Motion Planning for Autonomous Vehicles

Michal Cap

Autonomous Car Architecture

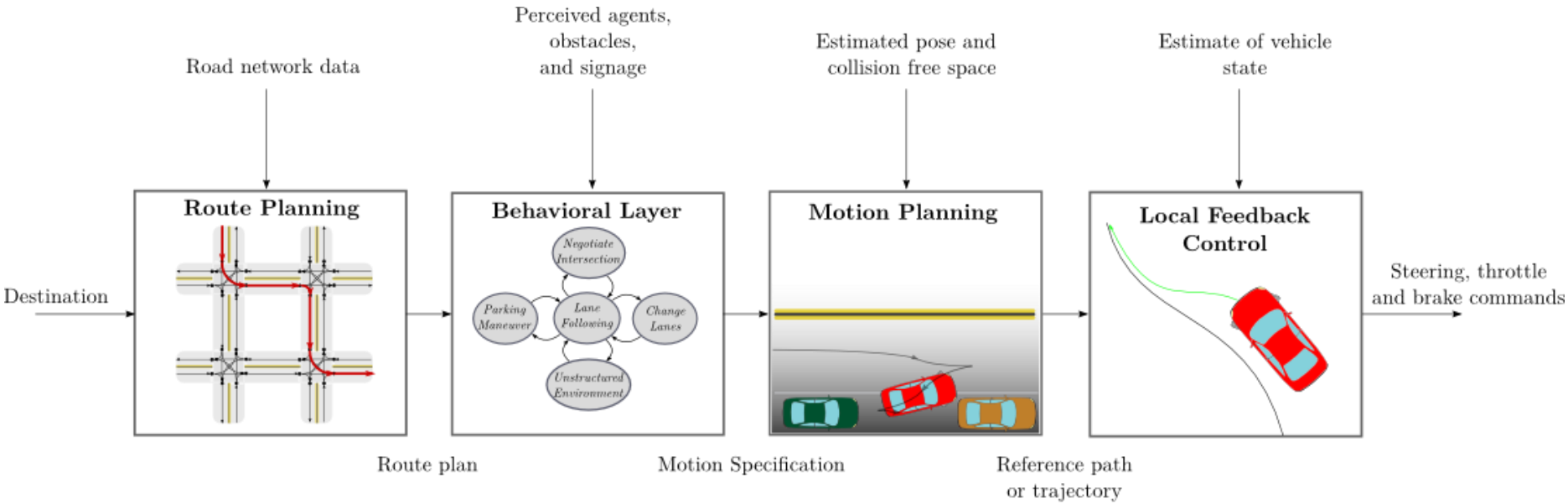
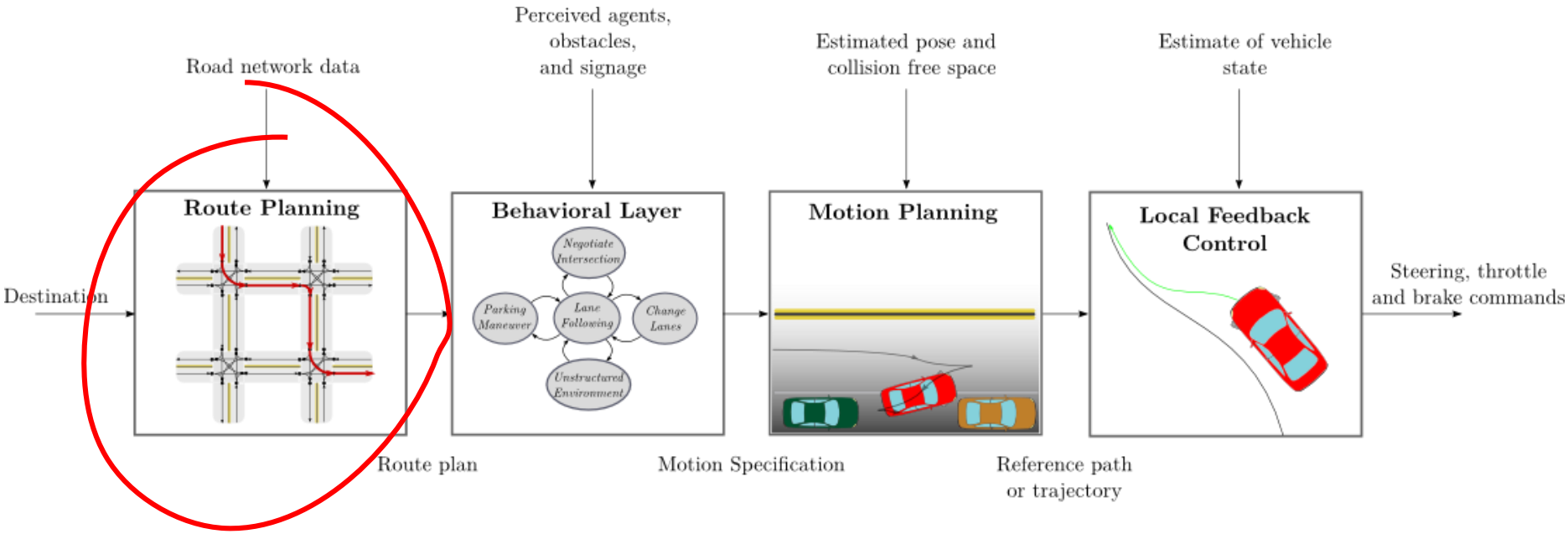


