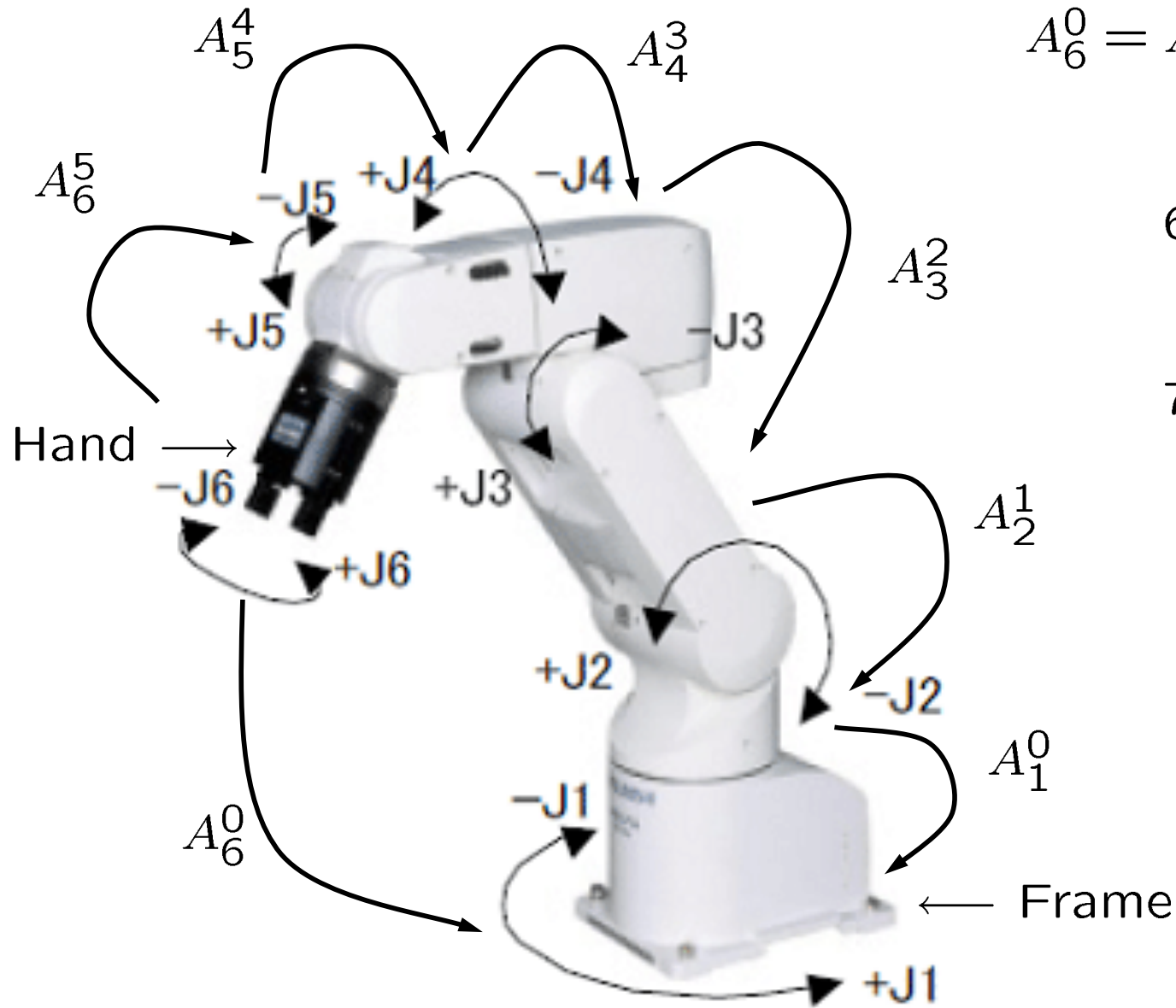


Advanced Robotics

Kinematics of serial manipulators

Serial manipulator kinematics

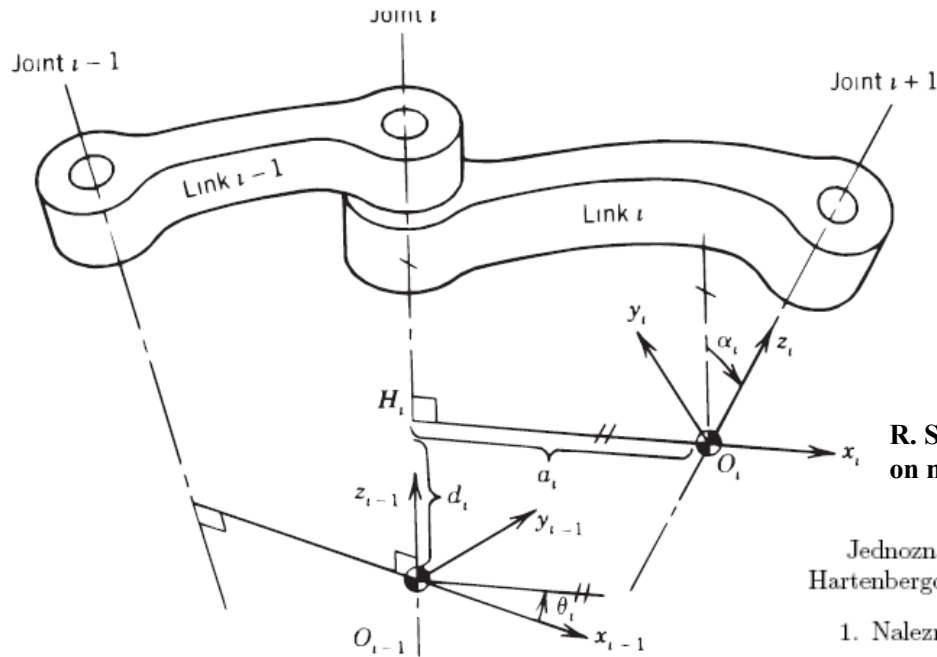


$$A_6^0 = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$

6 transformations

7 coordinate systems
indices 0 ... 6

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R. S. Hartenberg and J. Denavit, "A kinematic notation for lower pair mechanisms based on matrices," *Journal of Applied Mechanics*, vol. 77, pp. 215–221, June 1955.

Jednoznačný a efektivní popis jednotlivých transformací můžeme nalézt metodou Denavitovou-Hartenbergovou (Denavitova-Hartenbergova notace). Viz Obr. 9. Popisujeme kloub i .

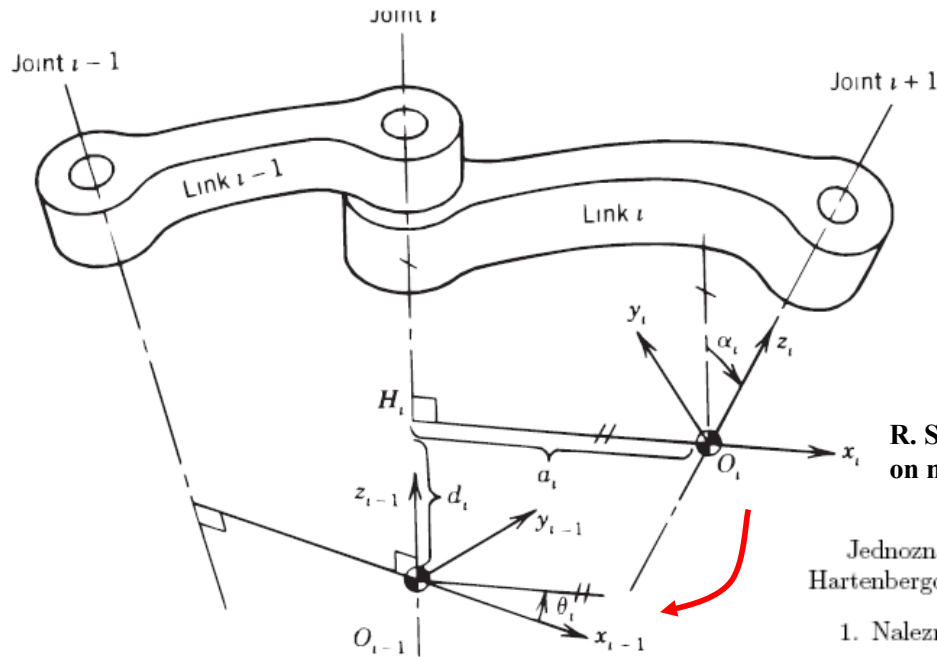
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Serial manipulator kinematics in the Denavit-Hartenberg convention



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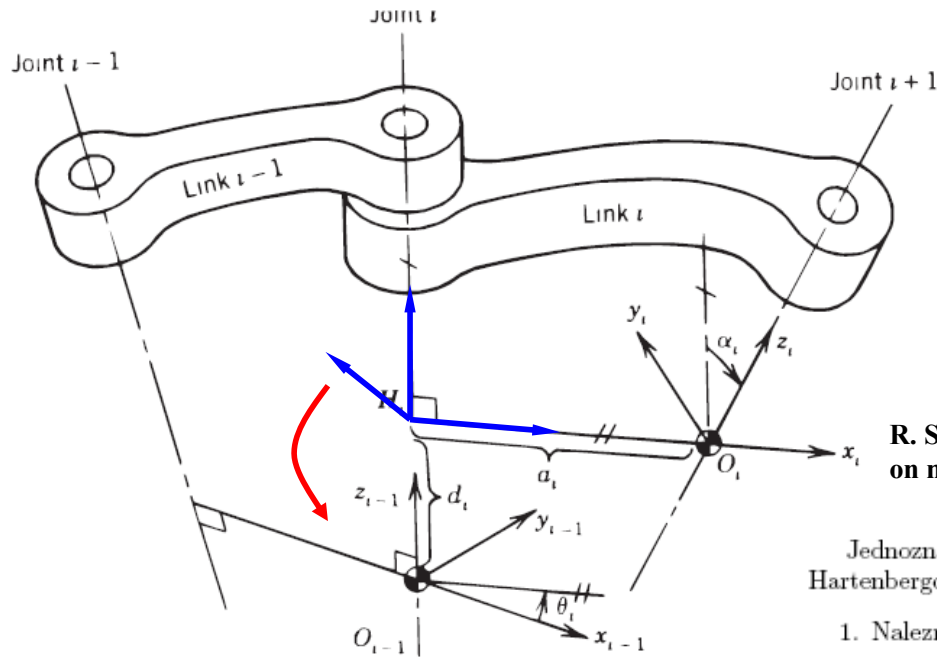
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Serial manipulator kinematics in the Denavit-Hartenberg convention



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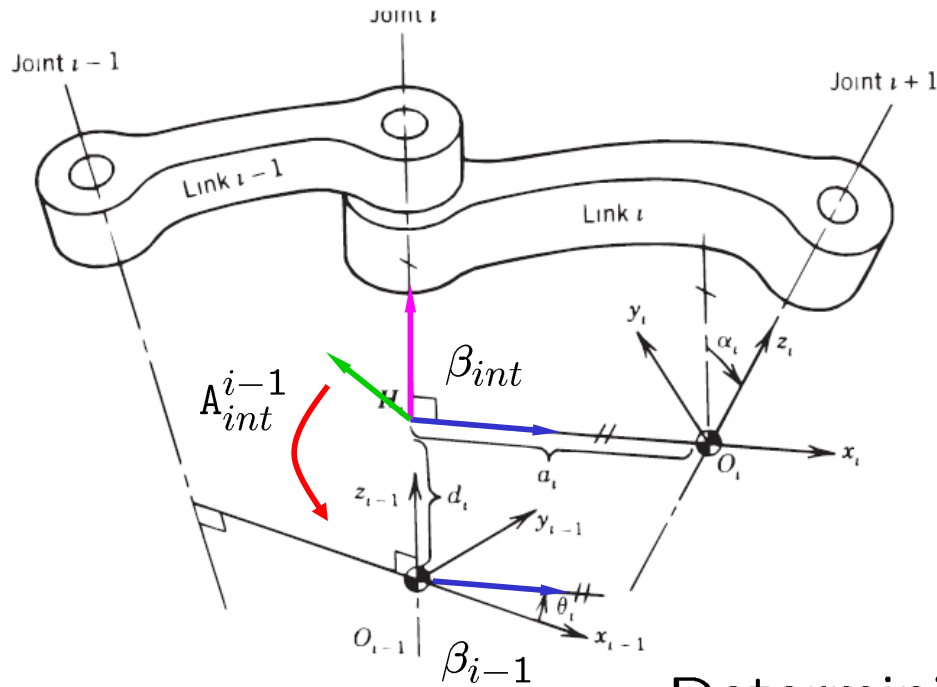
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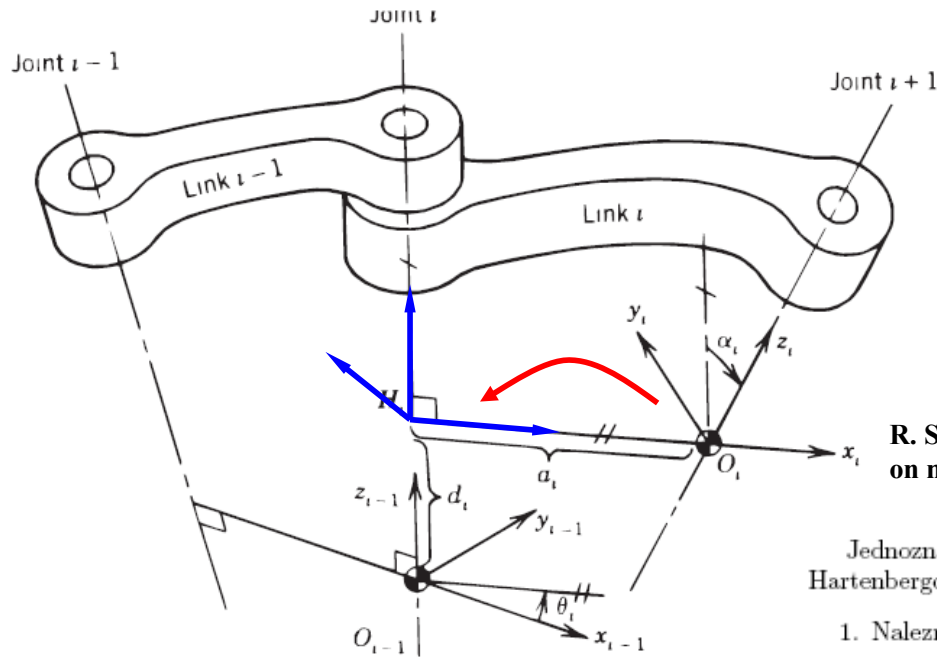
Serial manipulator kinematics in the Denavit-Hartenberg convention



Determining θ_i

$$\begin{aligned}
 \mathbf{X}^{i-1} &= A_{int}^{i-1} \mathbf{X}_{int} \\
 &= \begin{bmatrix} \vec{e}_{int1} \beta_{i-1} & \vec{e}_{int2} \beta_{i-1} & \vec{e}_{int3} \beta_{i-1} & \vec{d}_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{X}_{int} \\
 &= \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 & 0 \\ \sin \theta_i & \cos \theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{X}_{int}
 \end{aligned}$$

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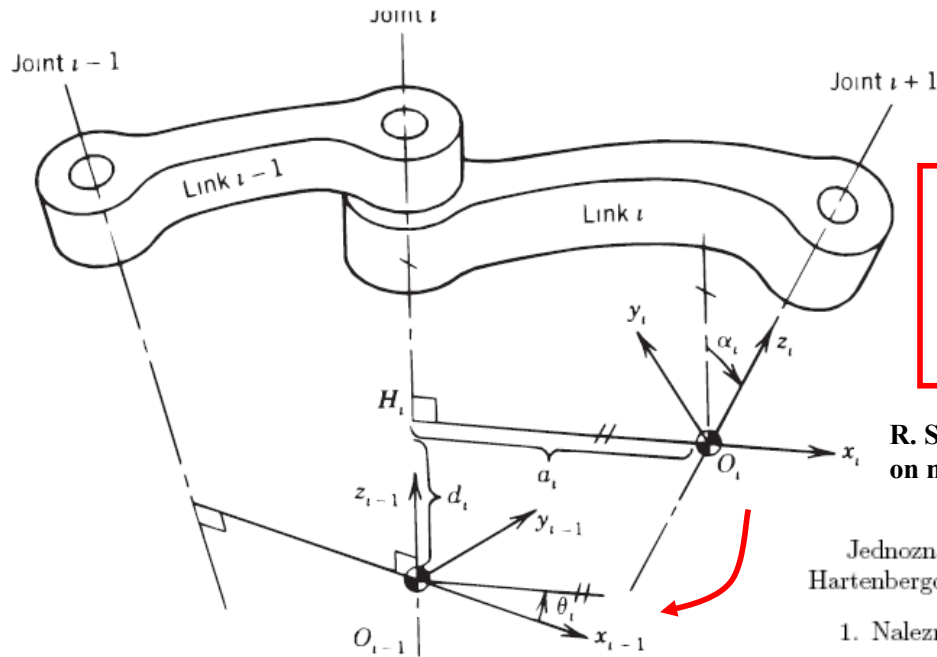
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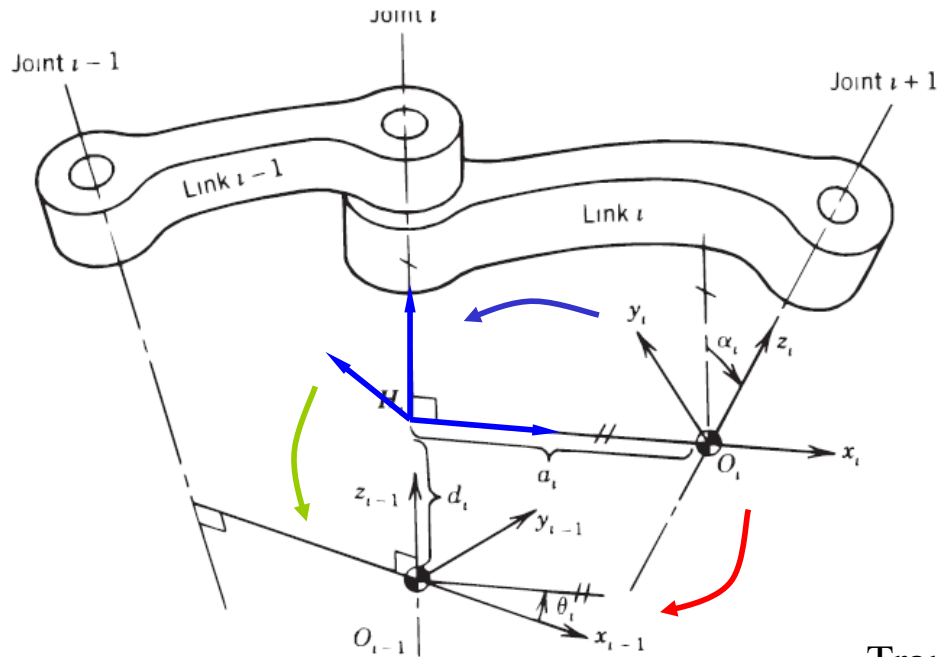
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Denavit-Hartenberg convention - step by step

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Serial manipulator kinematics in the Denavit-Hartenberg convention



