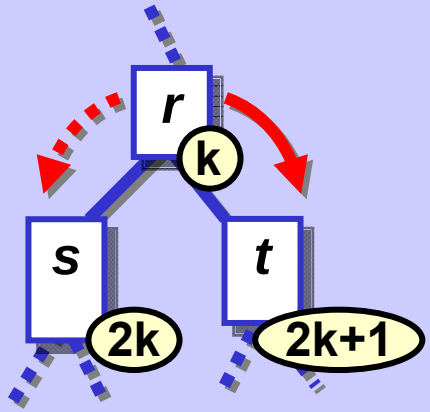
... a vytvoř haldu v ⑤ ...

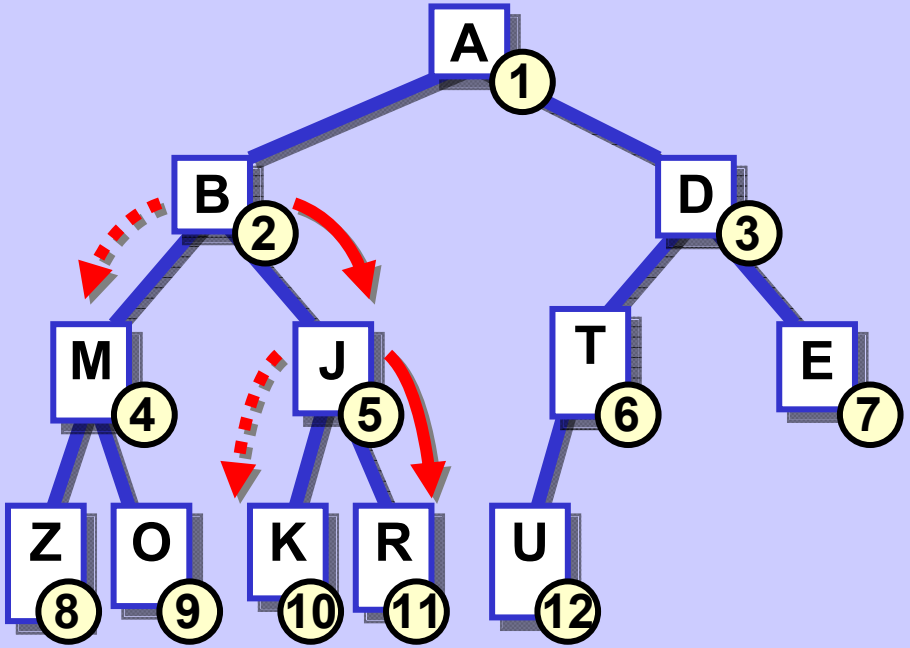
... a vytvoř haldu v ⑥ ...

… a celá halda je hotova.

