GRAPHICAL MARKOV MODELS (WS2017)
3. SEMINAR

Assignment 1. According to Definition 1b of Sec. 1 of the lecture, any Markov chain model can be specified in the form

$$p(s) = \frac{1}{Z} \prod_{i=2}^{n} g_i(s_{i-1}, s_i)$$

with arbitrary functions $g_i \colon K^2 \to \mathbb{R}_+$ and the normalisation constant Z. Find an algorithm for computing the pairwise marginal probabilities $p(s_{i-1} = k, s_i = k')$ for all $k, k' \in K$ and all $i = 2, \ldots, n$ from the given functions g_i, $i = 2, \ldots, n$.

Assignment 2.* Suppose that a regular language $\mathcal{L}$ of strings over the finite alphabet Σ is described by a non-deterministic finite-state machine. Given a string $y \notin \mathcal{L}$, the task is to find the string $x \in \mathcal{L}$ with smallest Hamming distance to y, i.e.

$$x^* = \arg\min_{x \in \mathcal{L}} d_h(x, y),$$

where d_h denotes the Hamming distance. Construct an efficient algorithm for solving this task.

Assignment 3. Let $\boldsymbol{x}$ be a grey value image of size $n \times m$, where x_{ij} denotes the grey value of the pixel with coordinates (i, j). The task is to segment such images into an upper and lower part by a boundary represented as a sequence of height values $s_j \in \{1, 2, \ldots, n\}$ for all $j = 1, 2, \ldots, m$.

The prior probability for boundaries is assumed to be a homogeneous Markov chain such that $p(s_j \mid s_{j-1}) = 0$ if $|s_j - s_{j-1}| > 1$. The appearance model for columns $\boldsymbol{x}_j$ given the boundary value s_j, is assumed to be conditional independent

$$p(\boldsymbol{x}_j \mid s_j) = \prod_{i \leqslant s_j} p_1(x_{ij}) \cdot \prod_{i > s_j} p_2(x_{ij}),$$

where $p_1()$ and $p_2()$ are two distributions for grey values.

a) Deduce an efficient algorithm for determining the most probable boundary.

b) Suppose that the loss function $\ell(s, s')$ for incorrectly recognised boundaries is defined by

$$\ell(s, s') = \sum_{j=1}^{m} (s_j - s'_j)^2.$$

Formulate the segmentation task for this case. Deduce an efficient inference algorithm.

c*) When applying the inference rule derived in b), it may happen that the inferred boundary has zero probability in the model. Propose an augmented loss function which prevents this and derive the corresponding inference algorithm.