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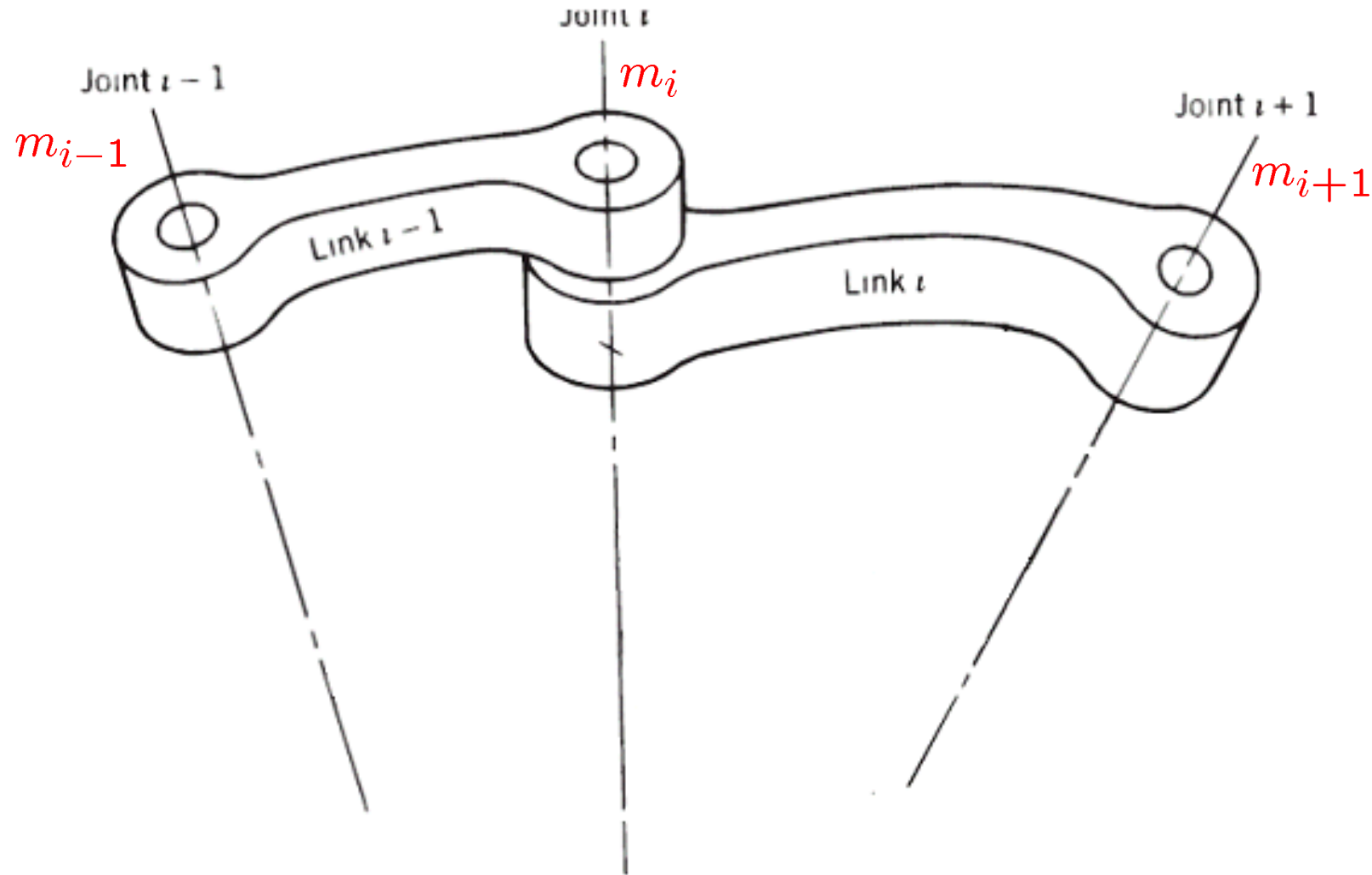
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Denavit-Hartenberg convention (for the joint i)

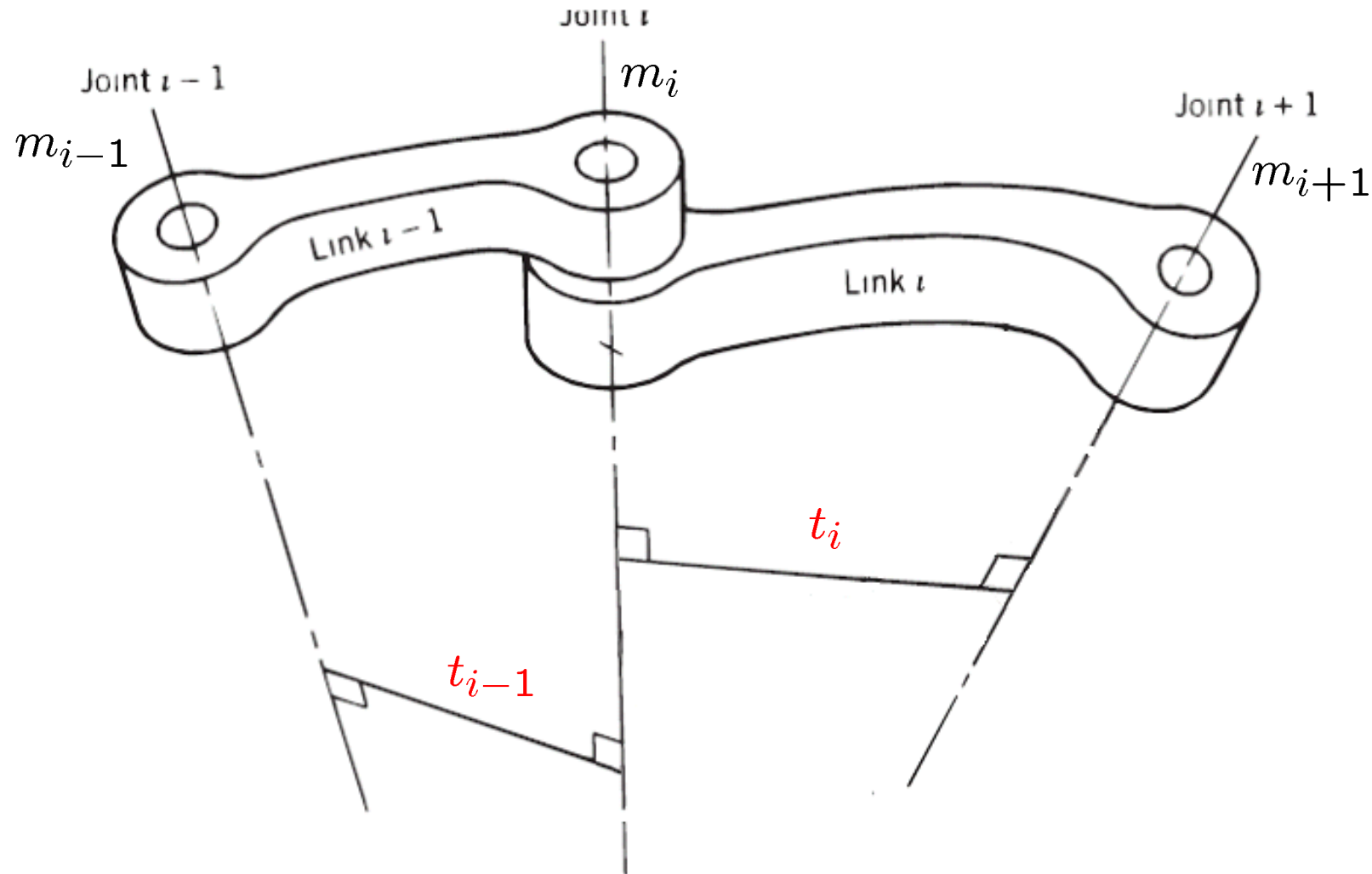
1. Find all motion axes $m_1, \dots, m_{i-1}, m_i, m_{i+1}, \dots$
- 2.1 Find the shortest transversals t_i between m_i and m_{i+1} .
- 2.2 If m_i is parallel to m_{i+1} , then t_i can be chosen arbitrarily, but the simplest is to make t_i intersect t_{i-1} .
- 2.3 If m_i intersects m_{i+1} , the t_i becomes the intersection point and the direction perpendicular to m_i, m_{i+1} .
- 3.2 O_0 can, in principle, be placed anywhere on m_1 , but the simplest choice is $O_0 = H_1$.
- 3.3 Find origins $O_i = t_i \wedge m_{i+1}$ and points $H_i = m_i \wedge t_i$.
- 3.4 If m_i intersects m_{i+1} , then $O_i = H_i$
- 3.5 O_N can, in principle, be placed anywhere, but the simplest choice is $O_N = H_N = O_{N-1}$.
- 4.1 Choose axis \vec{z}_0 along the motion axis m_1 . There are two choices of the orientation, which are equivalent and can be chosen at will
- 4.2 Choose axis \vec{x}_0 along t_1 in the direction from O_0 to O_1 . at will.
- 4.3 Choose axis \vec{y}_0 to form a right-handed coordinate system.
- 4.4 Place \vec{z}_i axis along the m_{i+1} axis, preferably to contain a sharp angle with the \vec{z}_{i-1} .
- 4.5 Place \vec{x}_i axis along t_i in the direction from H_i to O_i .
- 4.6 If m_i intersects m_{i+1} , then place \vec{x}_i in the direction perpendicular to m_i, m_{i+1} , preferably to contain a sharp angle with \vec{x}_{i-1} .
- 4.7 Choose axis \vec{y}_i to form a right-handed coordinate system.
- 4.8 Construct the *intermediate* coordinate system ($H_i, \vec{x}_{int} = \vec{x}_i, \vec{y}_{int} = \vec{z}_{i-1} \times \vec{x}_i, \vec{z}_{int} = \vec{z}_{i-1}$) and define α_i such that $\vec{y}_i = \cos(\alpha_i) \vec{y}_{int} + \sin(\alpha_i) \vec{z}_{int}$.
- 4.9 Define θ_i such that $\vec{x}_{int} = \cos(\theta_i) \vec{x}_{i-1} + \sin(\theta_i) \vec{y}_{i-1}$
- 5.1 Define a_i such that $O_i = H_i + a_i \vec{x}_{int}$
- 5.2 Define d_i such that $H_i = O_{i-1} + d_i \vec{z}_{i-1}$

Serial manipulator kinematics in the Denavit-Hartenberg convention



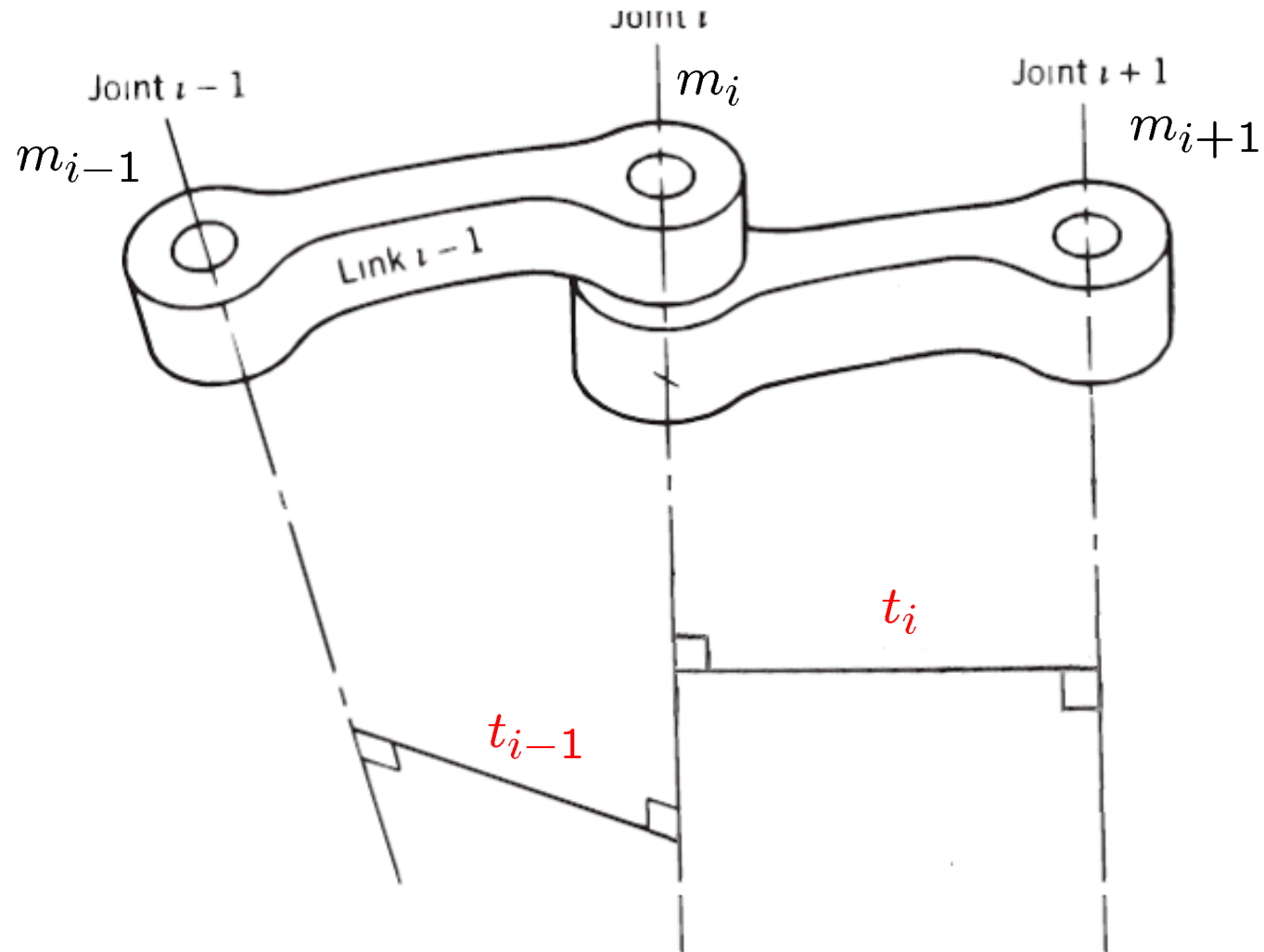
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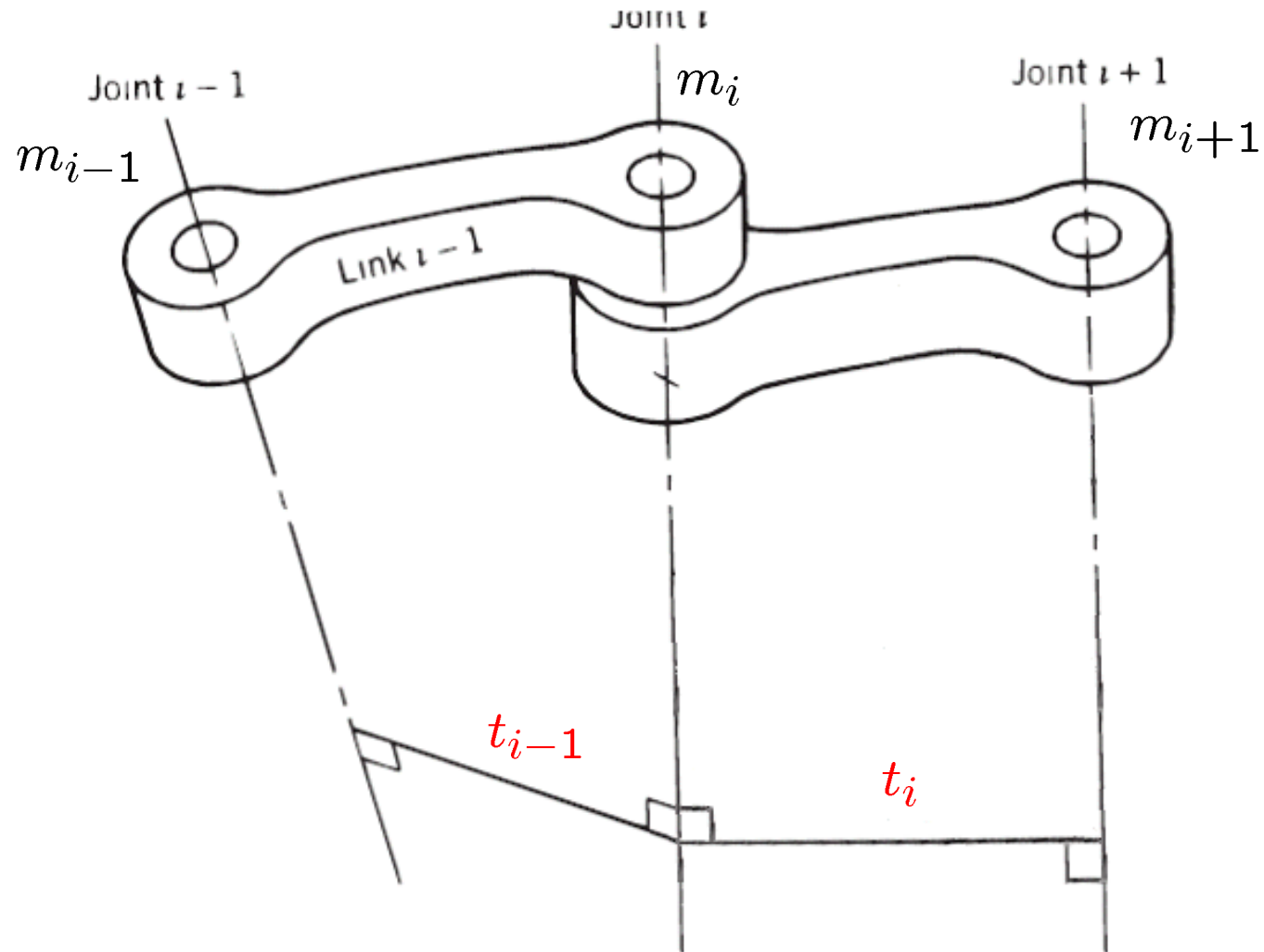
2.1 Find the shortest transversals t_i between m_i and m_{i+1} .

