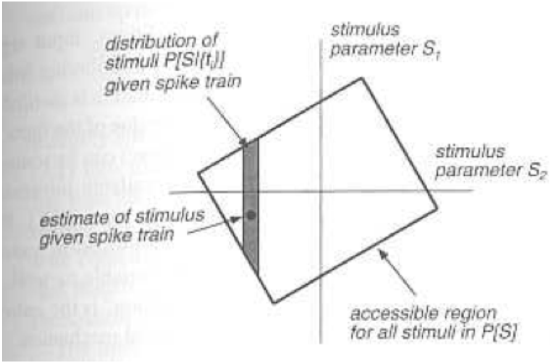
# Neuroinformatics

May 13, 2020

Lecture 10: Decoding and Encoding

# Why information theory

- quantifying information that sensory neuron convey about the world
- how much information is spike train $t_i$transmitting, is this transmission large or small?
- stimulation estimate (dot) estimated by ML or Bayes



# Communication channel as studied by Shanon

- (i) information depends on frequency of messages $p_i = P(y_i)$, (ii) independent information should be additive $(f(x,y) = f(x) + f(y))$: $p(x,y) = p(x)p(y)$
- logarithm has these characteristics: $I(y_i) = -log_2(p_i)$, how much we can learn relative what is known a priori.
- ENTROPY: average amount of information: $S(X) = -\sum_i p_i log_2(p_i)$
- Gaussian probability: $p(x) = \frac{1}{\sqrt{(2\pi)}\sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}} \leftarrow$

  $S = \frac{1}{2}\log_2(2\pi e\sigma^2)$, depends only on variability (ENERGY)

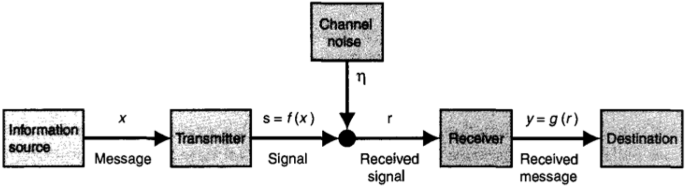
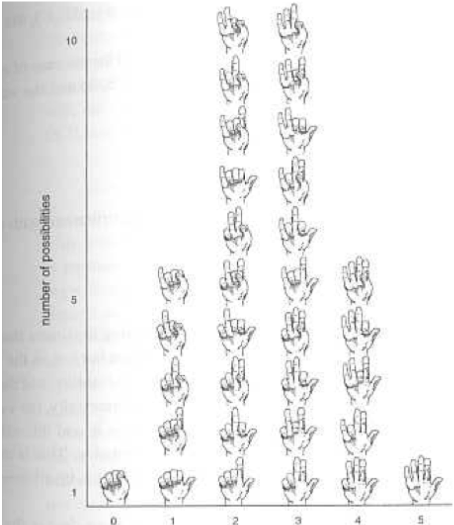



**Fig. 5.6** The communication channel as studied by Shannon. A message $x$ is converted into a signal $s = f(x)$ by a transmitter that sends this signal subsequently to the receiver. The receiver generally receives a distorted signal $r$ consisting of the sent signal $s$ convoluted by noise $\eta$. The received signal is then converted into a message $y = g(r)$ [adapted from C. Shannon, *The Bell System Technical Journal* 27: 379–423 (1948)].

