Additional Excercises

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Bayesian A (dfficulty 1)

A patient takes a lab test and the result comes back positive. It is known that the test returns a correct positive result in only 98% of the cases ($p(+|\text{cancer}) = 0.98$) and a correct negative result in only 97% of the cases ($p(-|\text{no cancer}) = 0.97$). Furthermore, only 0.001 of the entire population has this disease.

- What is the probability that this patient has cancer?
- What is the probability that he does not have cancer?
- What is the diagnosis?

Bayesian C (dfficulty 1)

Assume you calculated the posterior probabilities of the state $k \in \{1, \dots 4\}$ as $p_{K|X}(\cdot \mid x) = 0.4, 0.2, 0.2, 0.2$, respectively. The task is to decide whether $k = 1$. What is the optimal Bayesian decision in the following cases (explain):

- if the penalty for the wrong decision is constant;
- if mistakenly deciding $k = 1$ costs twice more than mistakenly deciding that $k \neq 1$.

Bayesian D (dfficulty 2)

A digital signal transmitting system reads 3 binary digits and for ith digit outputs the probability that the digit is 1, the resulting numbers are 0.3, 0.4, 0.7. It is know that the true digits form an error correcting code where the last digit is always the sum (modulo 2) of the first two digits.

- Recognize which number is encoded by the first two digits.
- Decide whether this packet of 3 digits has to be requested again considering that the cost of skipping an error is 100 more than requesting to repeated the packet.

Bayesian D (dfficulty 4)

A student prepares to the exam. There are K tickets in total, one for each lecture. Because the lectures are sequential, he prepares sequentially. He learns the first ticket with probability q. If he already learned k tickets, he learns the next one again with probability q or otherwise stops preparing.

At the exam a ticket is drawn randomly. Assume the student answered well the ticket with number x. The task is to recognize whether he has prepared at least half of the tickets (assume K is even). Model the problem as a Bayesian decision:

1. in this problem what is the hidden state, observation, decision?
2. What is the probability that he learned at least half of the tickets?
3. Derive the optimal Bayesian decision strategy.