Serial manipulator kinematics in the Denavit-Hartenberg convention



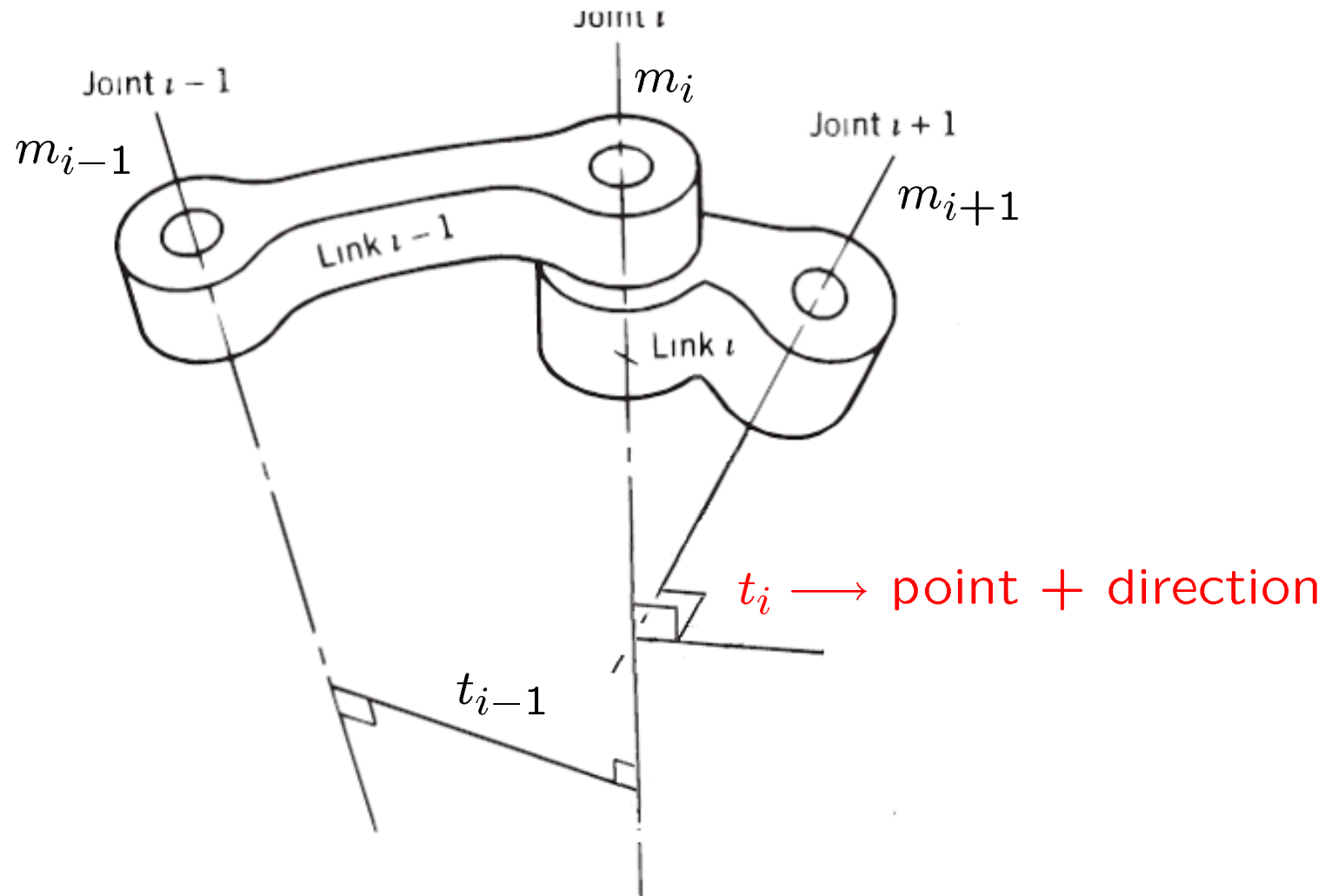
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Serial manipulator kinematics in the Denavit-Hartenberg convention



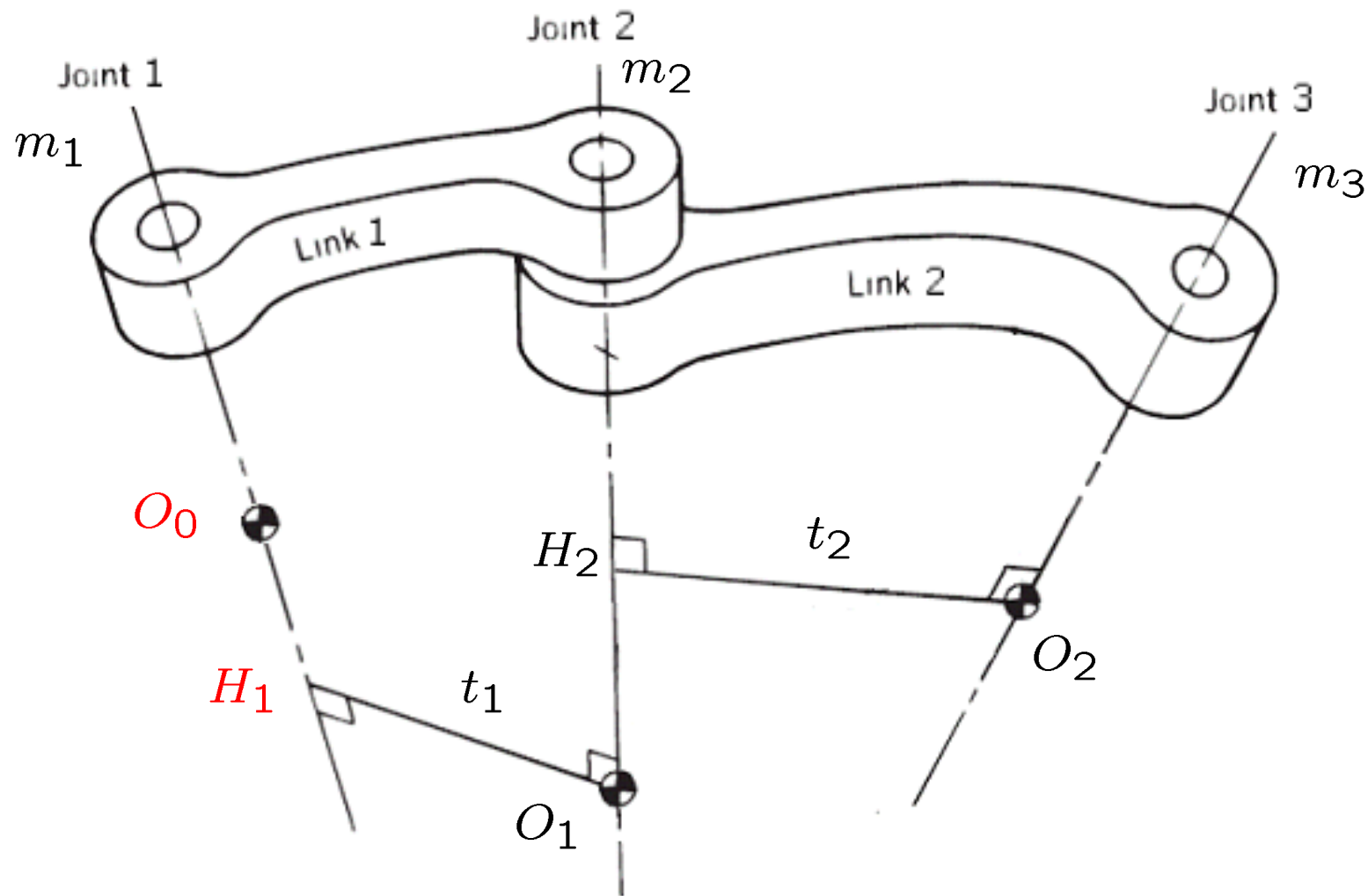
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Serial manipulator kinematics in the Denavit-Hartenberg convention



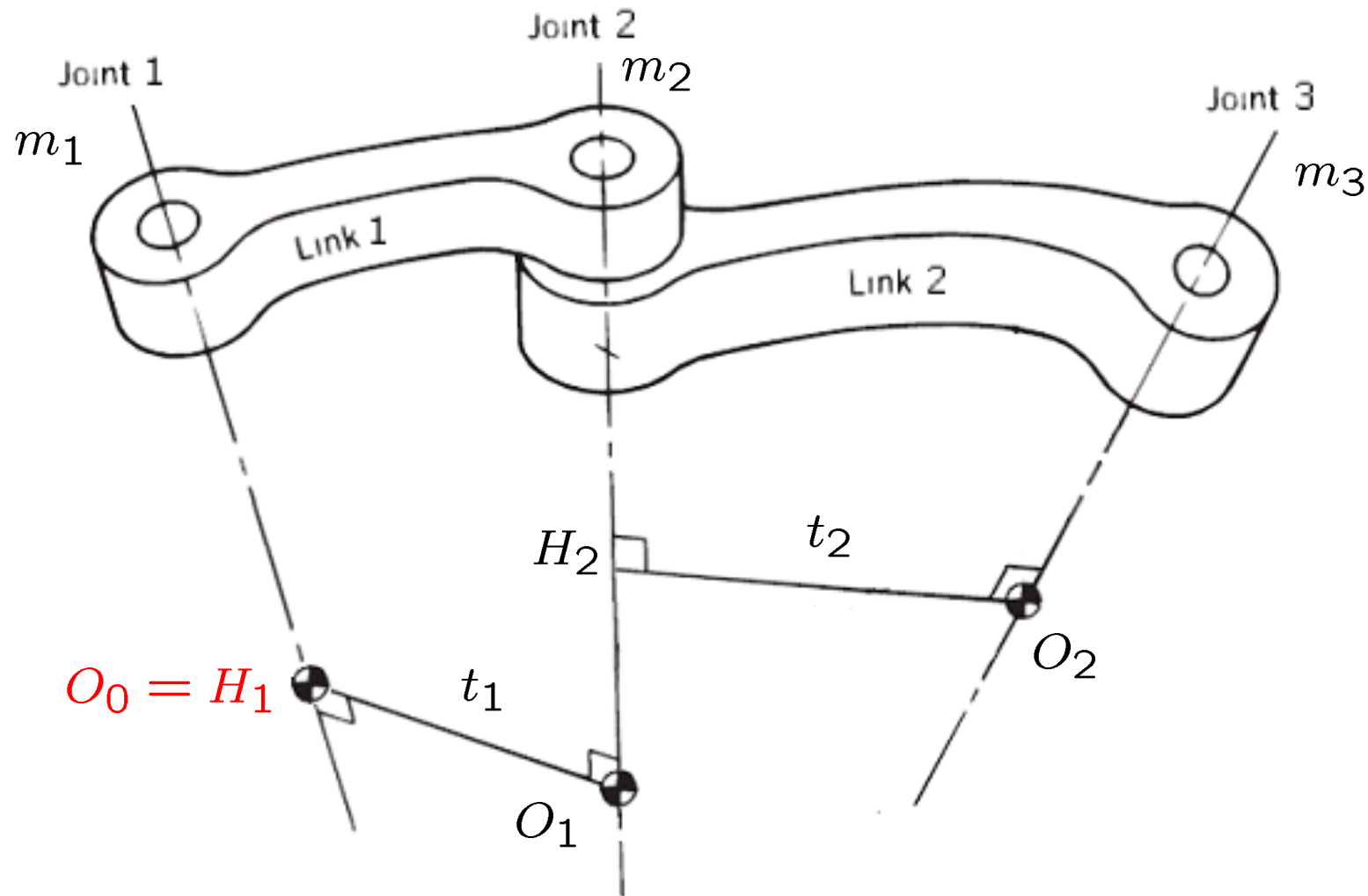
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Serial manipulator kinematics in the Denavit-Hartenberg convention



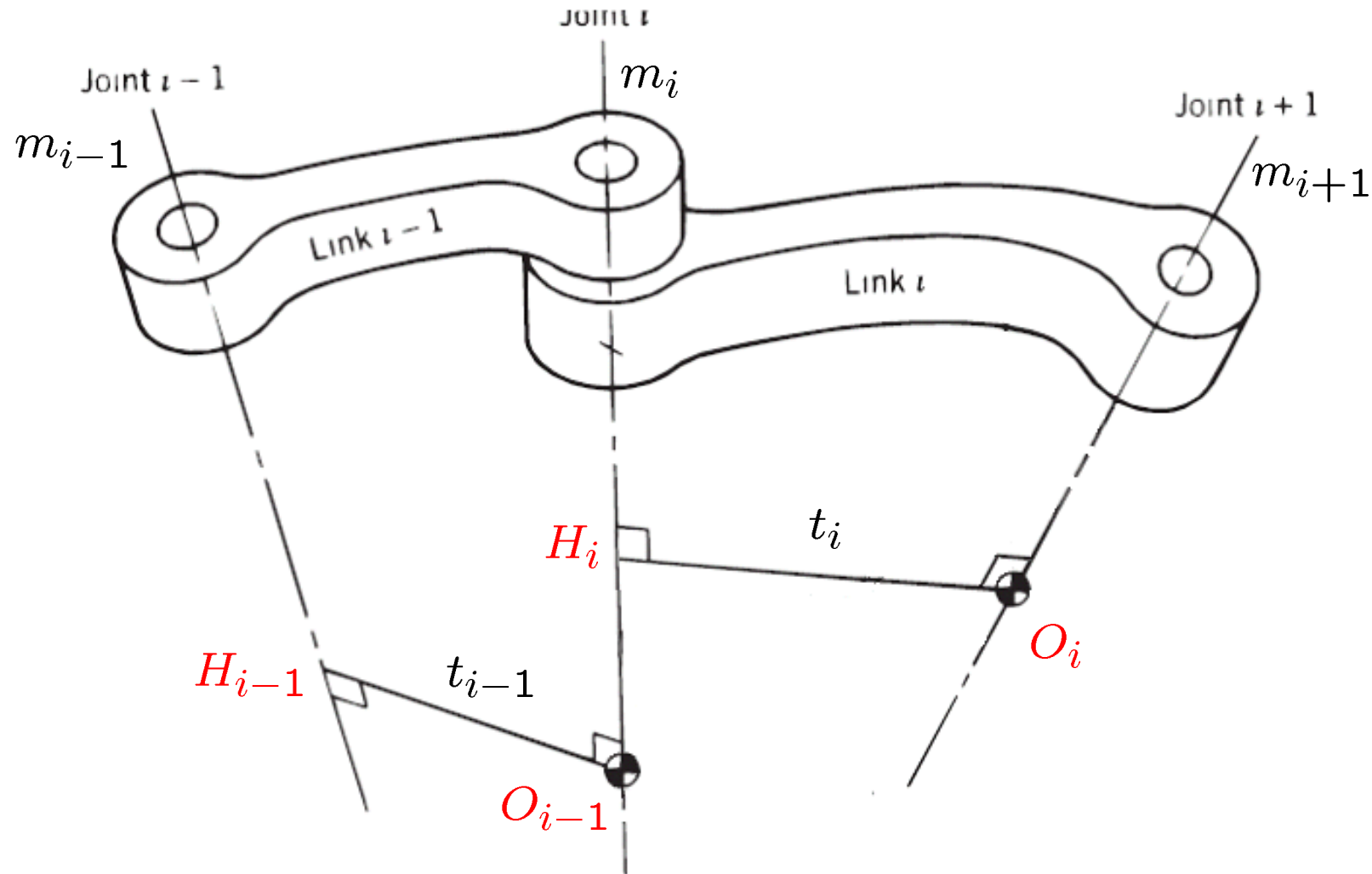
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Serial manipulator kinematics in the Denavit-Hartenberg convention



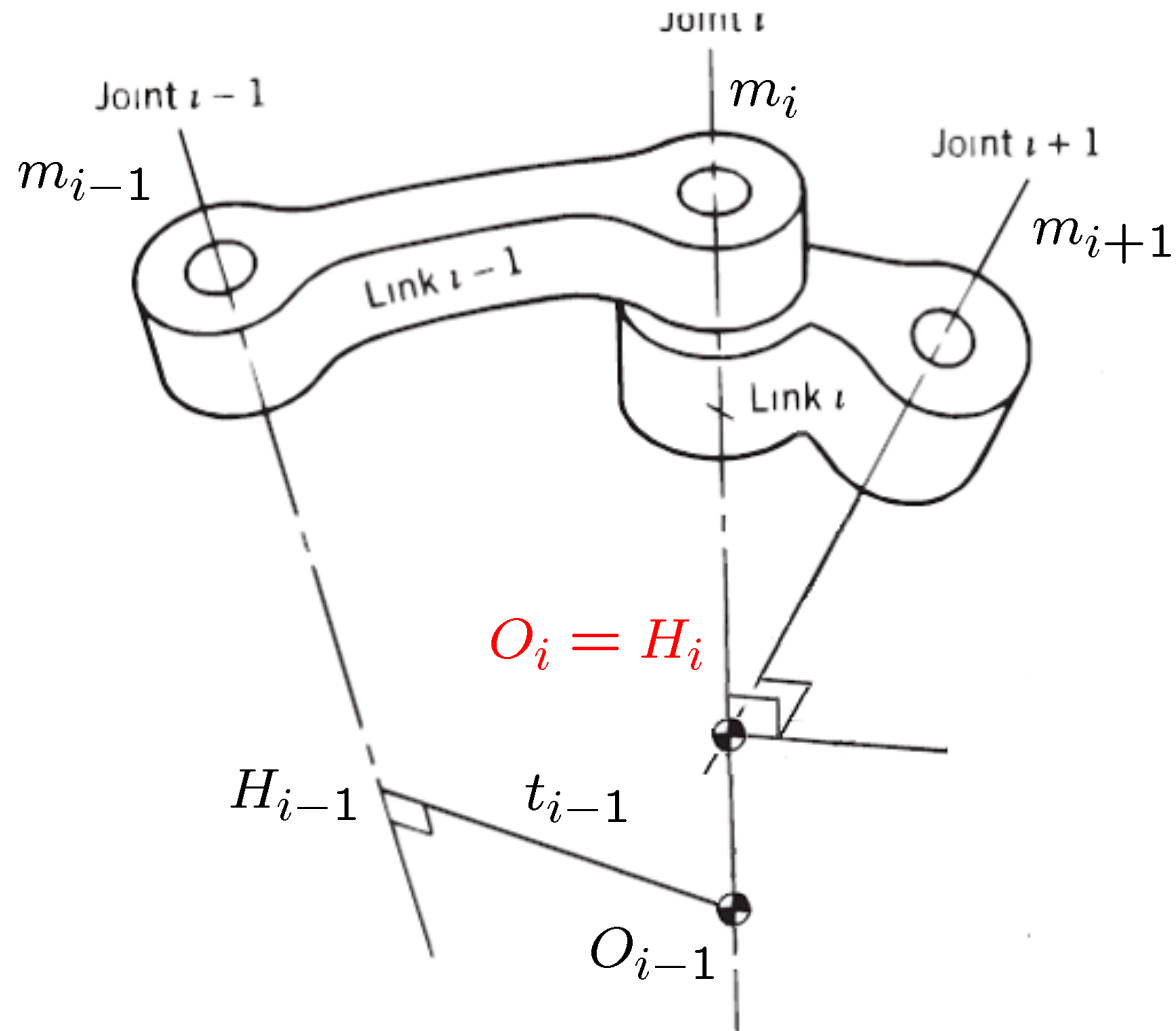
... the simplest choice is $O_0 = H_1$.

Serial manipulator kinematics in the Denavit-Hartenberg convention



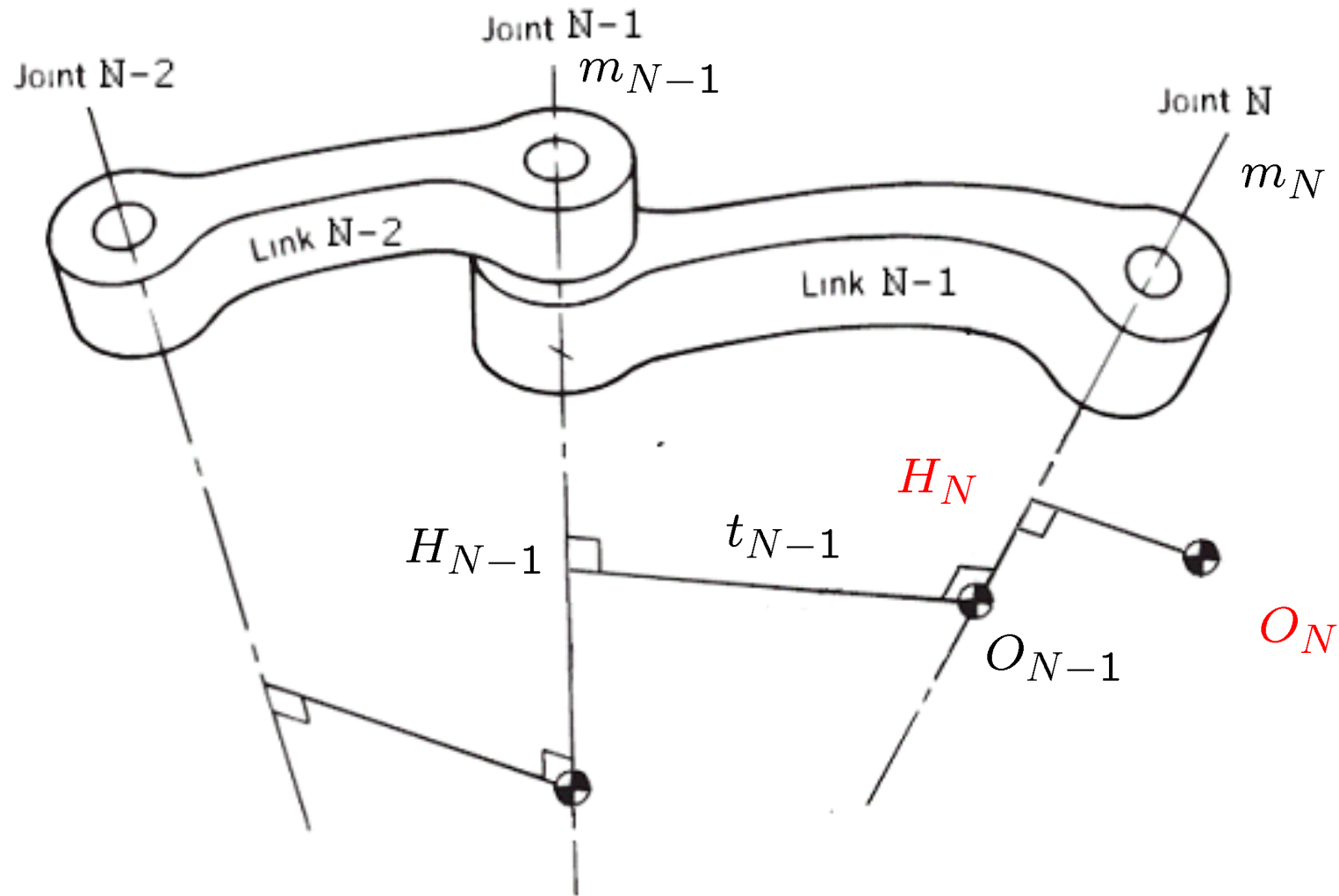
3.3 Find origins $O_i = t_i \wedge m_{i+1}$ and points $H_i = m_i \wedge t_i$.