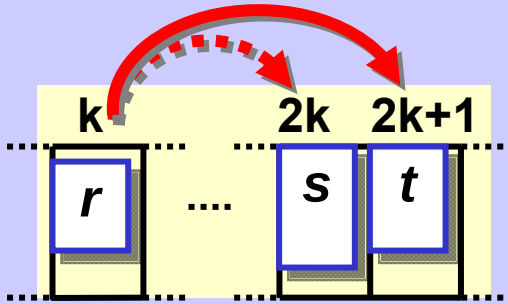
Halda v poli

Halda uložená v poli





následníci



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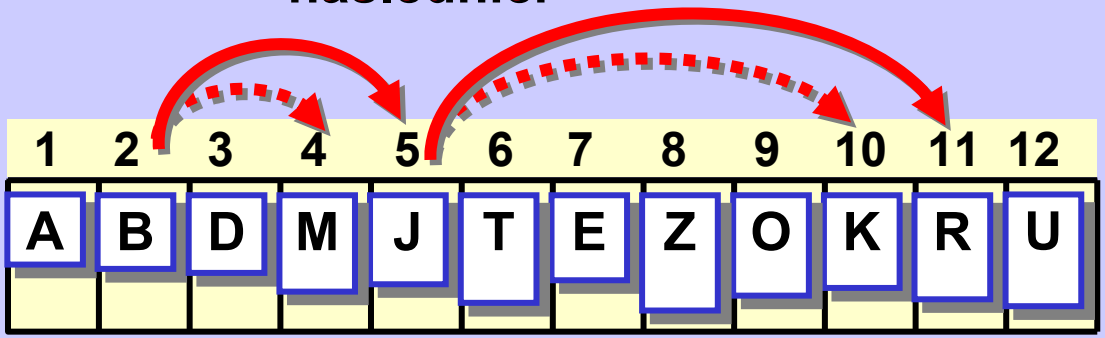
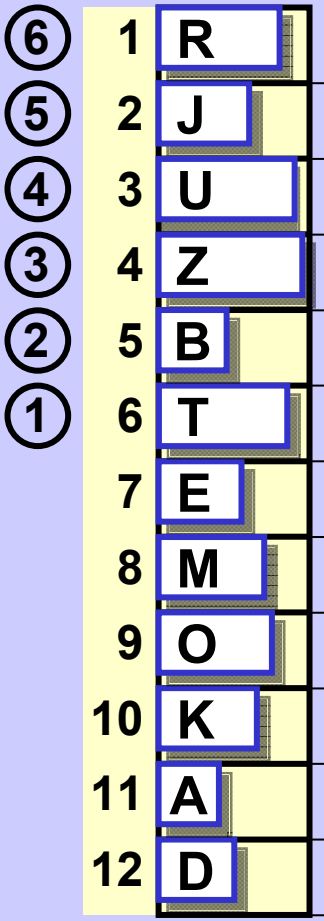
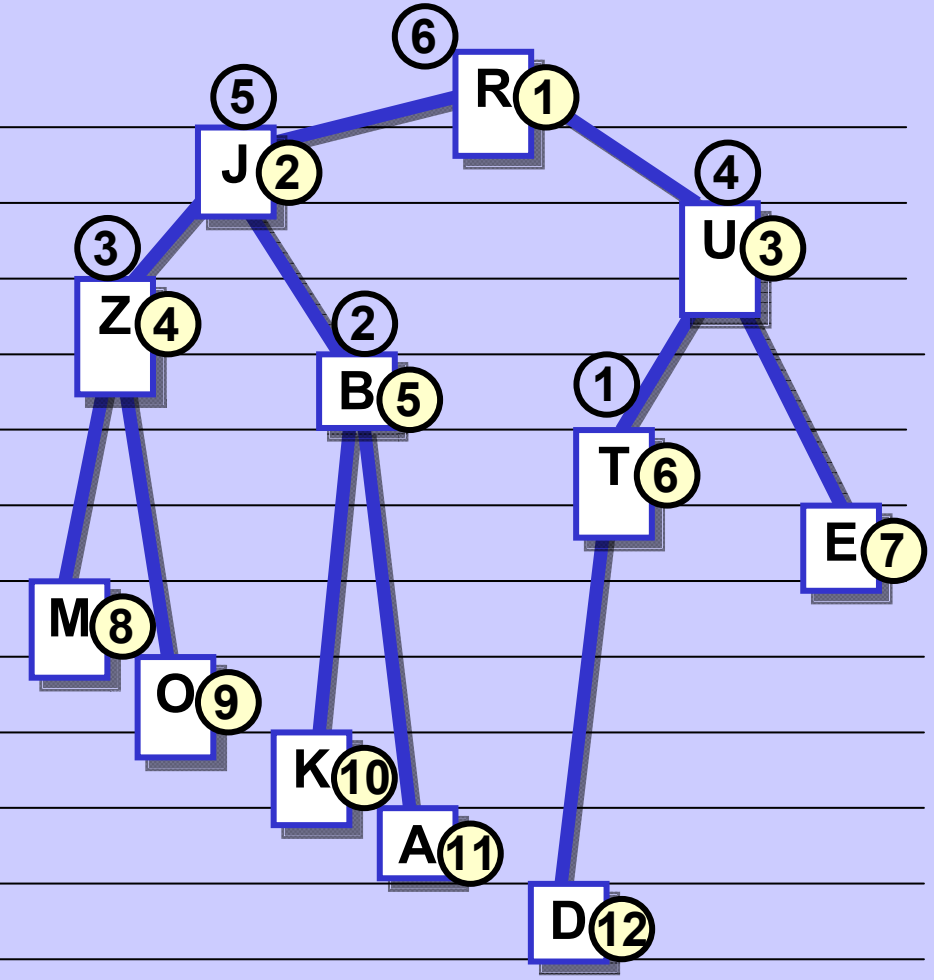