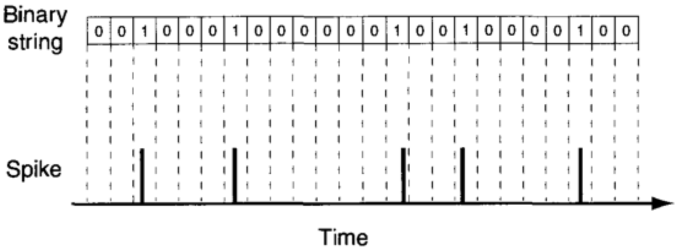
# Example: ordering drink in restaurant

- rate coding: one hand can carry $log_2(6) = 2.58$bits of information, MORE ROBUST TO NOISE
- temporal coding: one hand can convey $2^5 = 32$ distinct messages, 5 bits of information



# Entropy of Spike Train with temporal coding

- estimated in 1952 by MacKay and McCulloch, first application of information theory to nervous system, 4 years after Shannon
- $\bar{r}$ : mean rate, $\Delta\tau$ : time resolution, $T$ : length of spike, occurrence of 1 $p = \bar{r}\Delta\tau$
- set of different strings, e.g. $1111111\ldots111111$
- counting the number of different spike trains that can be distinguished given our time resolution



# Spike Train Calculation

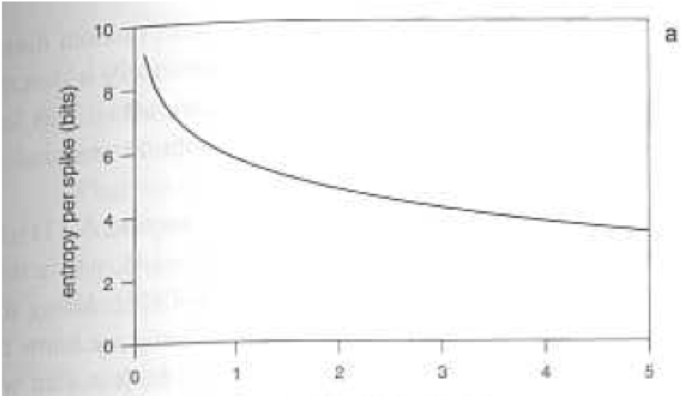
- total number of bins $N = N/\Delta\tau$, number of 1 (spikes) $N_1 = pN$, of 0 $N_0 = (1-p)N$, number of possible LARGE strings $N_{strings} = \frac{N!}{N_1!N_0!}$
- entropy $S = log_2 \frac{N!}{N_1!N_0!} = \frac{1}{ln2}(lnN! - lnN_1! - lnN_0!)$
- Stirling's approximation $lnx! = x(lnx - 1) + \ldots$, $ln_2(x) = ln(x)\ ln2$, all symbols K are equal, $S = -\sum_{i=1}^{K}(1/K)log_2(1/K) = log_2 K$

$$
\begin{aligned}
S &= \frac{1}{ln2}(lnN! - lnN_1! - lnN_0!) \\
&= \frac{1}{ln2}(NlnN - N_1 lnN_1 - N_0 lnN_0 - (N - N1 - N_0)), N = N_0 + N_1 \\
&= -\frac{1}{ln2}N(\frac{N1}{N}ln\frac{N1}{N} + \frac{N0}{N}ln\frac{N0}{N}) \\
&= \frac{N}{ln2}(plnp + (1-p)ln(1-p)) \\
&= -\frac{T}{\Delta\tau ln2}(\bar{r}\Delta\tau ln(\bar{r}\Delta\tau) + (1 - \bar{r}\Delta\tau)ln(1 - \bar{r}\Delta\tau)) \propto T, \bar{r}
\end{aligned}
$$

# Entropy rate S/T approximation

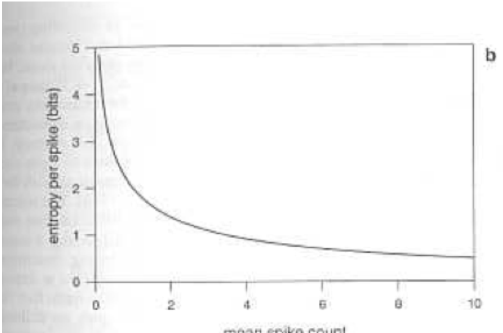
- approximating the entropy of spike trains
- limiting behaviour, $\Delta\tau$ is small, small bins, high resolution $\rightarrow$ Taylor series
- entropy is lager than 1 bits, $\bar{r} \sim 50s^{-}1$, $\Delta\tau \sim 1ms$, 5.76 bits per spike $\sim log_2(e/\bar{r}\Delta\tau)$, 288 bits/sec