Serial manipulator kinematics in the Denavit-Hartenberg convention



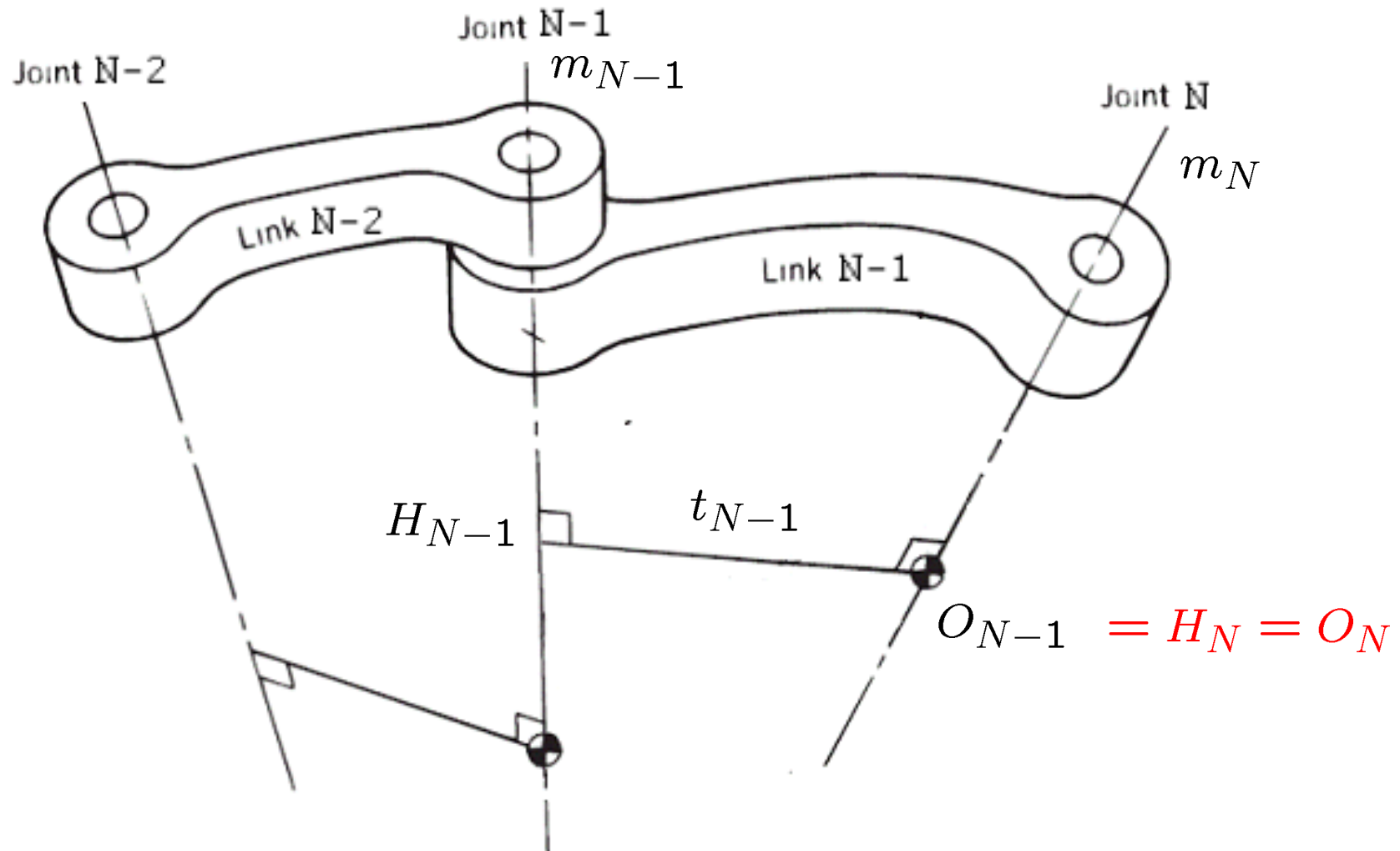
3.4 If m_i intersects m_{i+1} , then $O_i = H_i$

Serial manipulator kinematics in the Denavit-Hartenberg convention



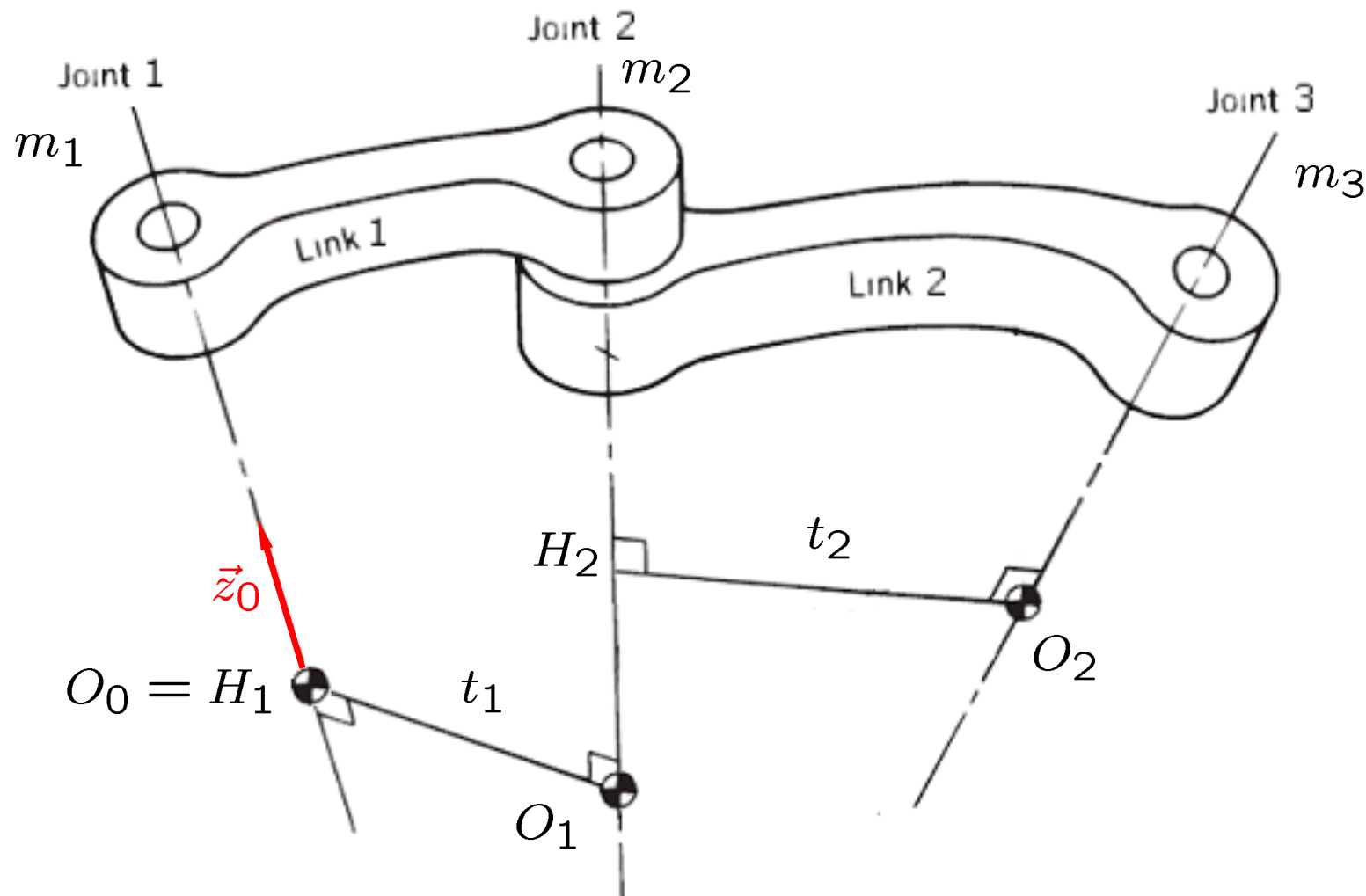
3.5 O_N can, in principle, be placed anywhere,
but ...

Serial manipulator kinematics in the Denavit-Hartenberg convention



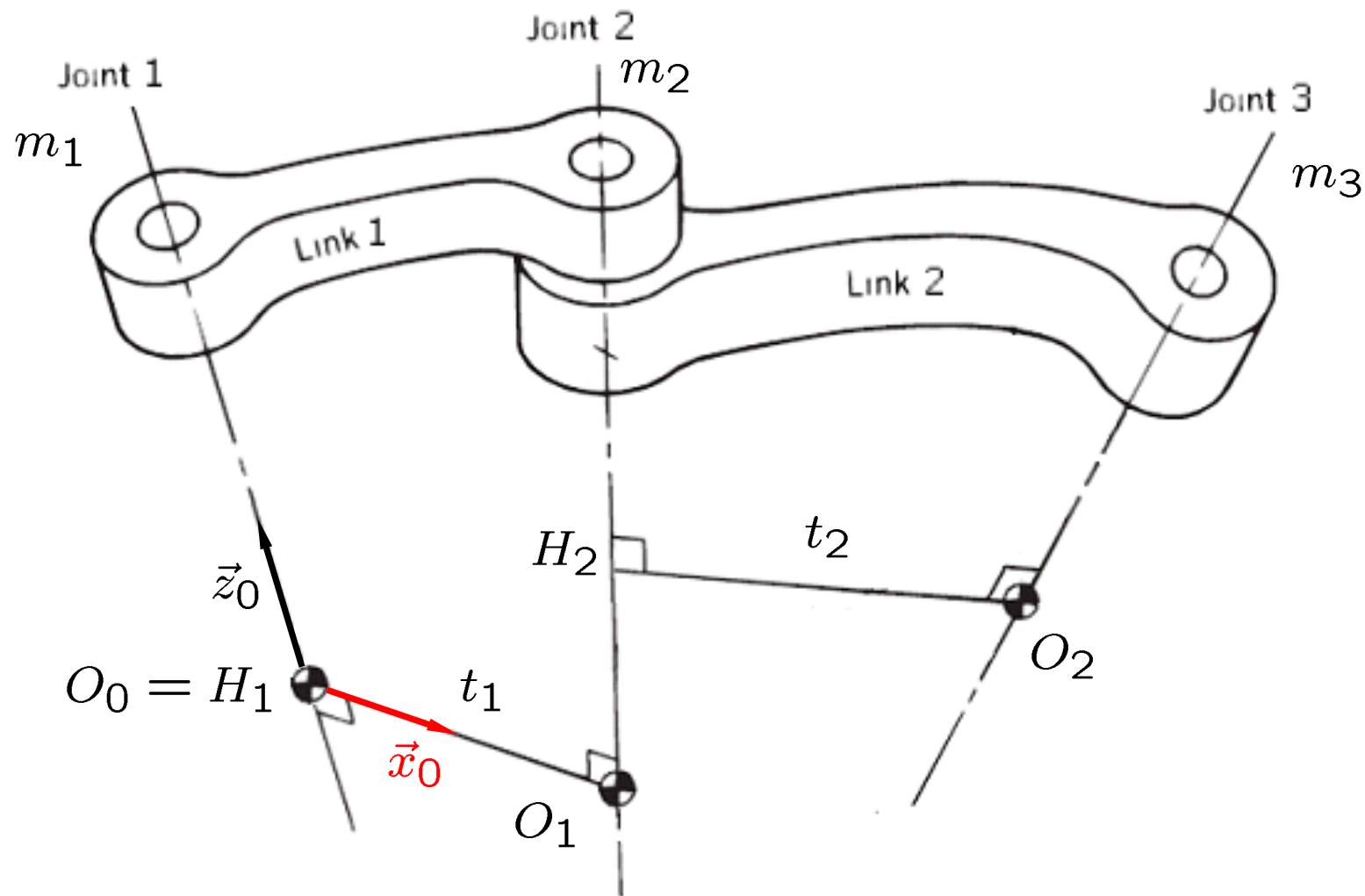
... the simplest choice is $O_N = H_N = O_{N-1}$.

Serial manipulator kinematics in the Denavit-Hartenberg convention



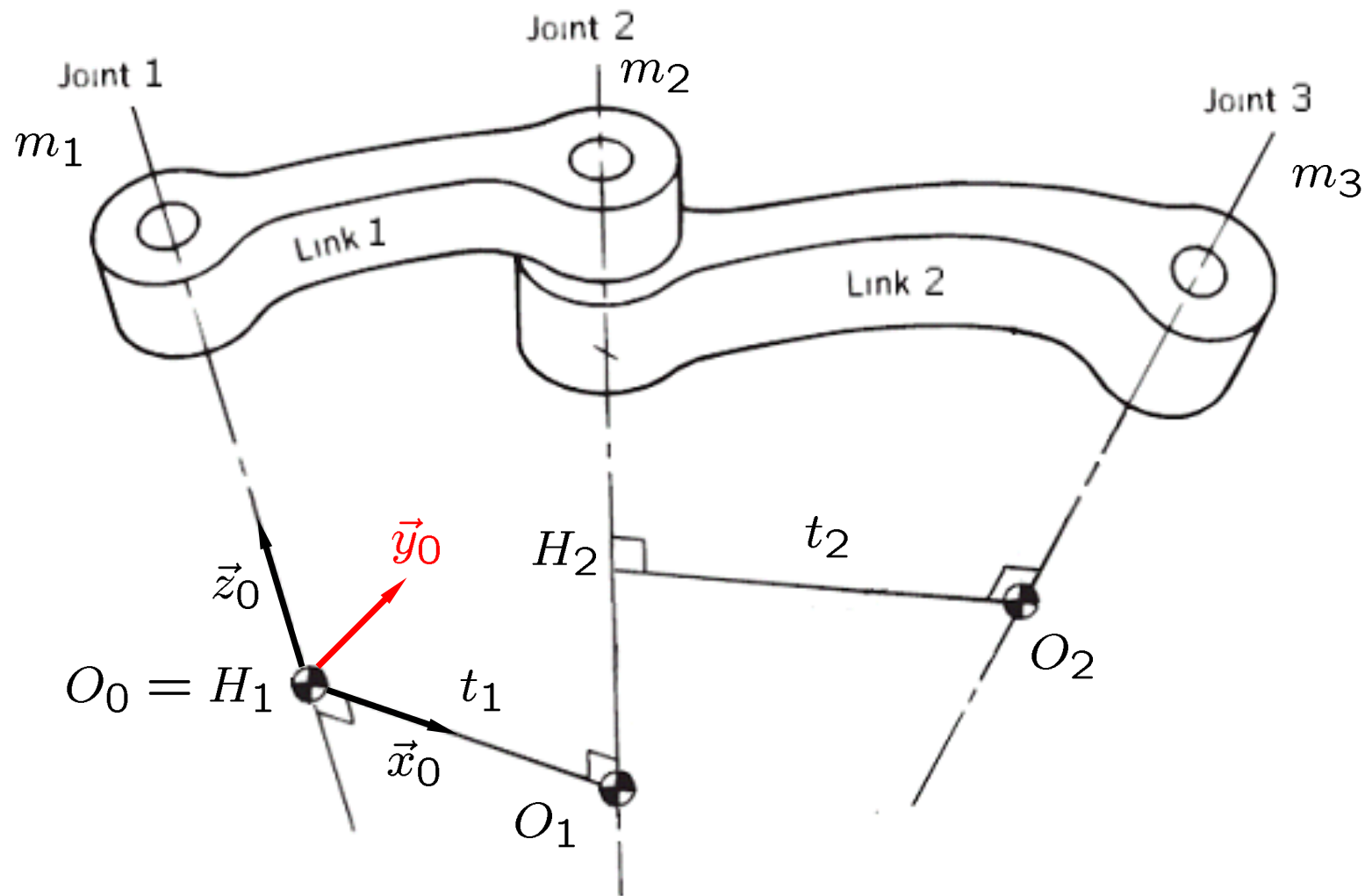
4.1 Choose axis \vec{z}_0 along the motion axis m_1 . There are two choices of the orientation, which are equivalent and can be chosen at will.

Serial manipulator kinematics in the Denavit-Hartenberg convention



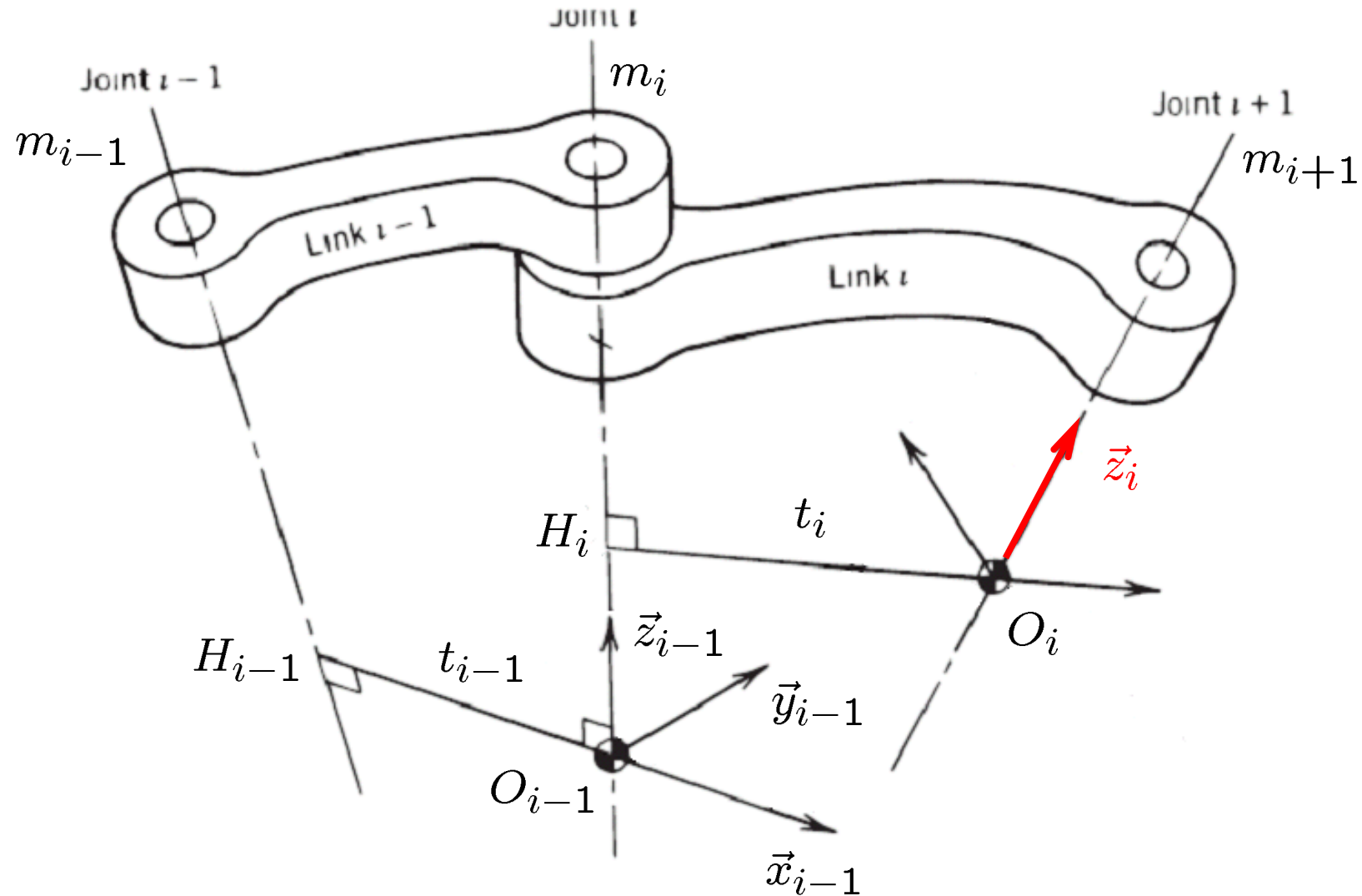
4.2 Choose axis \vec{x}_0 along t_1 in the direction from O_0 to O_1 . at will.