Tvorba haldy v poli

Neseřazeno

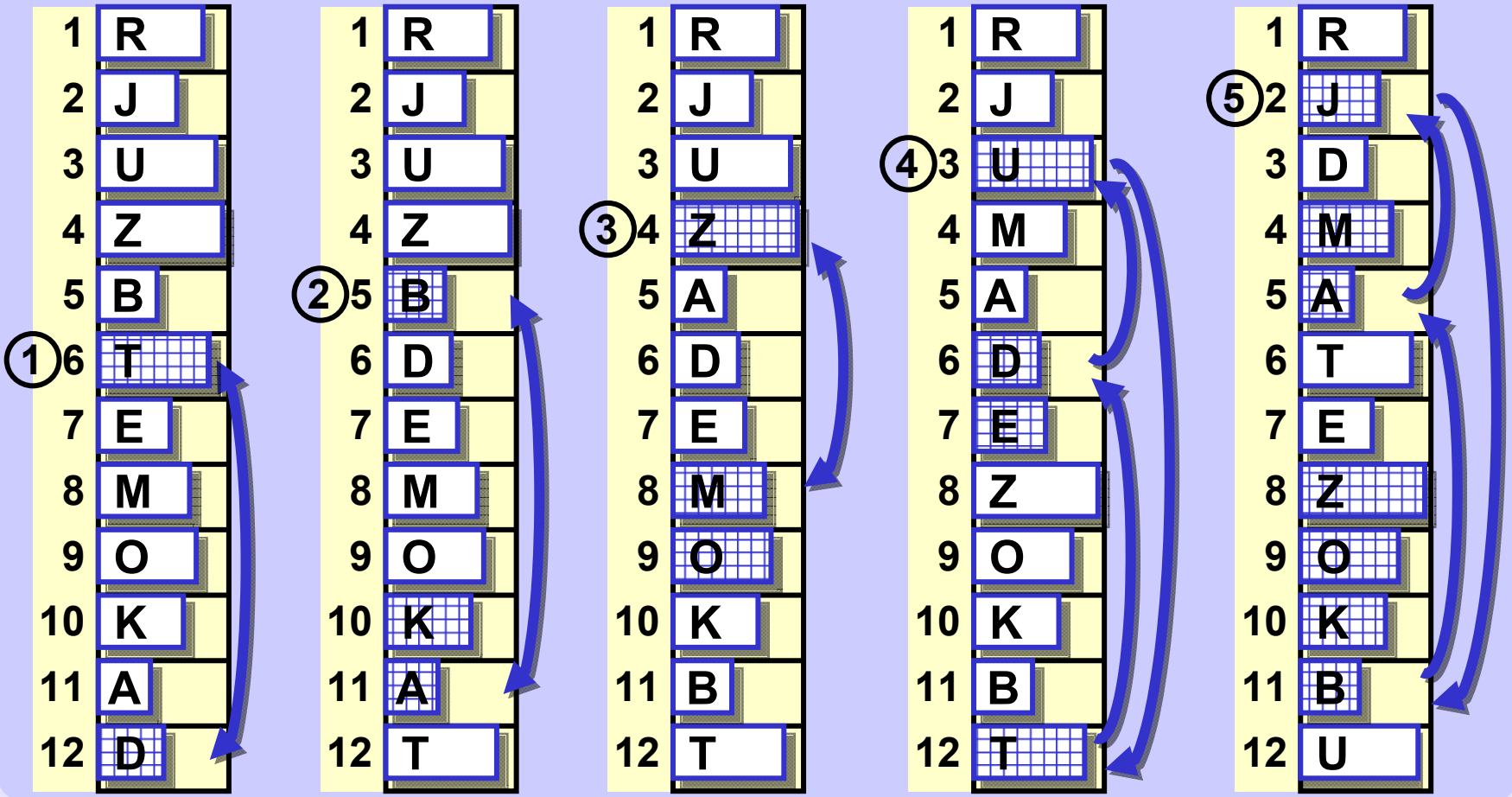


Není halda



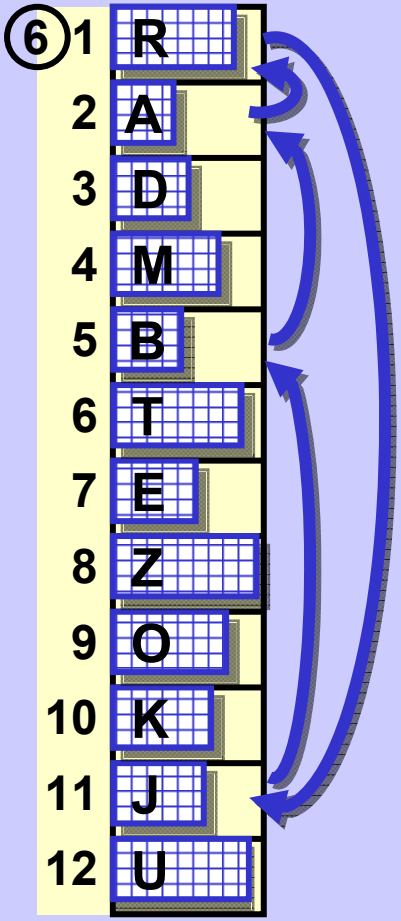
Tvorba haldy v poli

Tvorba haldy:

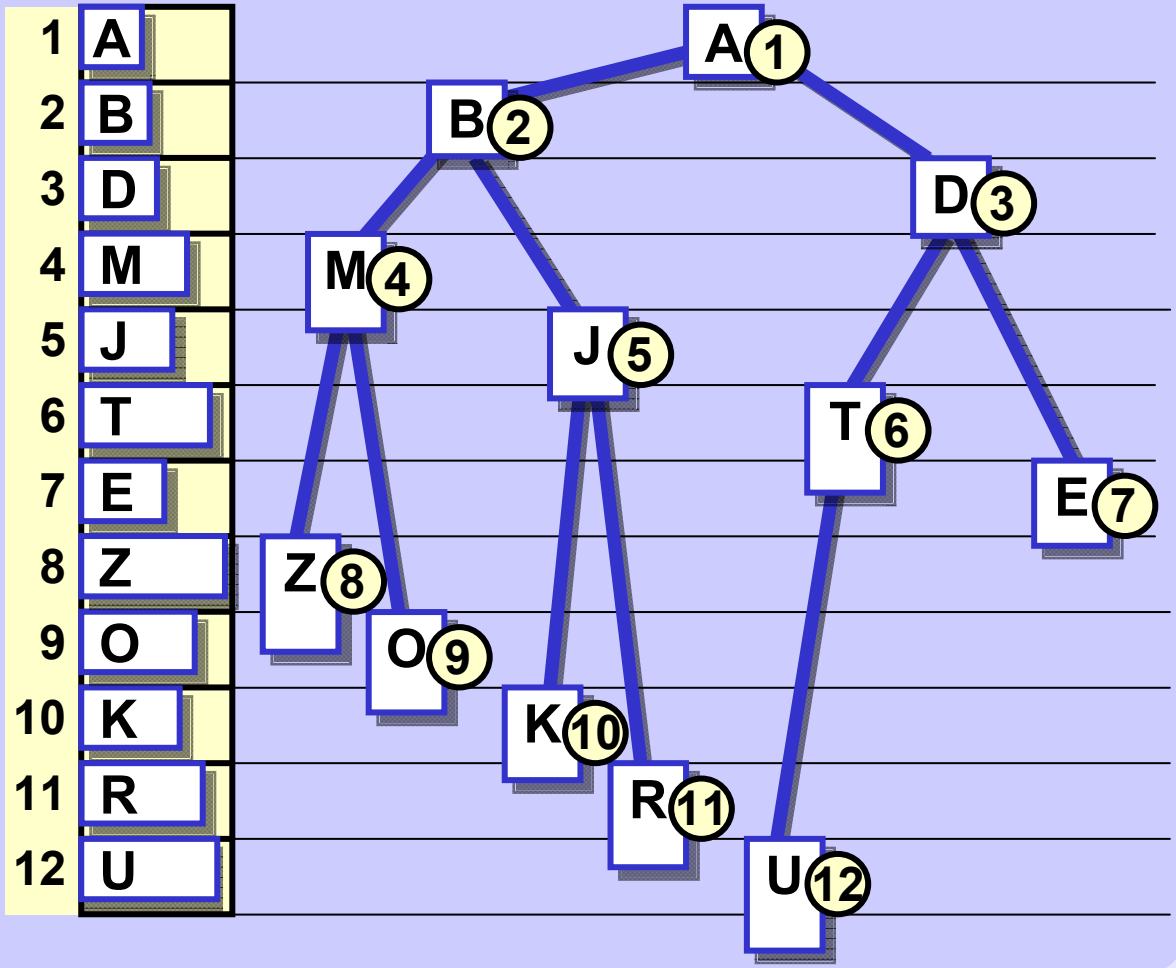


Tvorba haldy v poli

Tvorba haldy

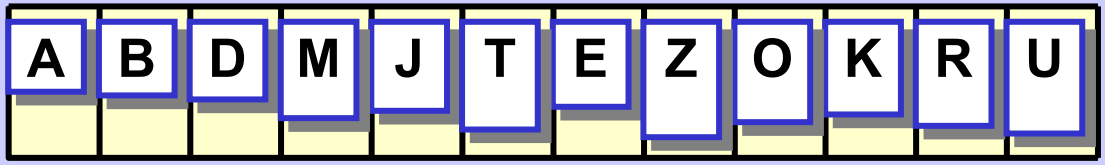


Halda

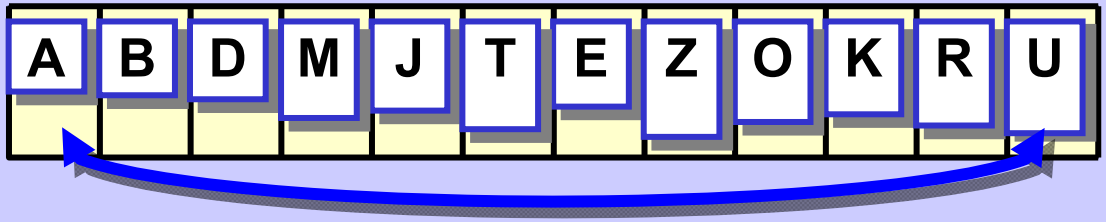


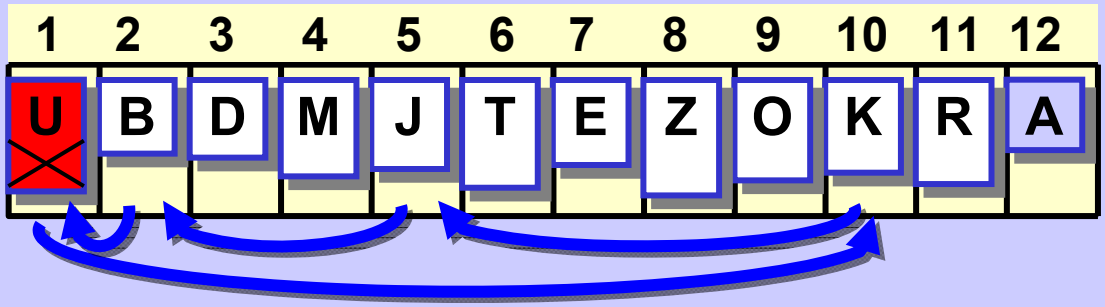
Heap sort

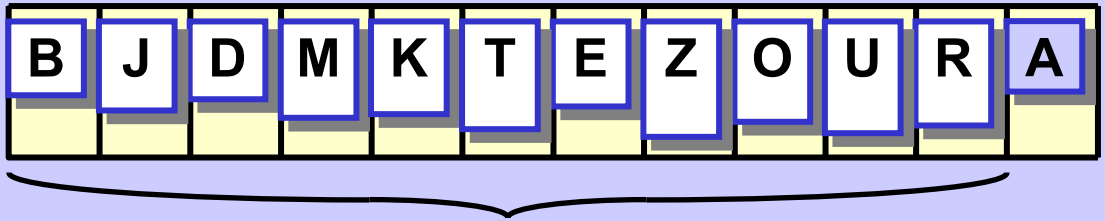
Halda



Krok 1



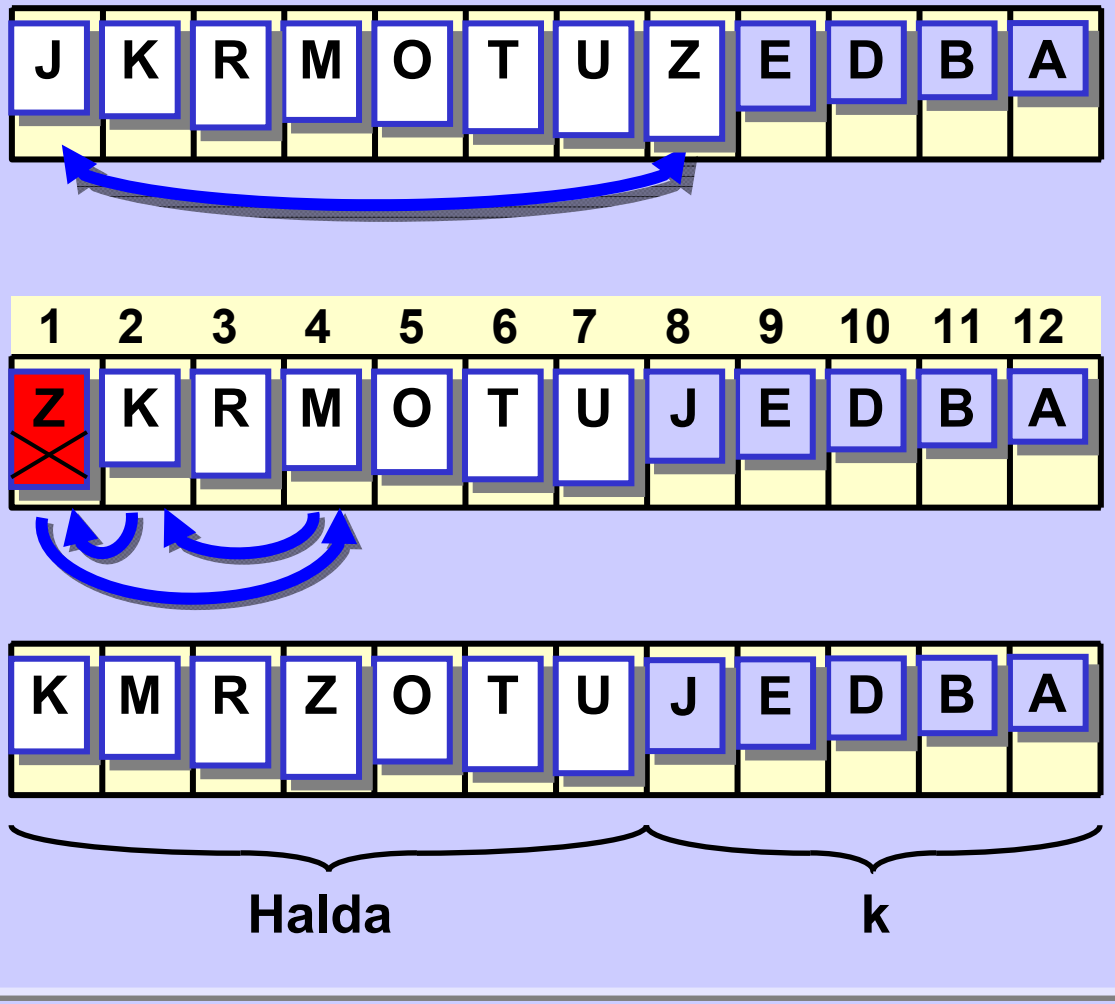




Halda

Heap sort

Krok k



Heap sort

```
                              // array: a[1]...a[n]  !!!!

void heapSort(Item a[], int n) {
  int i, j;
                              // create a heap
  for (i = n/2; i > 0; i--)
    repairTop(a, i, n);

                              // sort
  for (i = n; i > 1; i--) {
    swap(a, 1, i);