$$S/T \approx \bar{r}\,log_2\left(\frac{e}{\bar{r}\Delta\tau}\right)$$



# Entropy of spike rate count

- different coding scheme - before the position was relevant! Now we want to calculate $S(spikecount) = -\sum_n p(n) log_2 p(n)$
- we are counting spikes in some large window $T \rightarrow$ measuring rate of spiking, $p(n)$ is probability of observing $n$ spikes in window of length $T$
- $p(n) =?$, $\sum_n p(n) = 1$, average spike count $\langle n \rangle = \bar{r}T$, MAXIMAZING spiking count ENTROPY
- $p(n) \propto \exp(-\lambda n)$, $\lambda = ln(1 + (\bar{r}T)^{-}1)$, substituing
- $S(spikecount) \leq log_2(1 + \langle n \rangle) + \langle n \rangle log_2(1 + 1/\langle n \rangle)$ bits
- capacity 1 per bit, $\langle n \rangle \leq 3.4$ bits

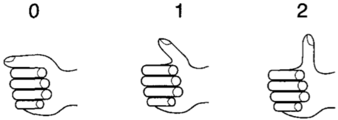


# Channel capacity

- Mutual information $I_{mutual} = S(X) + S(Y) - S(X, Y)$, model of channel $y = gs + \eta$, where $\eta$ is normal distribution and $g$ is gain
- adding the noise to the signal itself and than transducing, $y = g(s + n_{eff})$, $n_{eff} = \eta / g$
- Example:resolution of our visual system, noise introduced by the motor system
- information transmission can be increased by increasing variability of the input signals. High variability of spike trains is well suited for transmission in noisy neural systems

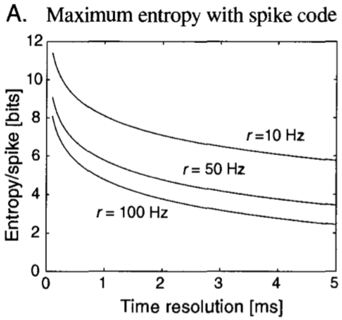
$$I = \frac{1}{2} log_2 \left( 1 + \frac{\langle s^2 \rangle}{\langle \eta^2 \rangle / \langle g^2 \rangle} \right)$$

$$I = \frac{1}{2} log_2 \left( 1 + \frac{\langle s^2 \rangle}{\langle n_{eff}^2 \rangle} \right) = \frac{1}{2} log_2 (1 + SNR)$$



# Summary

- measuring entropy is difficult → estimating probability distributions
- small events in the entropy → large factor in entropy (log). Realiable measurements of rare events
- overestimating entropy due to potential miss of rare events with high information content