Figure from Brian Paden

Autonomous Car Architecture



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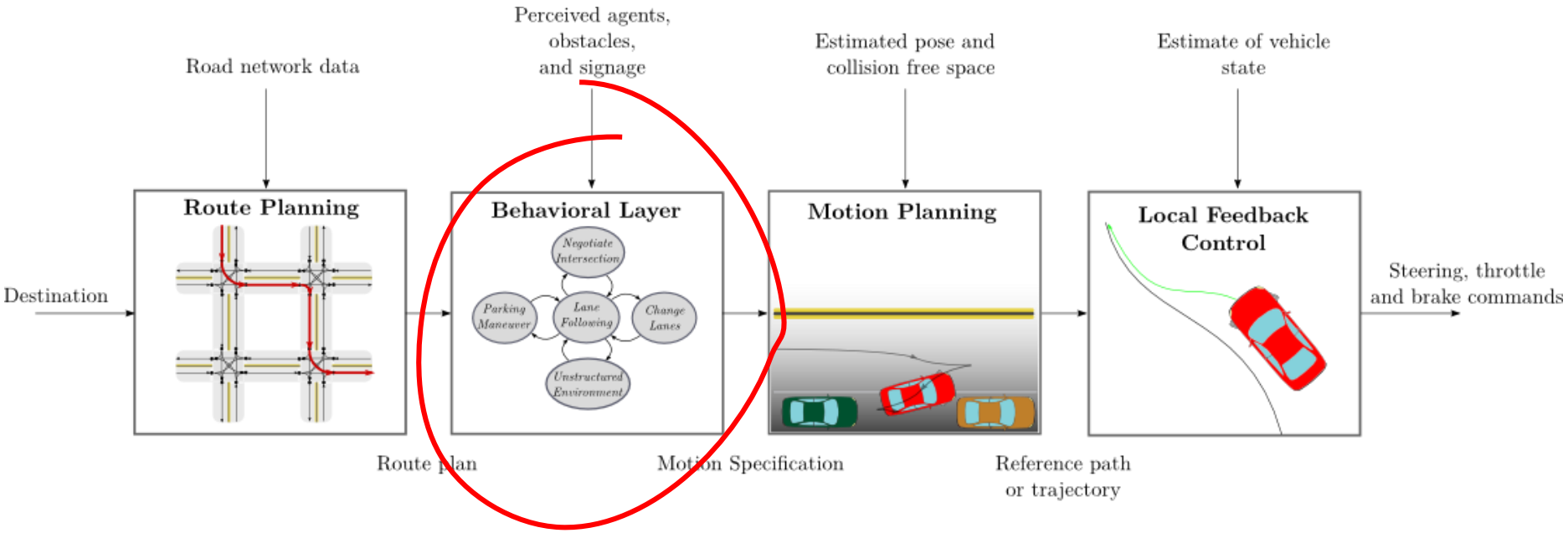