Serial manipulator kinematics in the Denavit-Hartenberg convention



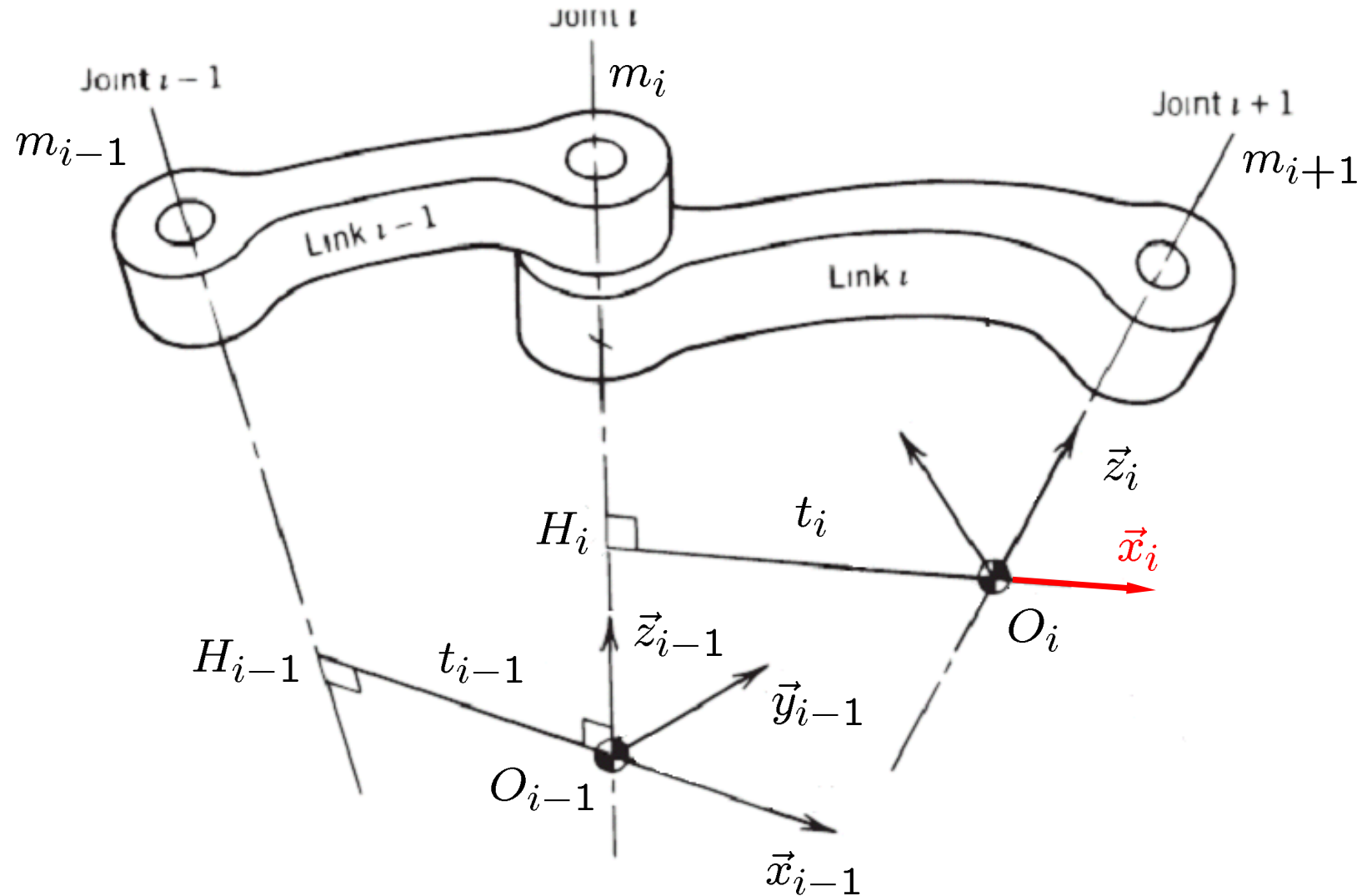
4.3 Choose axis \vec{y}_0 to form a right-handed coordinate system.

Serial manipulator kinematics in the Denavit-Hartenberg convention



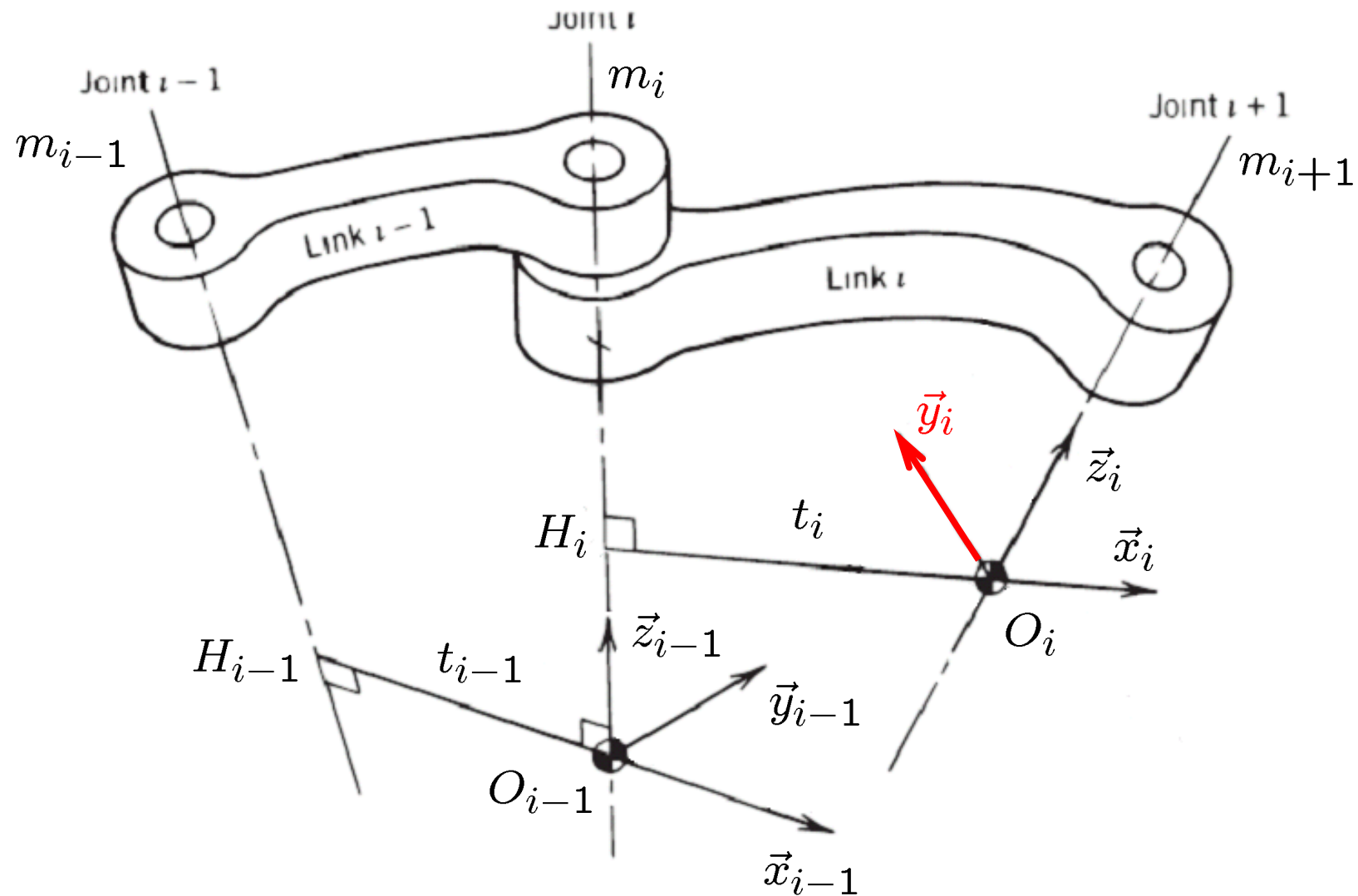
4.4 Place \vec{z}_i axis along the m_{i+1} axis, preferably to contain a sharp angle with the \vec{z}_{i-1} .

Serial manipulator kinematics in the Denavit-Hartenberg convention



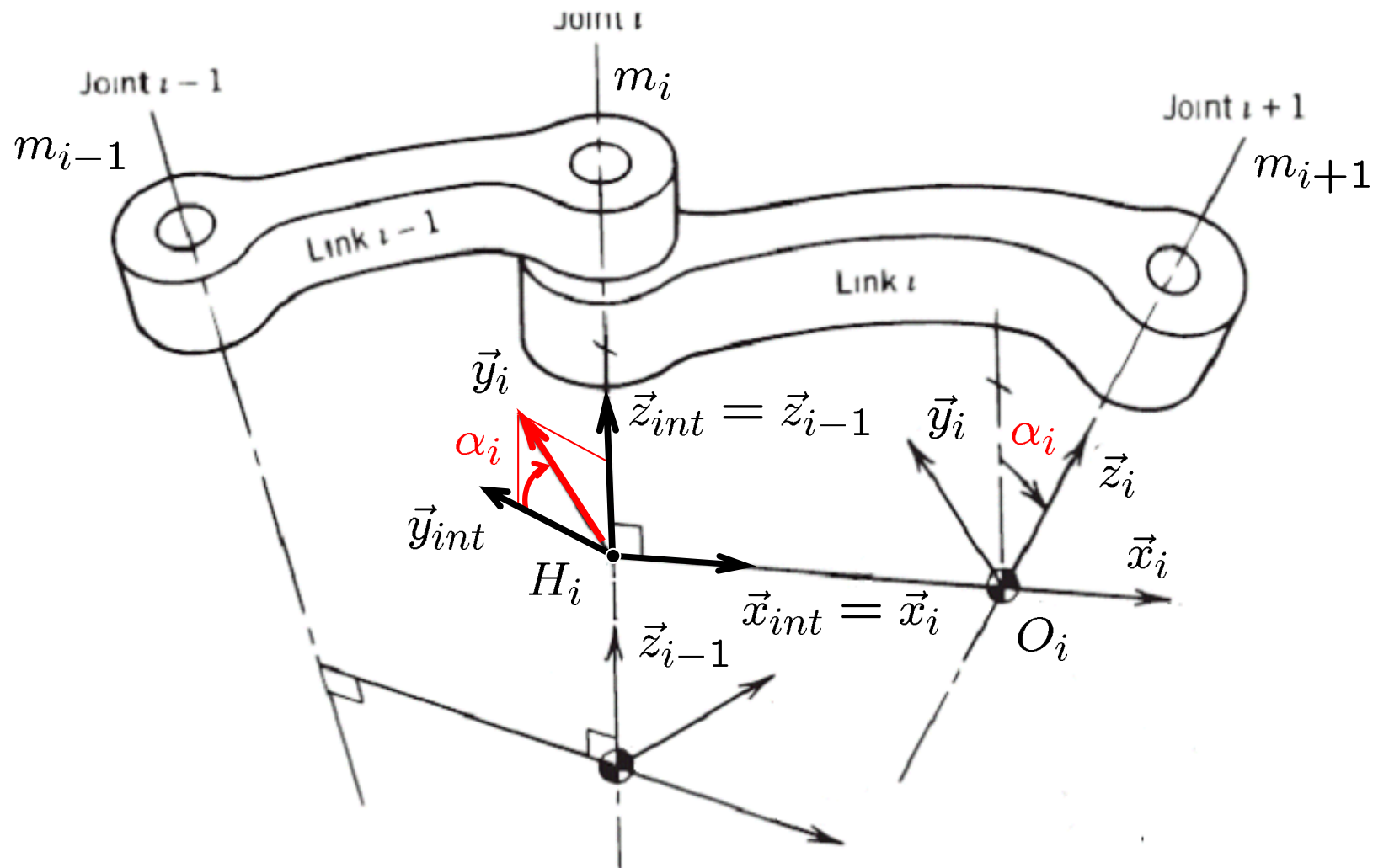
4.5 Place \vec{x}_i axis along t_i in the direction from H_i to O_i .

Serial manipulator kinematics in the Denavit-Hartenberg convention



4.7 Choose axis \vec{y}_i to form a right-handed coordinate system.

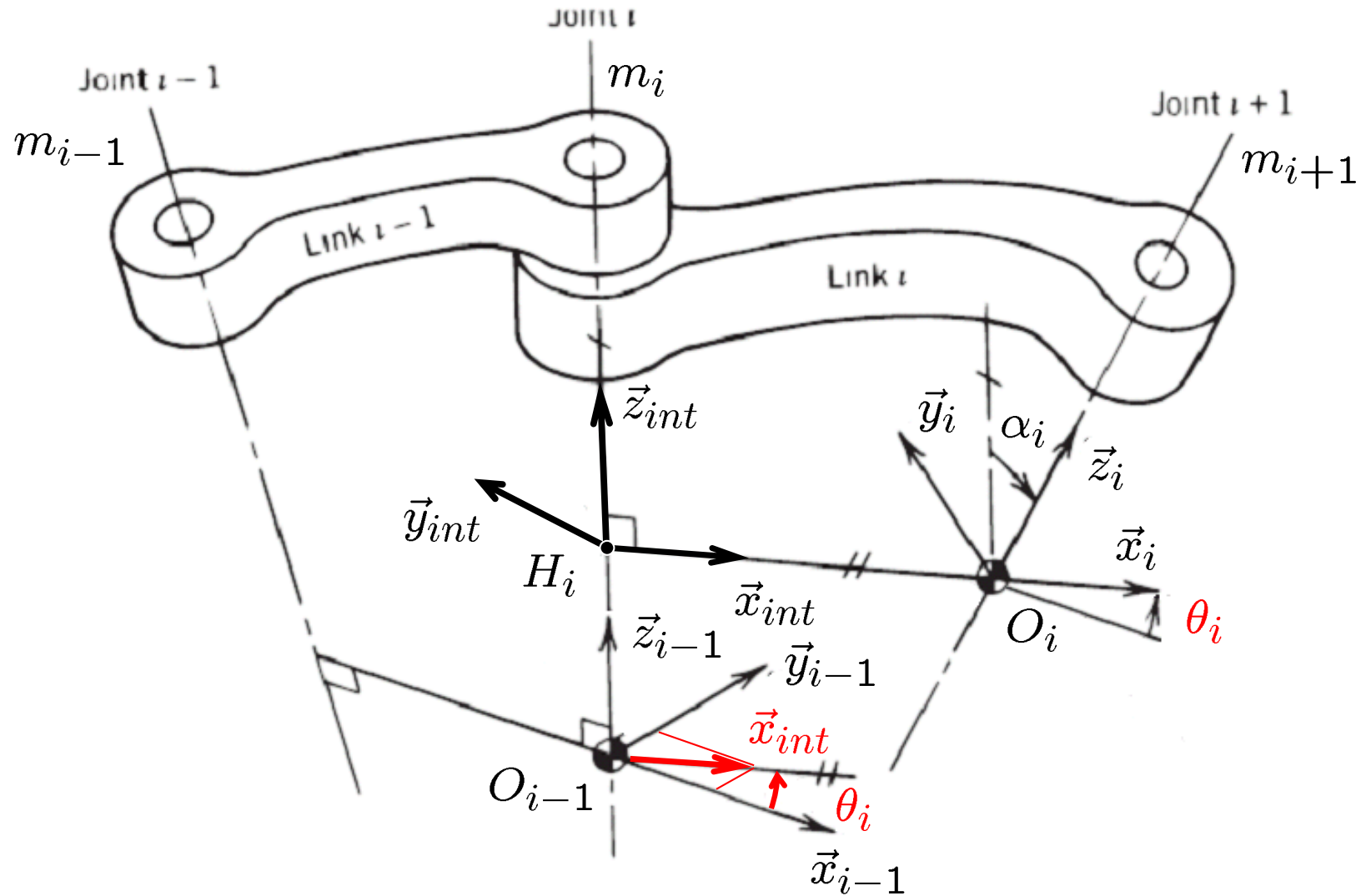
Serial manipulator kinematics in the Denavit-Hartenberg convention