    repairTop(a, 1, i-1);
  }
}
```

Heap sort

```
                          // array:  a[1]...a[n]  !!!!!!
void repairTop(Item a[], int top, int bott) {
  int i = top;            // a[2*i] and a[2*i+1]
  int j = i*2;            // are successors of a[i]

  Item topVal = a[top];

                          // try to find a successor < topVal
  if ((j < bott) && (a[j] > a[j+1]))  j++;

                         // while (successors < topVal)
                         //         move successors up
  while ((j <= bott) && (topVal > a[j])) {
    a[i] = a[j];
    i = j;  j = j*2; // skip to next successor
    if ((j < bott) && (a[j] > a[j+1])) j++;
  }
  a[i] = topVal;    // put the topVal
}
```

Heap sort

repairTop operace nejhorší případ ... $\log_2(n)$ (n=velikost haldy)

vytvoř haldu ... n/2 repairTop operací

$\log_2(n/2) + \log_2(n/2+1) + ... + \log_2(n) \leq (n/2)(\log_2(n)) = \underline{O(n \cdot \log_2(n))}$

seřaď haldy ... n-1 repairTop operací, nejhorší případ:

$\log_2(n) + \log_2(n-1) + ... + 1 \leq n \cdot \log_2(n) = O(n \cdot \log_2(n))$

ale i nejlepší případ = $\underline{\Theta(n \cdot \log_2(n))}$

celkem ... vytvoř haldu + seřaď haldu = $\underline{\Theta(n \cdot \log_2(n))}$

Asymptotická složitost Heap sortu je $\Theta(n \cdot \log_2(n))$

Heap sort není stabilní