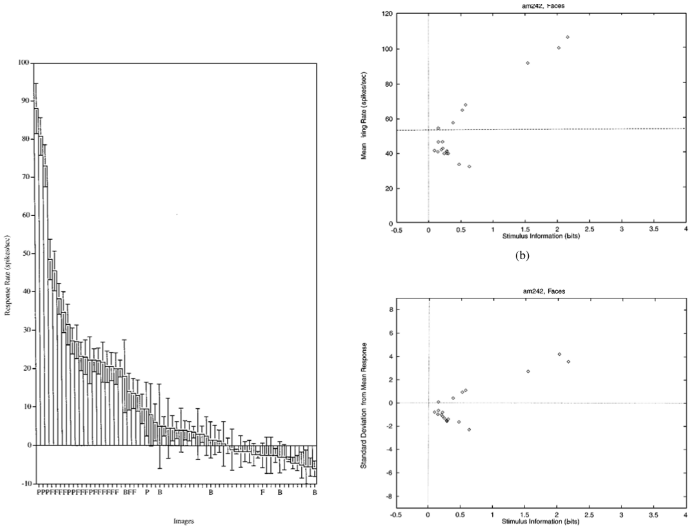
# Entropy measured from single neuron

- 65 visual stimuli in macaques performing a visual task (23 monkey and human faces and 42 nonfaces images from real word), 14 face-selective neurons
- how much information is available about each stimulus in the set
- measuring firing rate in poststimulus phase (100 . . . 500ms).
- defining information between stimulus $S = \{s_i\}$ and responce $R = \{r_i\}$, I(s,R): amount of information about stimulus s, I(S,R) -average information gain

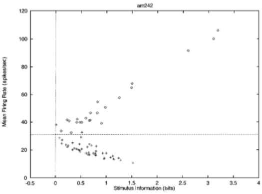
$$I(s, R) = \sum_r P(r|s) log_2 \frac{P(r|s)}{P(r)}$$

E. Rolls, Information in the Neuronal Representation of Individual Stimuli in the Primate Temporal Visual Cortex, Journal of Computational Neuroscience 4,309-333,1997

# AM242 - quantitative analyses



# AM242 - quantitative analyses

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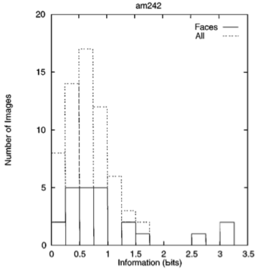


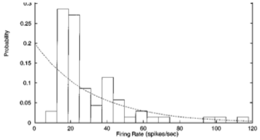


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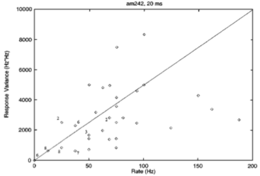


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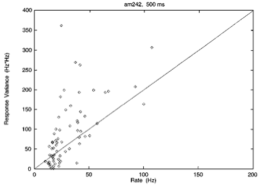

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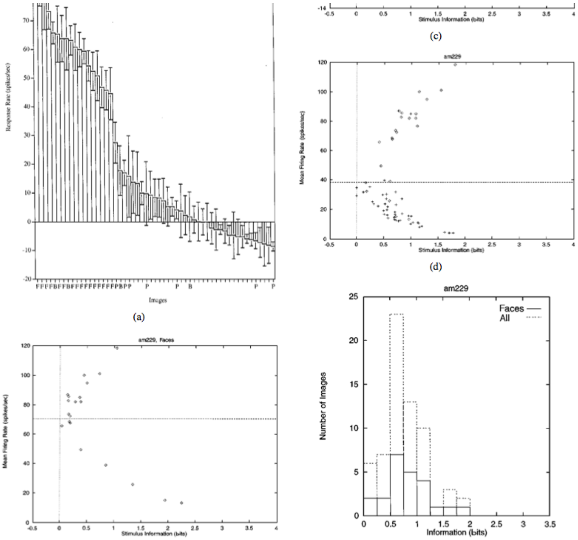
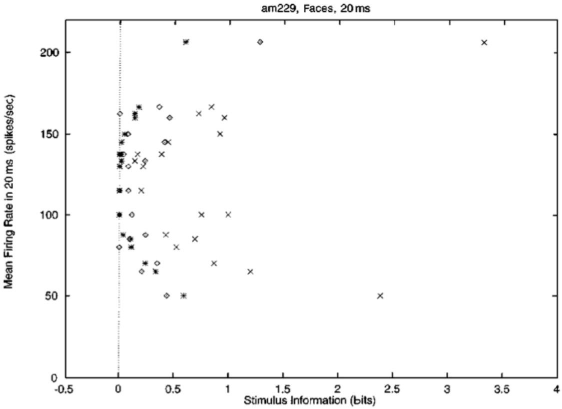