4.8 Construct the *intermediate* coordinate system

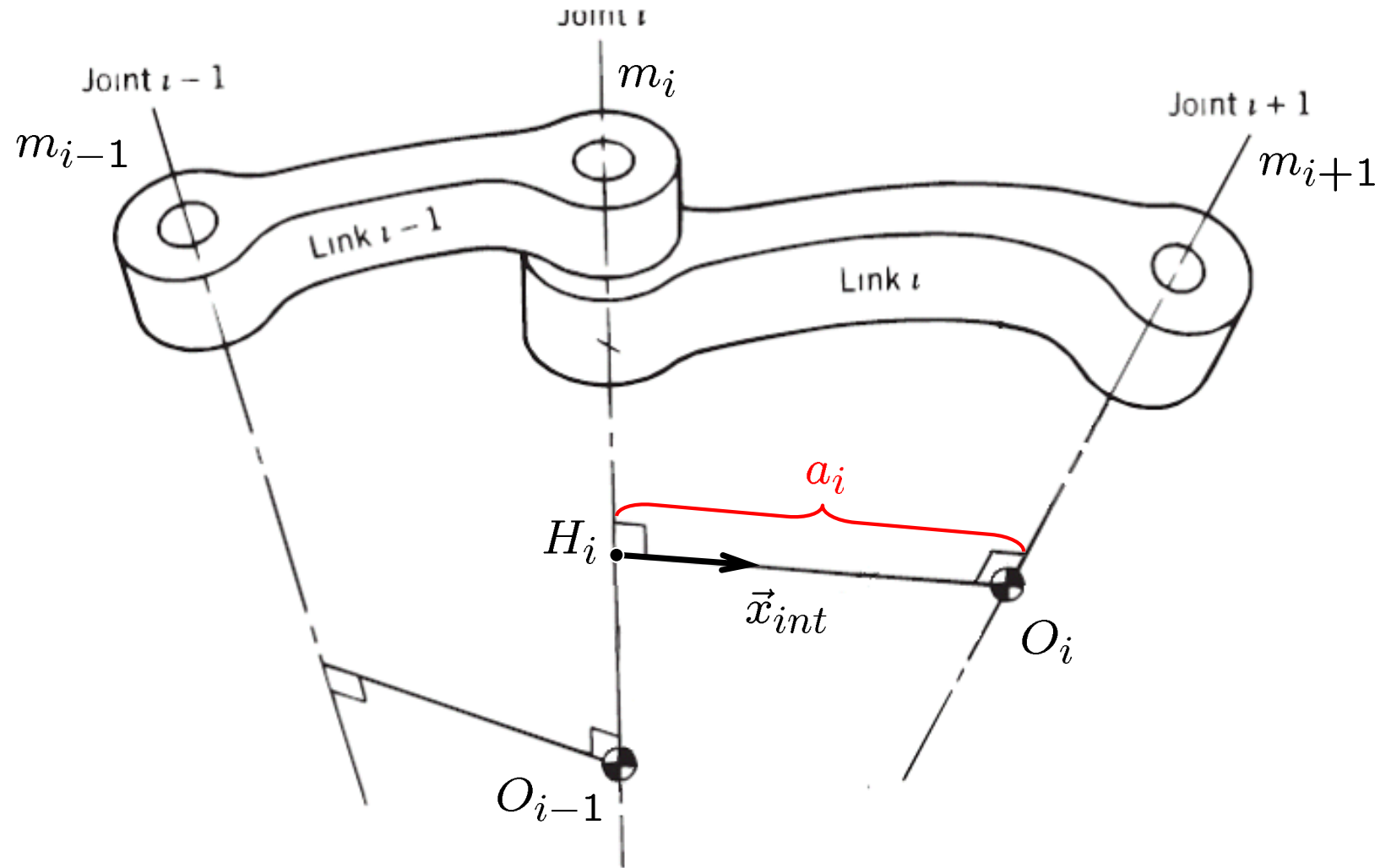
($H_i, \vec{x}_{int} = \vec{x}_i, \vec{y}_{int} = \vec{z}_{i-1} \times \vec{x}_i, \vec{z}_{int} = \vec{z}_{i-1}$) and define α_i such that $\vec{y}_i = \cos(\alpha_i) \vec{y}_{int} + \sin(\alpha_i) \vec{z}_{int}$

Serial manipulator kinematics in the Denavit-Hartenberg convention



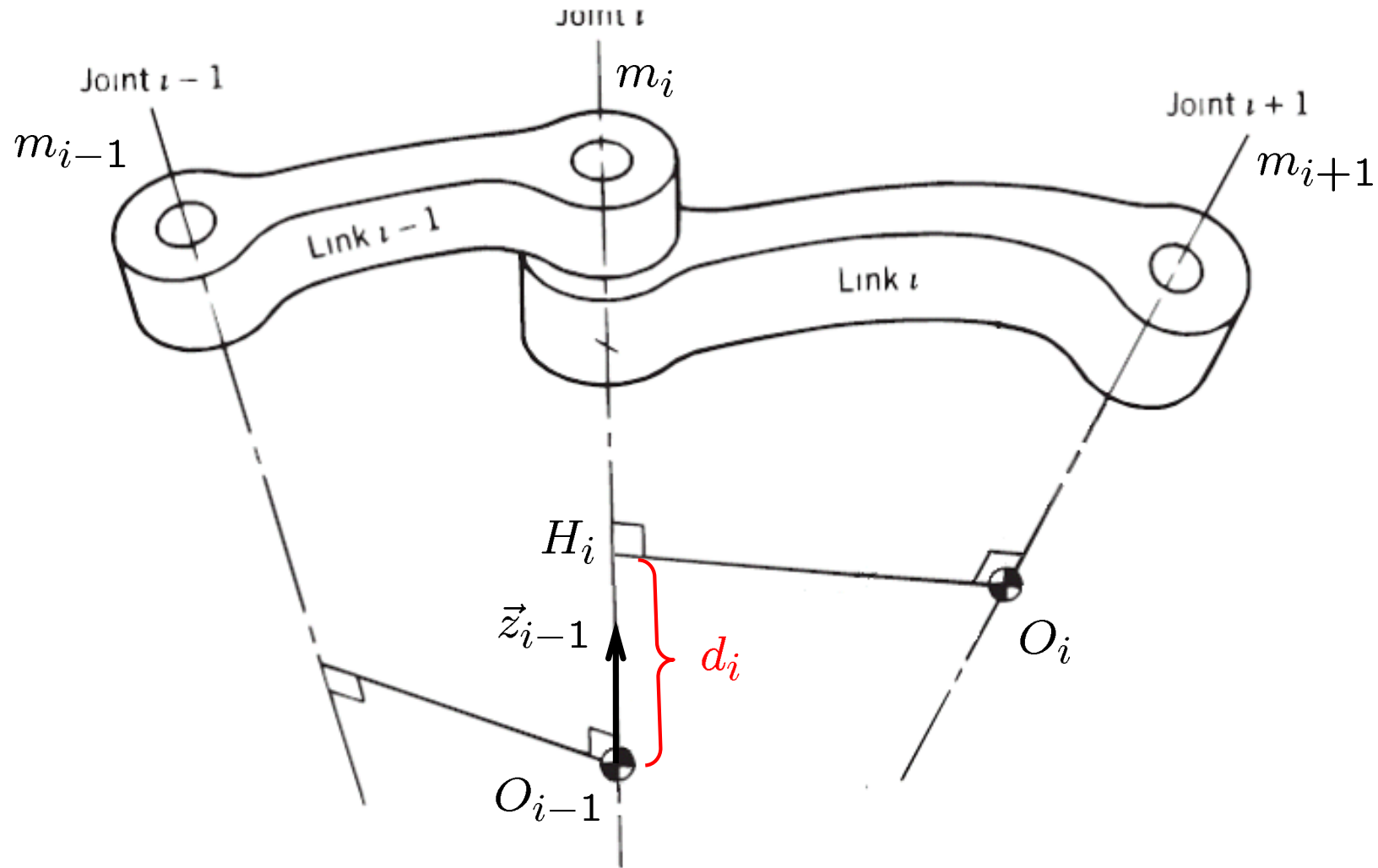
4.9 Define θ_i such that $\vec{x}_{int} = \cos(\theta_i) \vec{x}_{i-1} + \sin(\theta_i) \vec{y}_{i-1}$

Serial manipulator kinematics in the Denavit-Hartenberg convention



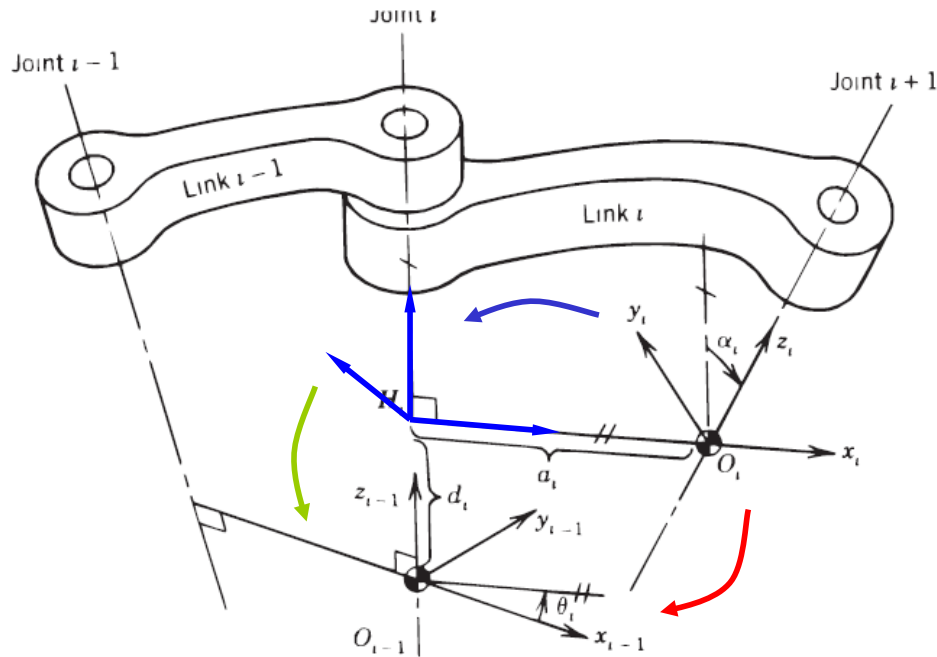
5.1 Define a_i such that $O_i = H_i + a_i \vec{x}_{int}$

Serial manipulator kinematics in the Denavit-Hartenberg convention



5.2 Define d_i such that $H_i = O_{i-1} + d_i \vec{z}_{i-1}$

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\mathbf{A}_i^{i-1} = \mathbf{A}_{int}^{i-1} \mathbf{A}_i^{int},$$

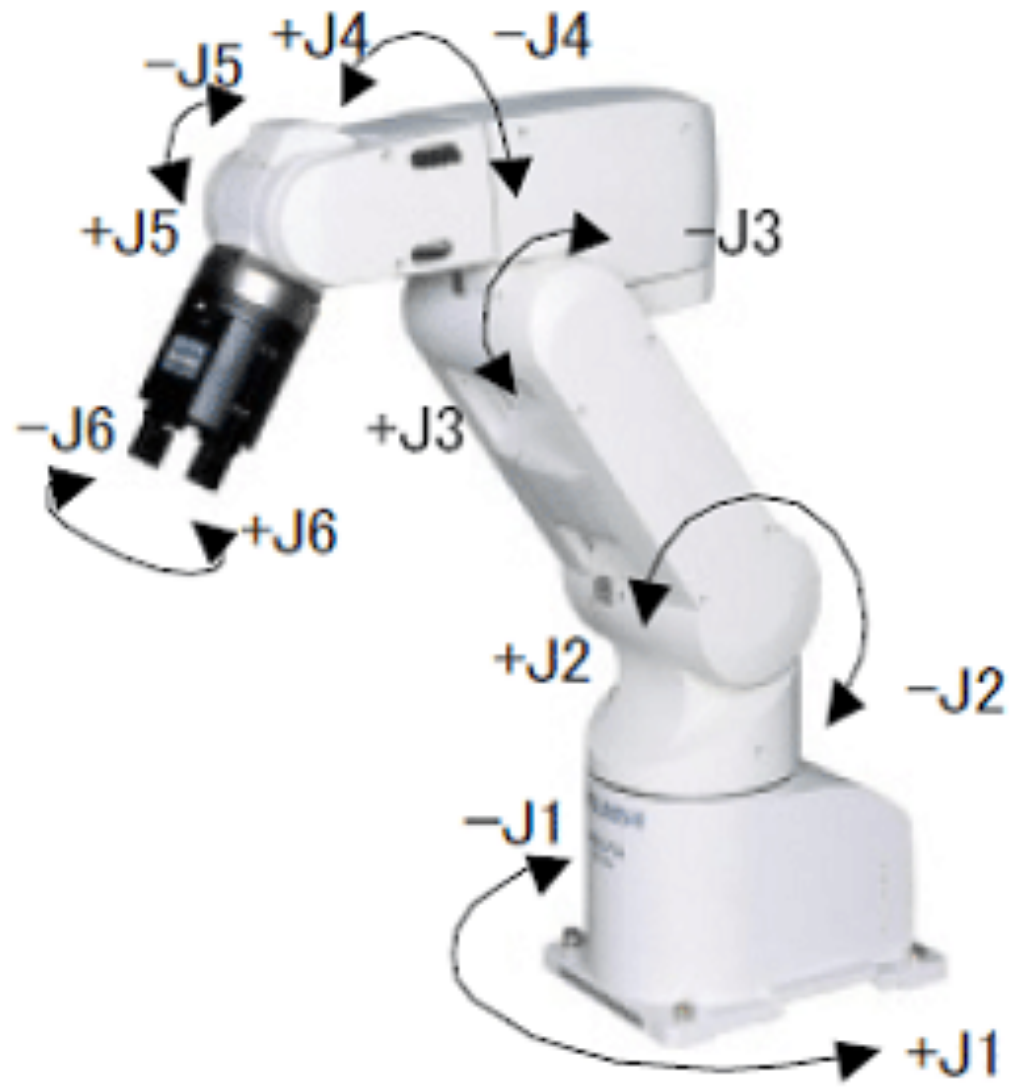
$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$\mathbf{A}_{int}^{i-1} = \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 & 0 \\ \sin \theta_i & \cos \theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_i^{int} = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_i^{int} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos \alpha_i & -\sin \alpha_i & 0 \\ 0 & \sin \alpha_i & \cos \alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Kinematics of Mitsubishi RV-6S/6SC Robot

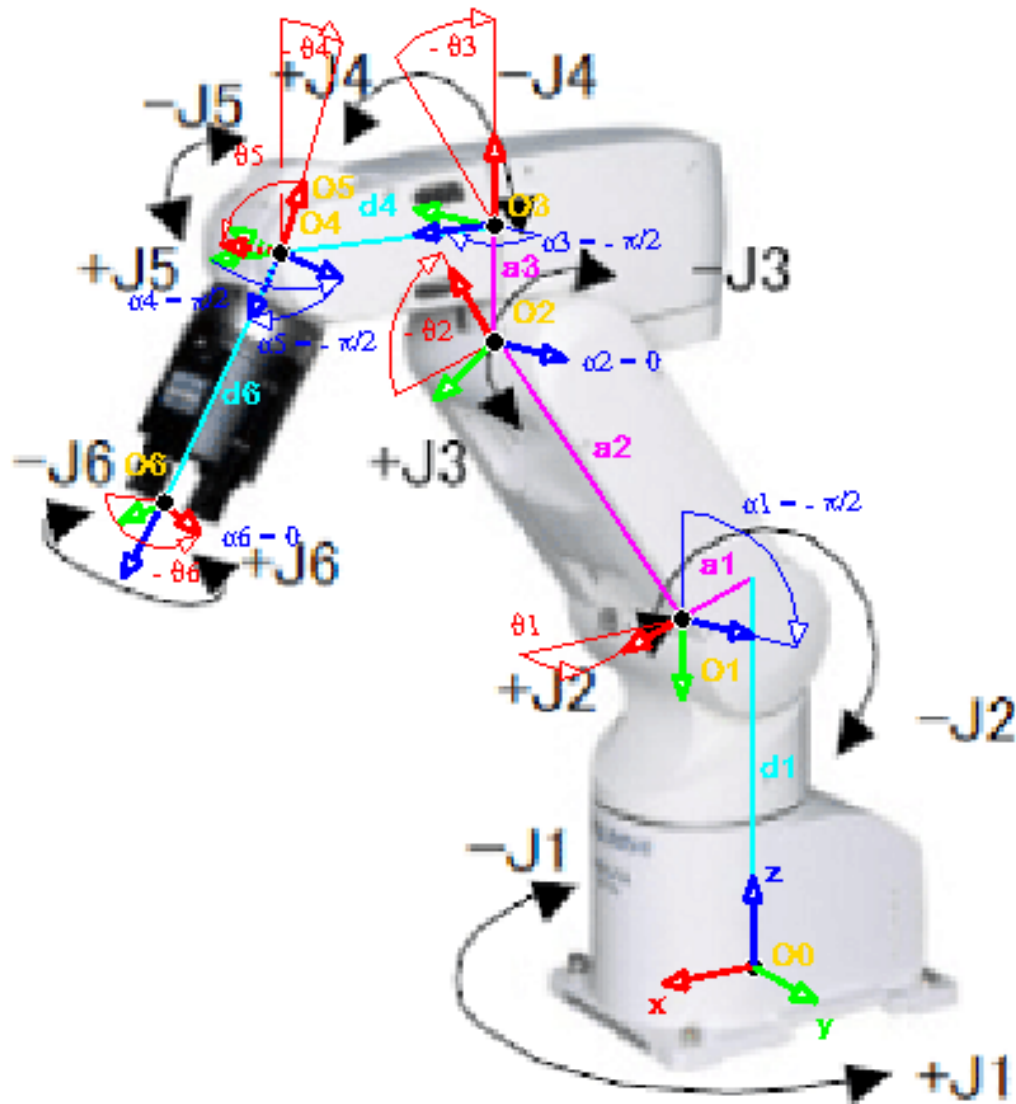


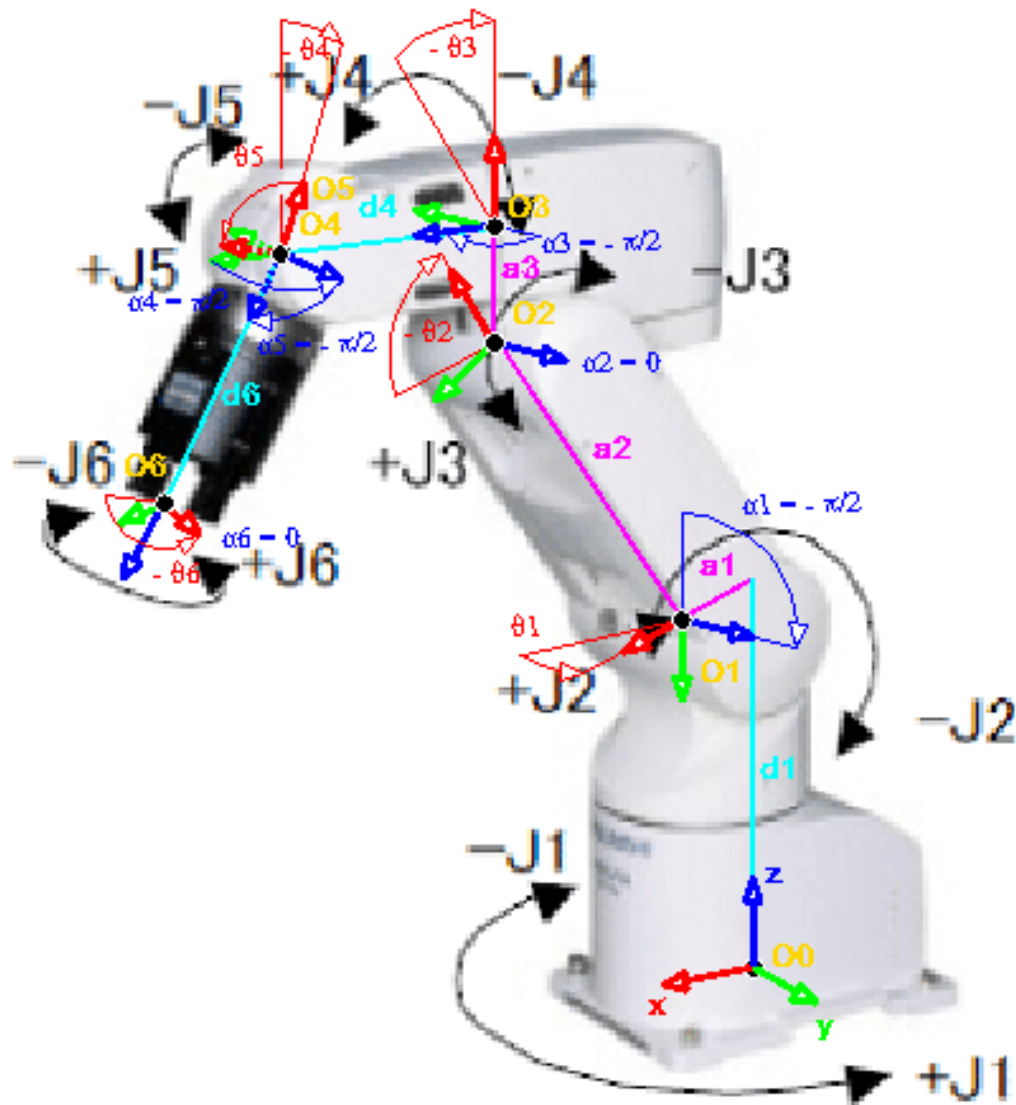
RV-6S/6SC

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X_{i-1} = A_i^{i-1} X_i$$



A_1^0


$$\begin{array}{c|c|c|c} \alpha_1 & a_1 & \theta_1 & d_1 \\ \hline -\frac{\pi}{2} & a_1 & \theta_1 & d_1 \end{array}$$

$$A_1^0 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 \cos \alpha_1 & \sin \theta_1 \sin \alpha_1 & a_1 \cos \theta_1 \\ \sin \theta_1 & \cos \theta_1 \cos \alpha_1 & -\cos \theta_1 \sin \alpha_1 & a_1 \sin \theta_1 \\ 0 & \sin \alpha_1 & \cos \alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