# AM242 - any coding at the beginning?



# Population coding (encoding and decoding)

**Probability of neural response for a sensory input (encoding):**

$$P(\mathbf{r}|s) = P(r_1^s, r_2^s, r_3^s, ...|s)$$

**Decoding:** $P(s|\mathbf{r}) = P(s|r_1^s, r_2^s, r_3^s, ...)$

**Stimulus estimate:** $\hat{s} = \arg\max_s P(s|\mathbf{r})$

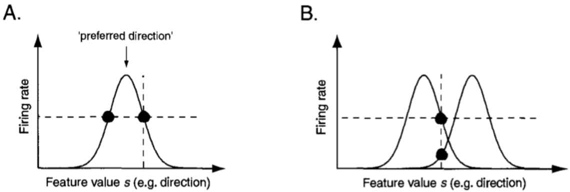
**Bayes's theorem**: $P(s|\mathbf{r}) = \frac{P(\mathbf{r}|s)P(s)}{P(\mathbf{r})}$

**Likelihood:** $P(\mathbf{r}|s), P = f(s)$

# Decoding with response tuning curves

- we need at least two tuning curves $r_i = f_i(s)$to estimate the stimulus
- responses of neurons $r_i$ are not correlated and tuning curves have Gaussian probability
- decoding using ML estimate - equivalent to least square fit

$$
\begin{aligned}
P(\mathbf{r}|s) &= \prod_i P(r_i|s) \\
P(r_i|s) &= \frac{1}{\sqrt{2\pi\sigma}} e^{-}(r_i - f_s(s))^2/2\sigma_i^2 \\
\hat{s} &= \text{argmin} \sum_i \left( \frac{r_i - f_i(s)}{\sigma_i} \right)^2
\end{aligned}
$$

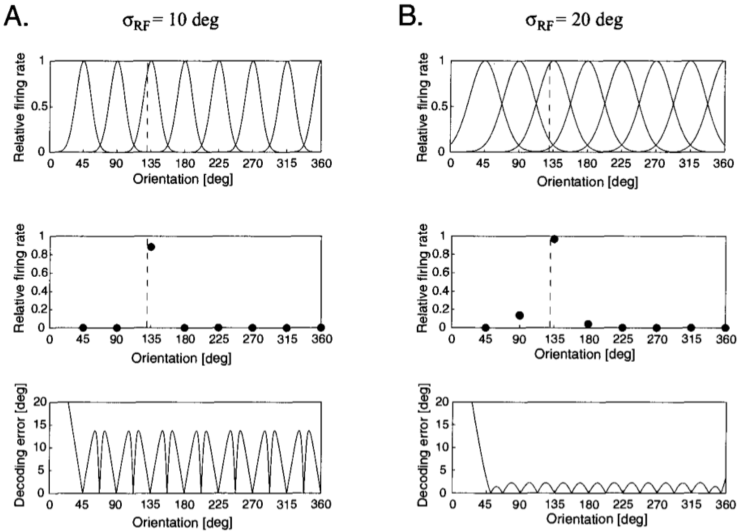


# Population vector decoding

- e.g. Gaussian or cosine tuning curve: $f_i(s) = e^{-}(s - s_i^{pref}/2\sigma_{RF}^2)$, $\sigma_R F$ is receptibe field size
- Easy implementation in brain: dot product, normalization needed

$$
\begin{aligned}
\hat{s} &= \sum_i r_i s_i^{pref} \\
\hat{r}_i &= \frac{r_i - r_i^{min}}{r_i^{max}} \\
\hat{s}_{pop} &= \sum_i \frac{\hat{r}_i}{\sum_j \hat{r}_j} s_i^{pref}
\end{aligned}
$$

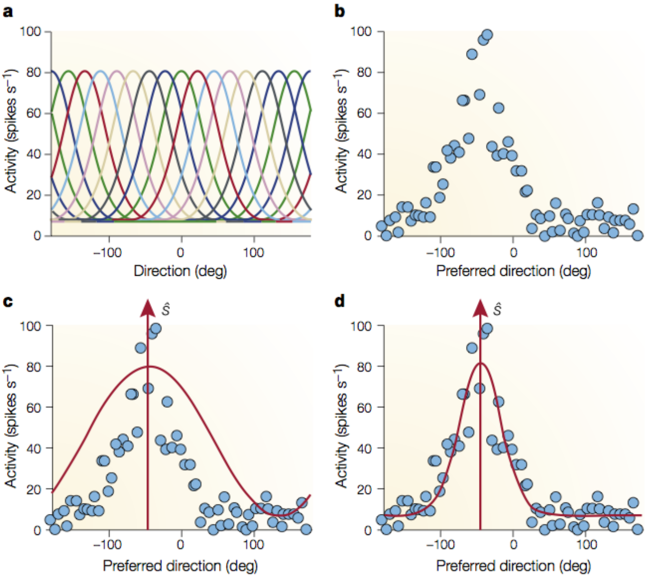
# Population vector decoding - example



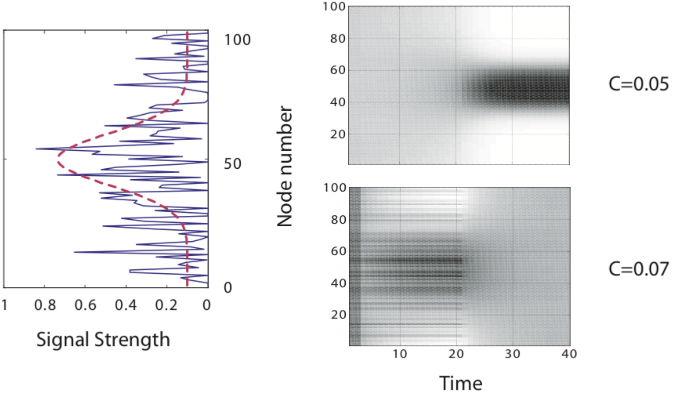
# Example of coding model

- noisy model is used: $r_i = f_i(s) + \eta_i$, $f_i(s) = e^{-}(s - s_i^{pref}/2\sigma_{RF}^2)$

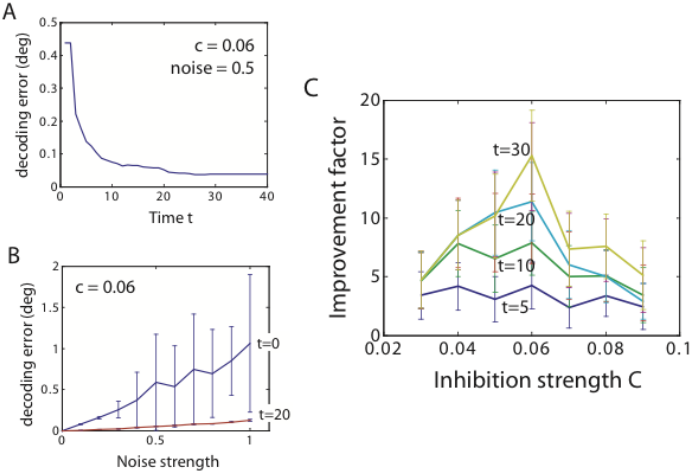
A. Pouget, Information Processing with population Codes, Nature Reviews, Neuroscience,2000



# Implementations of decoding mechanisms with DNF



# Quality of decoding



# Further Readings


Fred Rieke (1995), **Spikes, exploring the neural code**, The MIT Press, 3st edition.

E. Rolls, Information in the Neuronal Representation of Individual Stimuli in the Primate Temporal Visual Cortex, Journal of Computational Neuroscience 4,309-333,1997

S. Funahashi, C.J. Bruce and P.S. Goldman-Rakic, Mnemonic coding of visual space in the monkeys dorsolateral prefrontal cortex, J Neurophysiol 61:33149, 1989

A. Pouget, Information Processing with population Codes, Nature Reviews, Neuroscience,2000