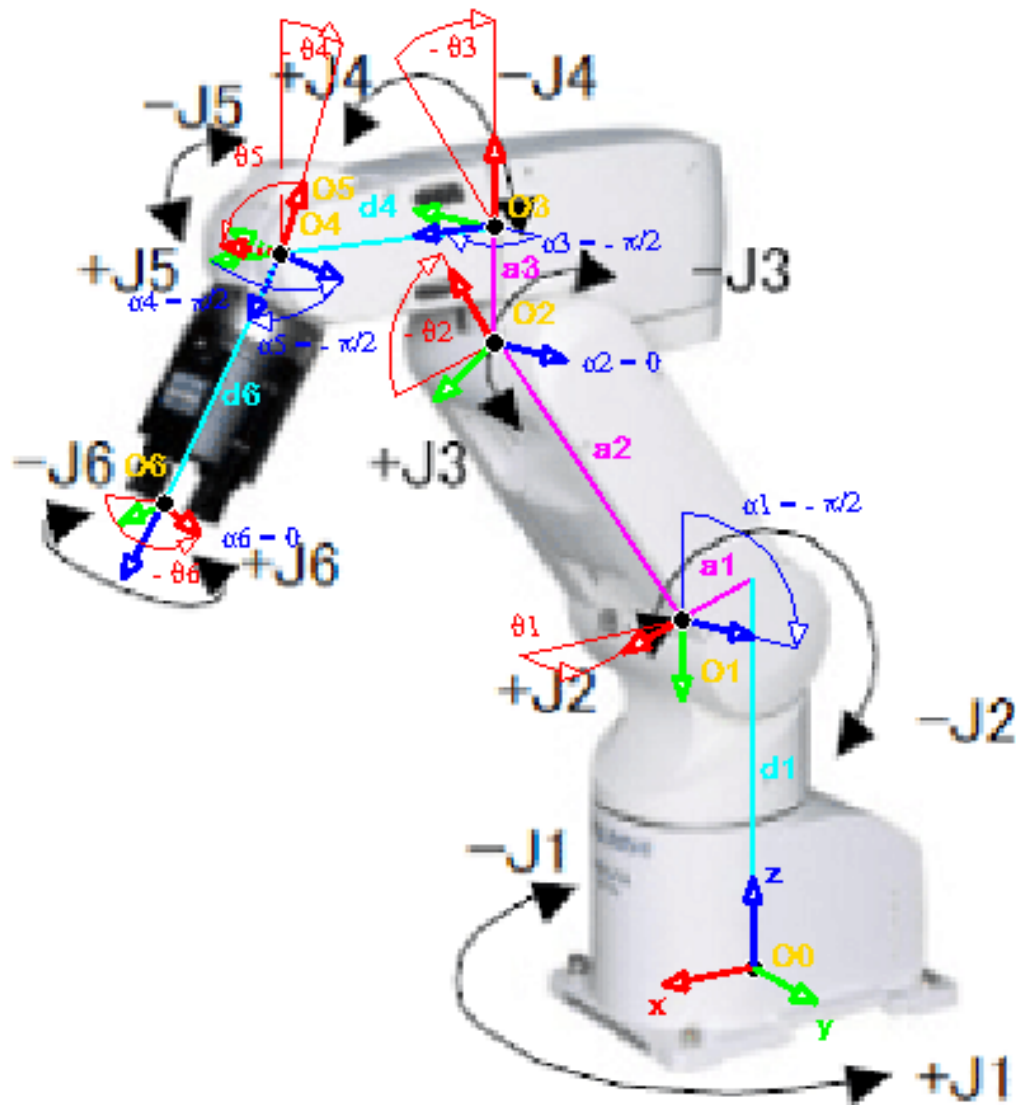
$$= \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 \cos -\pi/2 & \sin \theta_1 \sin -\pi/2 & a_1 \cos \theta_1 \\ \sin \theta_1 & \cos \theta_1 \cos -\pi/2 & -\cos \theta_1 \sin -\pi/2 & a_1 \sin \theta_1 \\ 0 & \sin -\pi/2 & \cos -\pi/2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

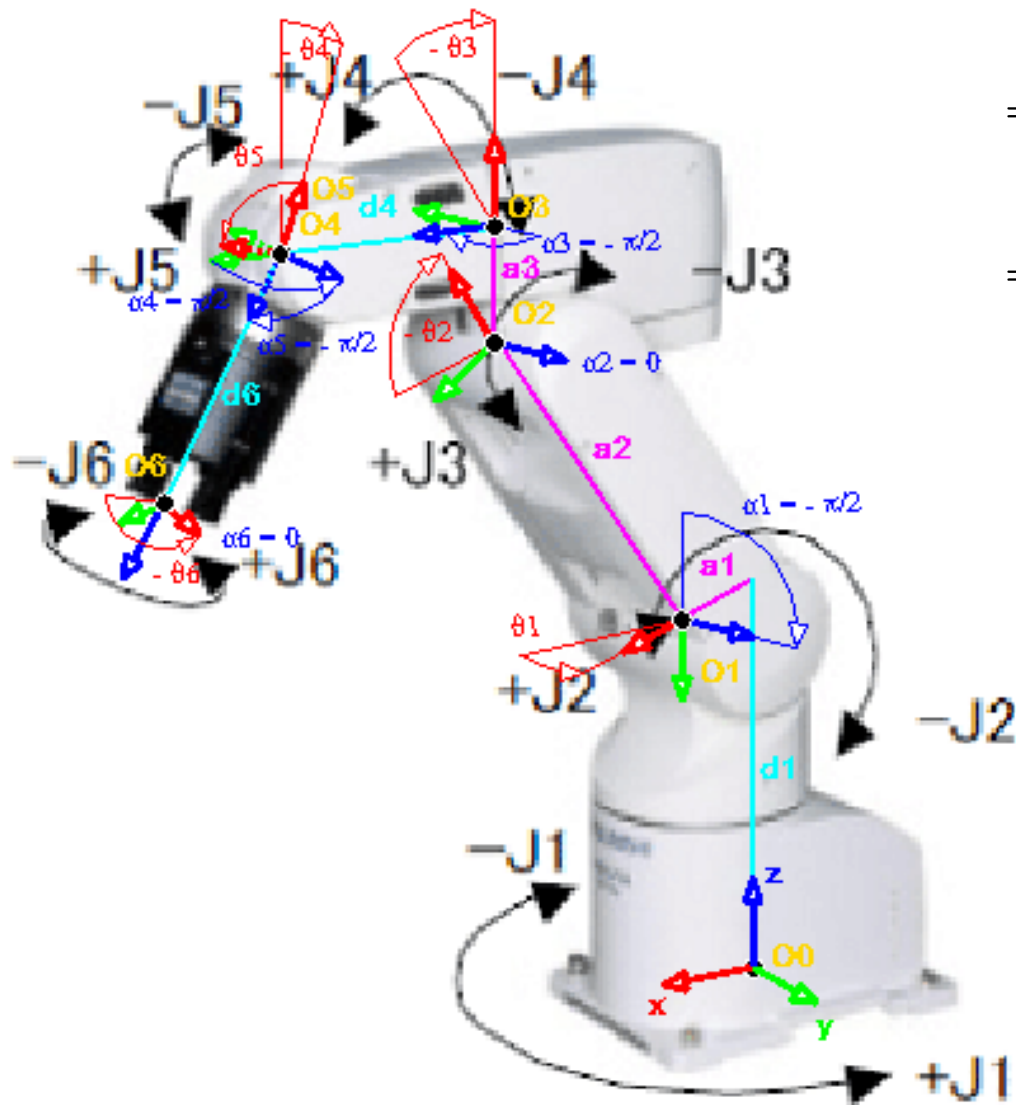
$$= \begin{bmatrix} \cos \theta_1 & 0 & -\sin \theta_1 & a_1 \cos \theta_1 \\ \sin \theta_1 & 0 & \cos \theta_1 & a_1 \sin \theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_2^1

α_2	a_2	θ_2	d_2
0	a_2	θ_2	0

$$\begin{aligned}
 A_2^1 &= \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 \cos \alpha_2 & \sin \theta_2 \sin \alpha_2 & a_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 \cos \alpha_2 & -\cos \theta_2 \sin \alpha_2 & a_2 \sin \theta_2 \\ 0 & \sin \alpha_2 & \cos \alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 \cos 0 & \sin \theta_2 \sin 0 & a_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 \cos 0 & -\cos \theta_2 \sin 0 & a_2 \sin \theta_2 \\ 0 & \sin 0 & \cos 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & a_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & a_2 \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$



A_3^2


$$\begin{array}{c|c|c|c} \alpha_3 & a_3 & \theta_3 & d_3 \\ \hline -\frac{\pi}{2} & a_3 & \theta_3 & 0 \end{array}$$

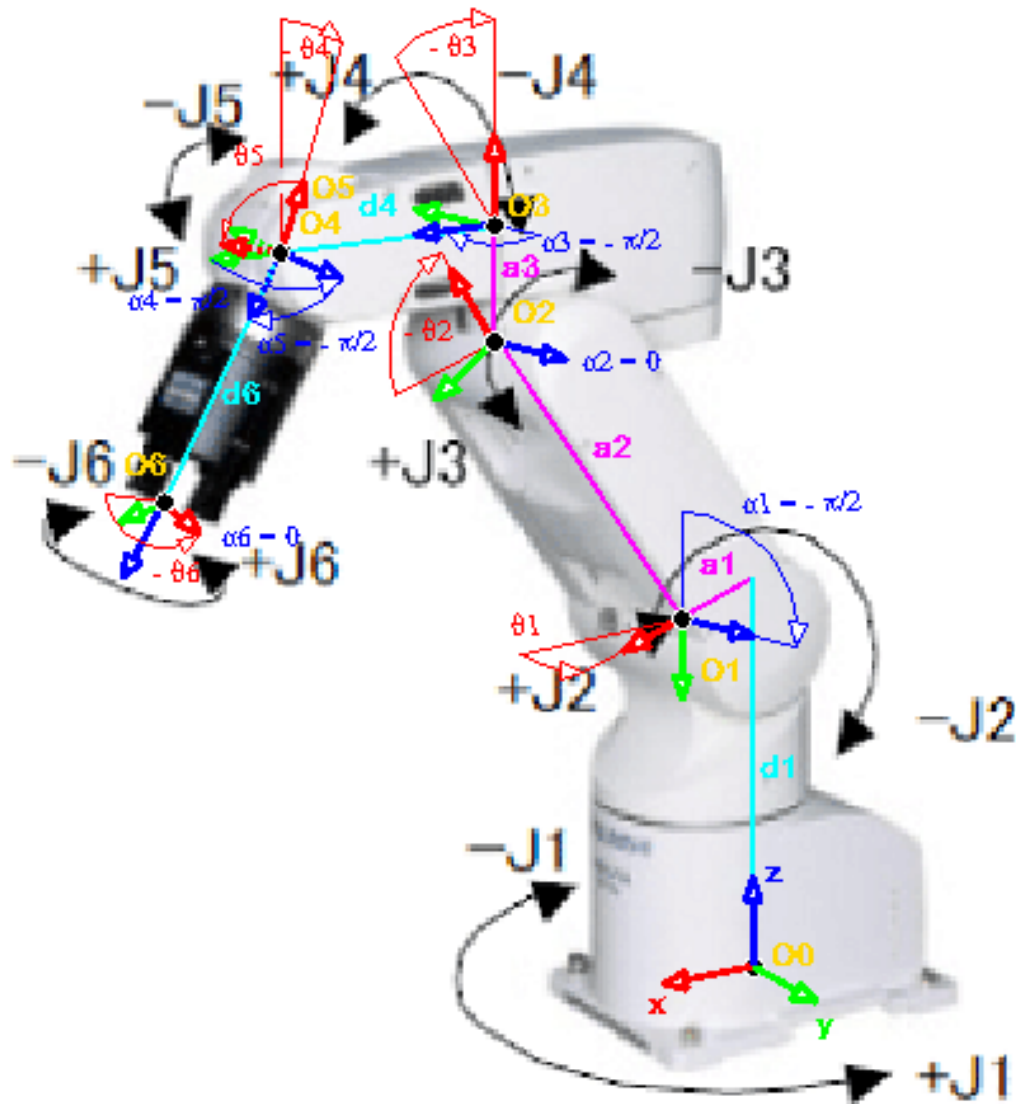
$$A_3^2 = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 \cos \alpha_3 & \sin \theta_3 \sin \alpha_3 & a_3 \cos \theta_3 \\ \sin \theta_3 & \cos \theta_3 \cos \alpha_3 & -\cos \theta_3 \sin \alpha_3 & a_3 \sin \theta_3 \\ 0 & \sin \alpha_3 & \cos \alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 \cos -\pi/2 & \sin \theta_3 \sin -\pi/2 & a_3 \cos \theta_3 \\ \sin \theta_3 & \cos \theta_3 \cos -\pi/2 & -\cos \theta_3 \sin -\pi/2 & a_3 \sin \theta_3 \\ 0 & \sin -\pi/2 & \cos -\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta_3 & 0 & -\sin \theta_3 & a_3 \cos \theta_3 \\ \sin \theta_3 & 0 & \cos \theta_3 & a_3 \sin \theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_4^3

α_4	a_4	θ_4	d_4
$\frac{\pi}{2}$	0	θ_4	d_4



$$A_4^3 = \begin{bmatrix} \cos \theta_4 & -\sin \theta_4 \cos \alpha_4 & \sin \theta_4 \sin \alpha_4 & a_4 \cos \theta_4 \\ \sin \theta_4 & \cos \theta_4 \cos \alpha_4 & -\cos \theta_4 \sin \alpha_4 & a_4 \sin \theta_4 \\ 0 & \sin \alpha_4 & \cos \alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

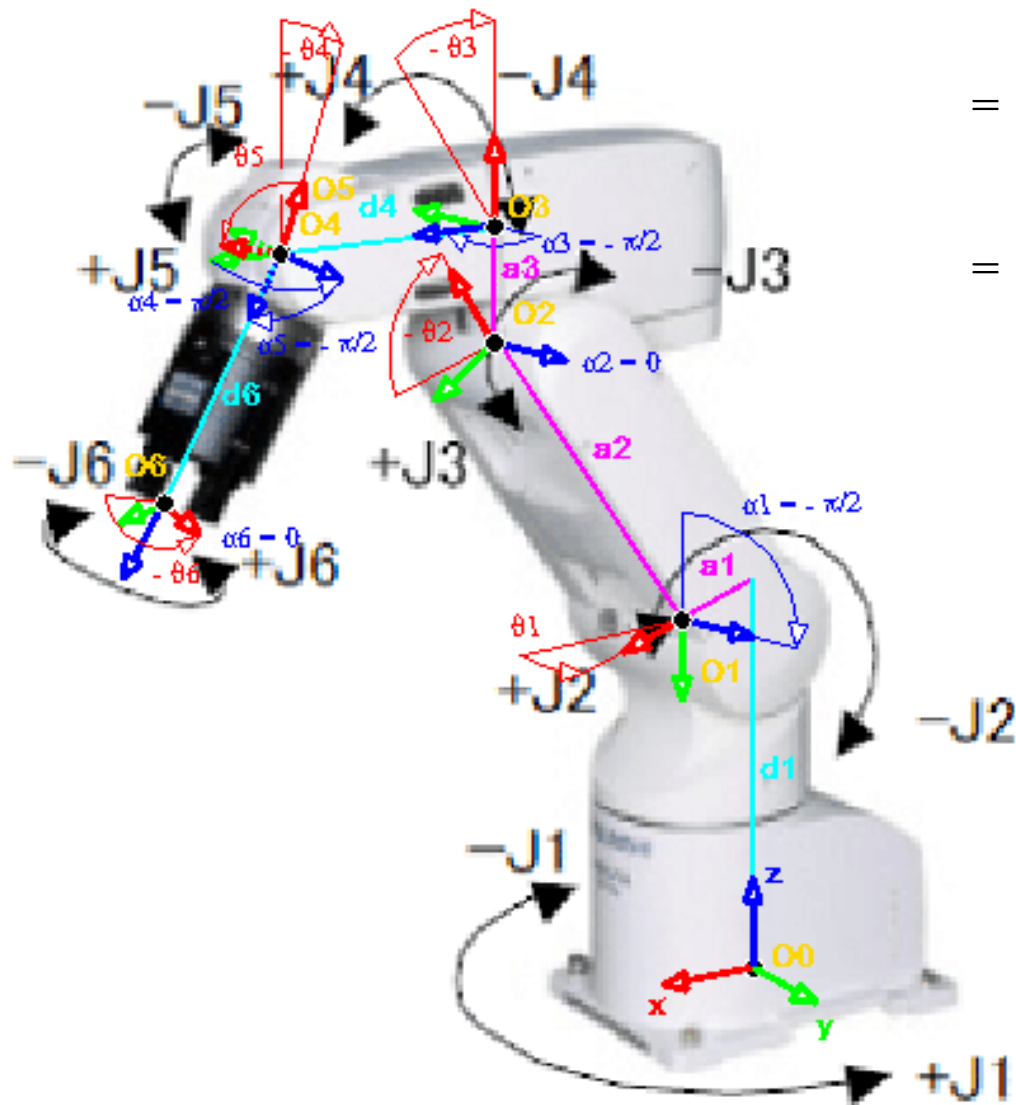
$$= \begin{bmatrix} \cos \theta_4 & -\sin \theta_4 \cos \pi/2 & \sin \theta_4 \sin \pi/2 & a_4 \cos \theta_4 \\ \sin \theta_4 & \cos \theta_4 \cos \pi/2 & -\cos \theta_4 \sin \pi/2 & a_4 \sin \theta_4 \\ 0 & \sin \pi/2 & \cos \pi/2 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta_4 & 0 & \sin \theta_4 & 0 \\ \sin \theta_4 & 0 & -\cos \theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_5^4

α_5	a_5	θ_5	d_5
$-\frac{\pi}{2}$	0	θ_5	0

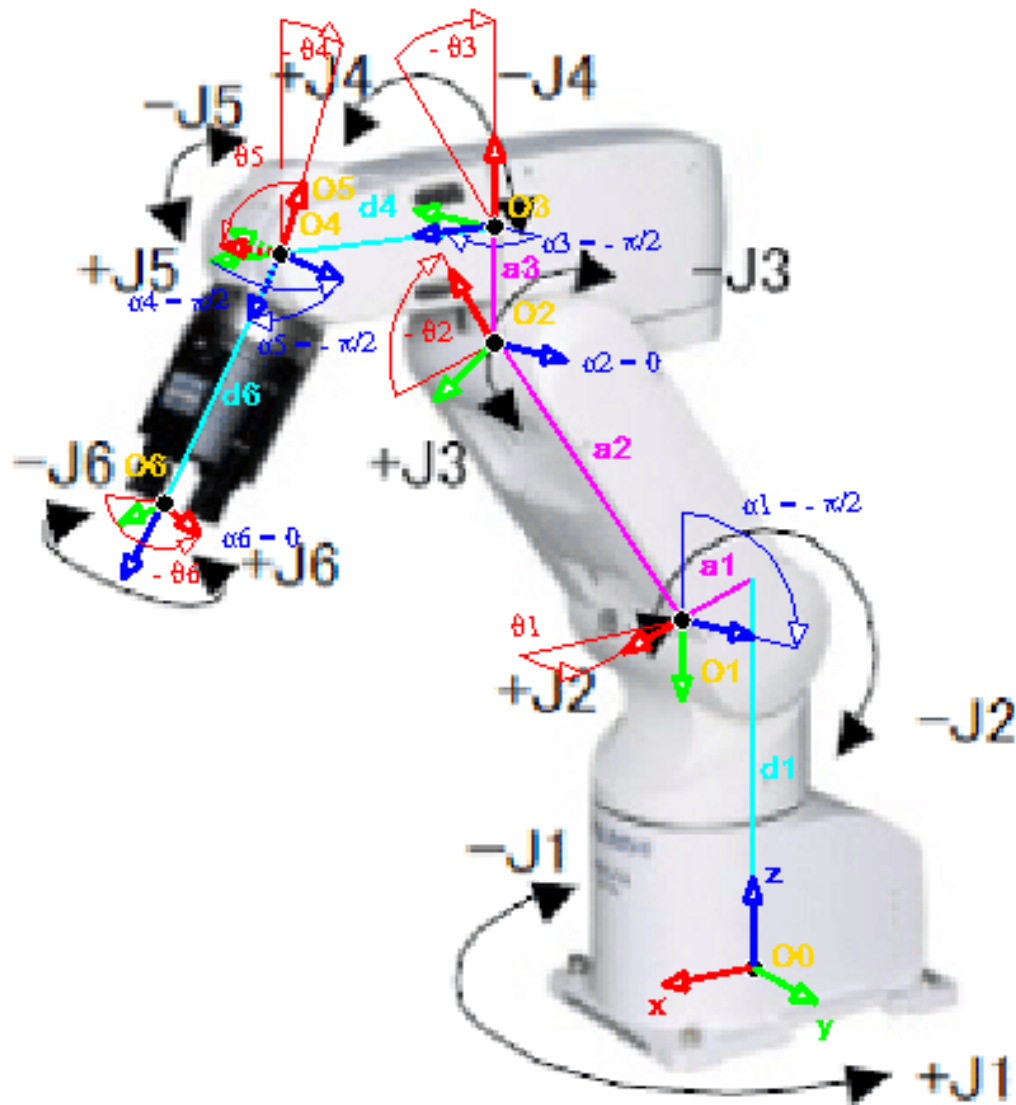
$$\begin{aligned}
 A_5^4 &= \begin{bmatrix} \cos \theta_5 & -\sin \theta_5 \cos \alpha_5 & \sin \theta_5 \sin \alpha_5 & a_5 \cos \alpha_5 \\ \sin \theta_5 & \cos \theta_5 \cos \alpha_5 & -\cos \theta_5 \sin \alpha_5 & a_5 \sin \alpha_5 \\ 0 & \sin \alpha_5 & \cos \alpha_5 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta_5 & -\sin \theta_5 \cos -\pi/2 & \sin \theta_5 \sin -\pi/2 & 0 \cos -\pi/2 \\ \sin \theta_5 & \cos \theta_5 \cos -\pi/2 & -\cos \theta_5 \sin -\pi/2 & 0 \sin -\pi/2 \\ 0 & \sin -\pi/2 & \cos -\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta_5 & 0 & -\sin \theta_5 & 0 \\ \sin \theta_5 & 0 & \cos \theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$



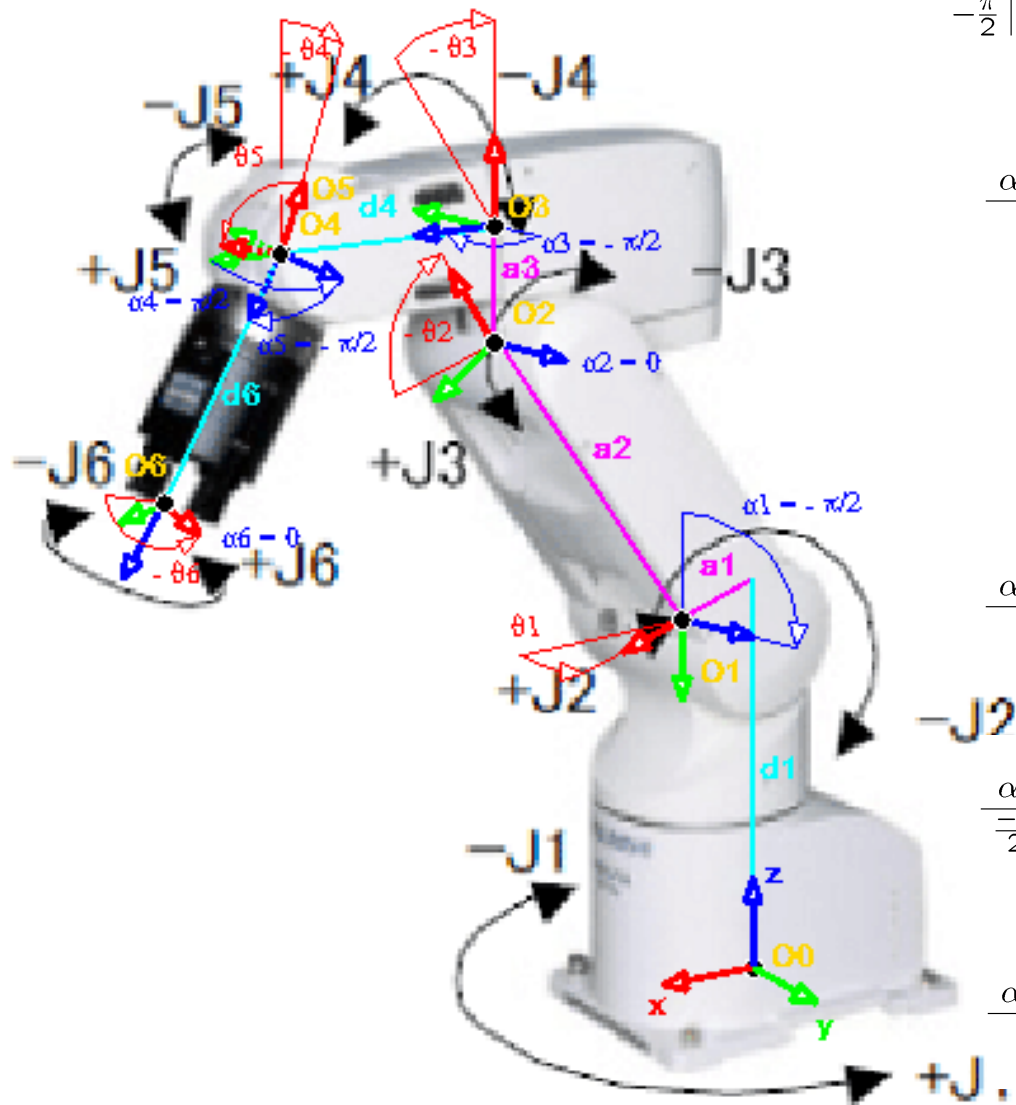
A_6^5

α_6	a_6	θ_6	d_6
0	0	θ_6	d_6

$$\begin{aligned}
 A_6^5 &= \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 \cos \alpha_6 & \sin \theta_6 \sin \alpha_6 & a_6 \cos \alpha_6 \\ \sin \theta_6 & \cos \theta_6 \cos \alpha_6 & -\cos \theta_6 \sin \alpha_6 & a_6 \sin \alpha_6 \\ 0 & \sin \alpha_6 & \cos \alpha_6 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 \cos 0 & \sin \theta_6 \sin 0 & 0 \cos 0 \\ \sin \theta_6 & \cos \theta_6 \cos 0 & -\cos \theta_6 \sin 0 & 0 \sin 0 \\ 0 & \sin 0 & \cos 0 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 & 0 & 0 \\ \sin \theta_6 & \cos \theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$



$$G = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$



$$\begin{array}{c|c|c|c} \alpha_1 & a_1 & \theta_1 & d_1 \\ \hline -\frac{\pi}{2} & a_1 & \theta_1 & d_1 \end{array} \quad A_1^0 = \begin{bmatrix} \cos \theta_1 & 0 & -\sin \theta_1 & a_1 \cos \theta_1 \\ \sin \theta_1 & 0 & \cos \theta_1 & a_1 \sin \theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c|c|c|c} \alpha_2 & a_2 & \theta_2 & d_2 \\ \hline 0 & a_2 & \theta_2 & 0 \end{array} \quad A_2^1 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & a_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & a_2 \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

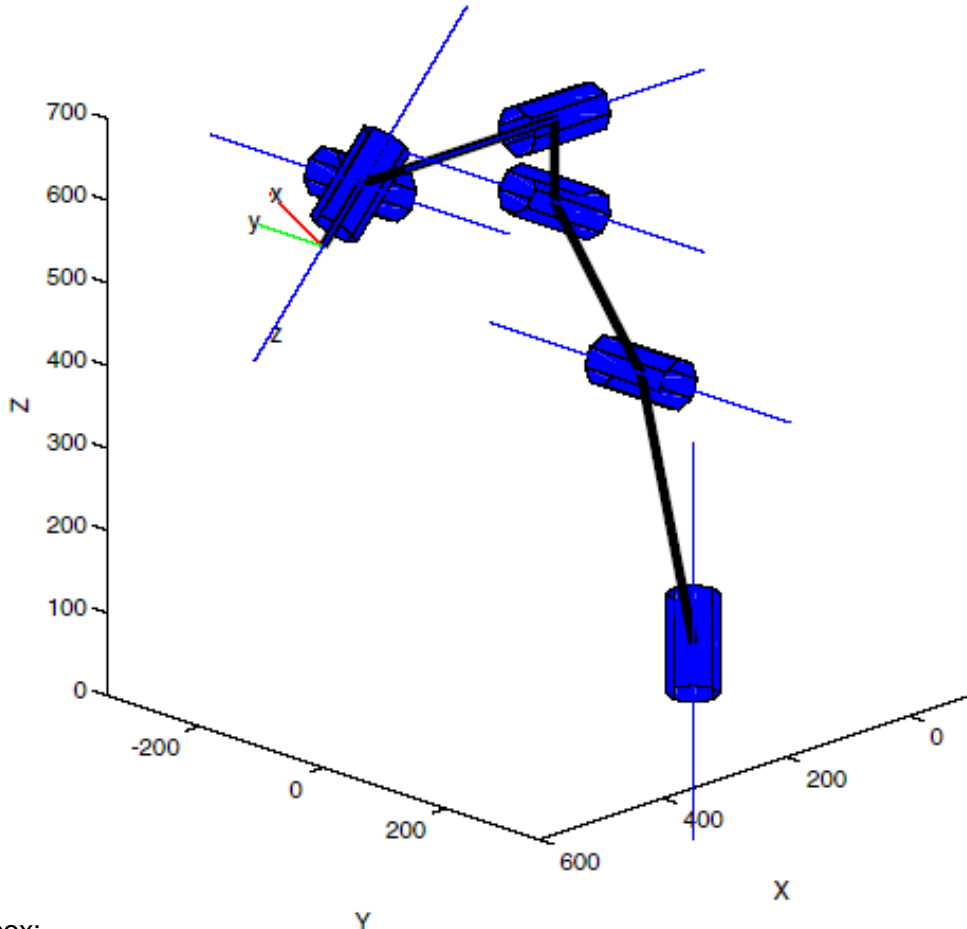
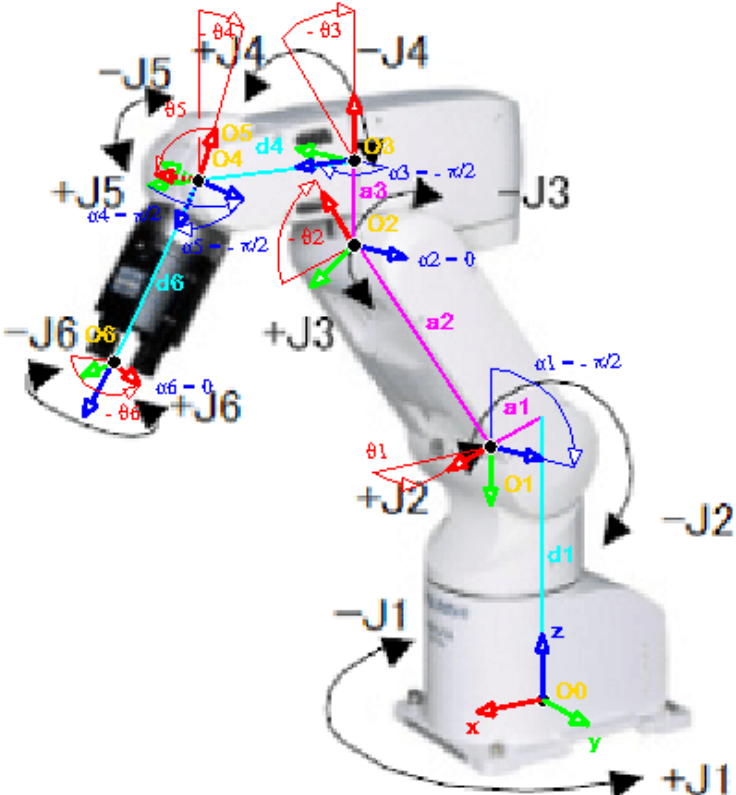
$$\begin{array}{c|c|c|c} \alpha_3 & a_3 & \theta_3 & d_3 \\ \hline -\frac{\pi}{2} & a_3 & \theta_3 & 0 \end{array} \quad A_3^2 = \begin{bmatrix} \cos \theta_3 & 0 & -\sin \theta_3 & a_3 \cos \theta_3 \\ \sin \theta_3 & 0 & \cos \theta_3 & a_3 \sin \theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c|c|c|c} \alpha_4 & a_4 & \theta_4 & d_4 \\ \hline \frac{\pi}{2} & 0 & \theta_4 & d_4 \end{array} \quad A_4^3 = \begin{bmatrix} \cos \theta_4 & 0 & \sin \theta_4 & 0 \\ \sin \theta_4 & 0 & -\cos \theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c|c|c|c} \alpha_5 & a_5 & \theta_5 & d_5 \\ \hline -\frac{\pi}{2} & 0 & \theta_5 & 0 \end{array} \quad A_5^4 = \begin{bmatrix} \cos \theta_5 & 0 & -\sin \theta_5 & 0 \\ \sin \theta_5 & 0 & \cos \theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c|c|c|c} \alpha_6 & a_6 & \theta_6 & d_6 \\ \hline 0 & 0 & \theta_6 & d_6 \end{array} \quad A_6^5 = \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 & 0 & 0 \\ \sin \theta_6 & \cos \theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Robot Description — Matlab Robotic Toolbox



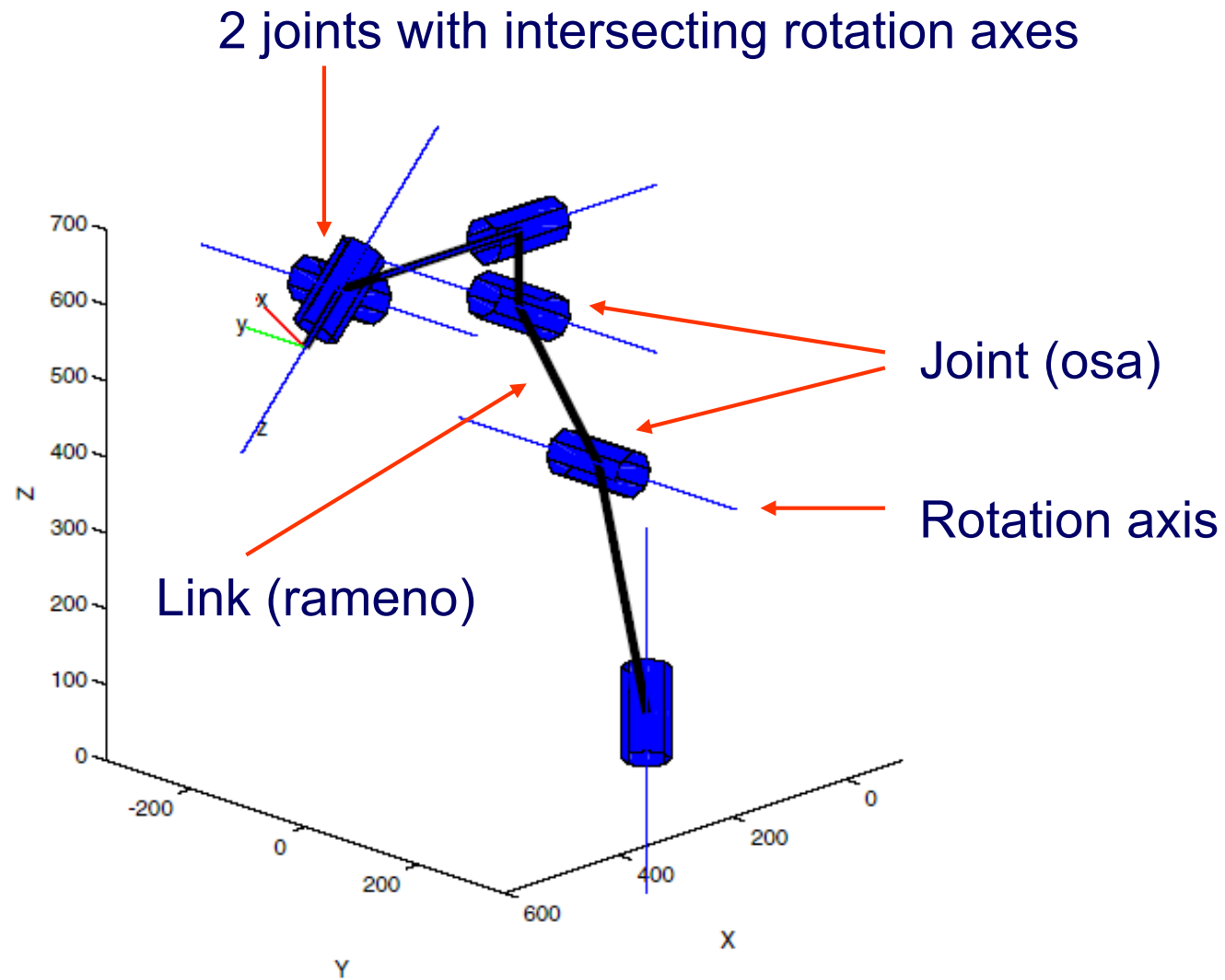
MATLAB simulation in ROBOT toolbox:

mRV6S =

RV-6S (6 axis, RRRRRR) [Mitsubishi] <home = [0.000000 -1.047198 -0.523599 0.000000 0.785398 0.000000]>
 grav = [0.00 0.00 9.81]

alpha_i	a_i	theta_i	d_i	R/P	standard D&H parameters
-1.570796	85.000	parameter	350.000	R	(std)
0.000000	280.000	parameter	0.000	R	(std)
-1.570796	100.000	parameter	0.000	R	(std)
1.570796	0.000	parameter	315.000	R	(std)
-1.570796	0.000	parameter	0.000	R	(std)
0.000000	0.000	parameter	85.000	R	(std)

Robot Description — Matlab Robotic Toolbox



Robot Description — Drawings