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Autonomous Car Architecture

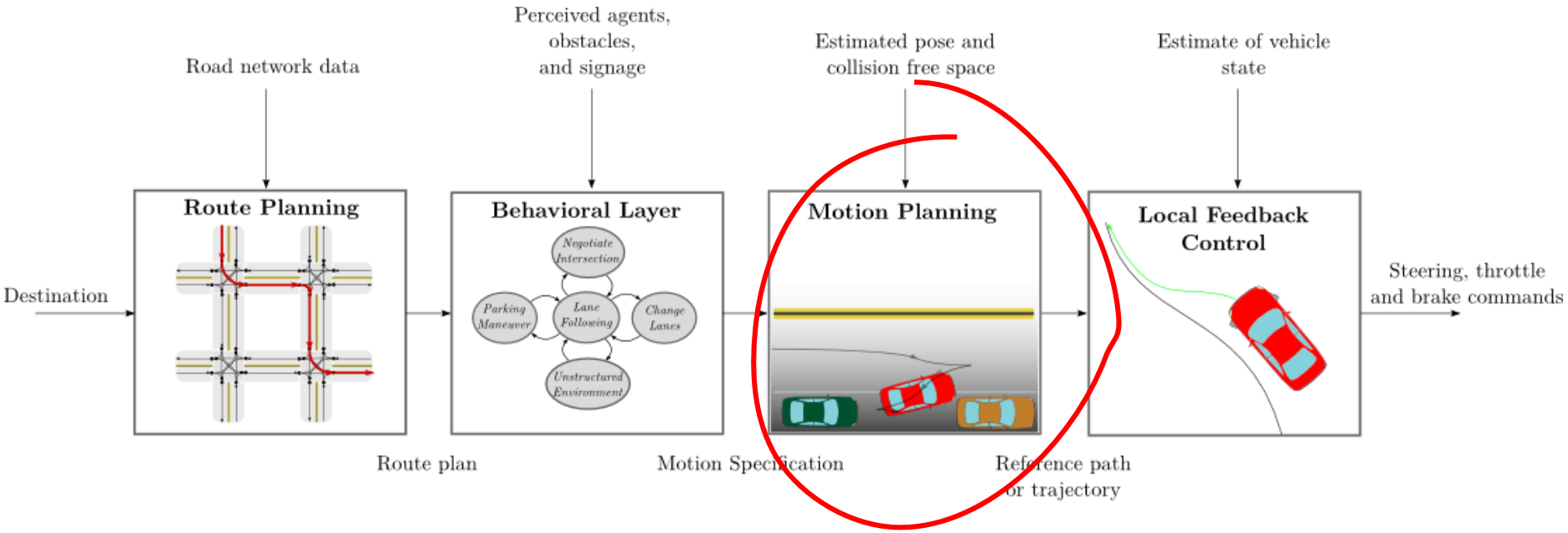


Figure from Brian Paden

Autonomous Car Architecture

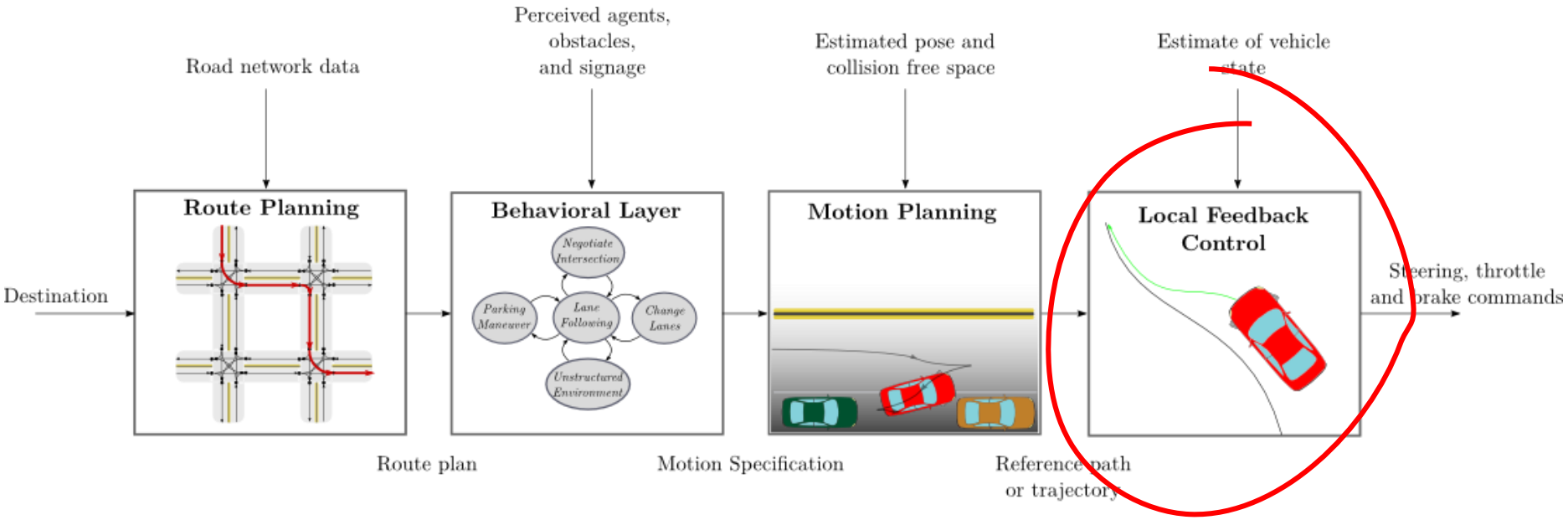


Figure from Brian Paden

Motivation

Motivation

Driving in highly structured environments;

Video by Ricco831

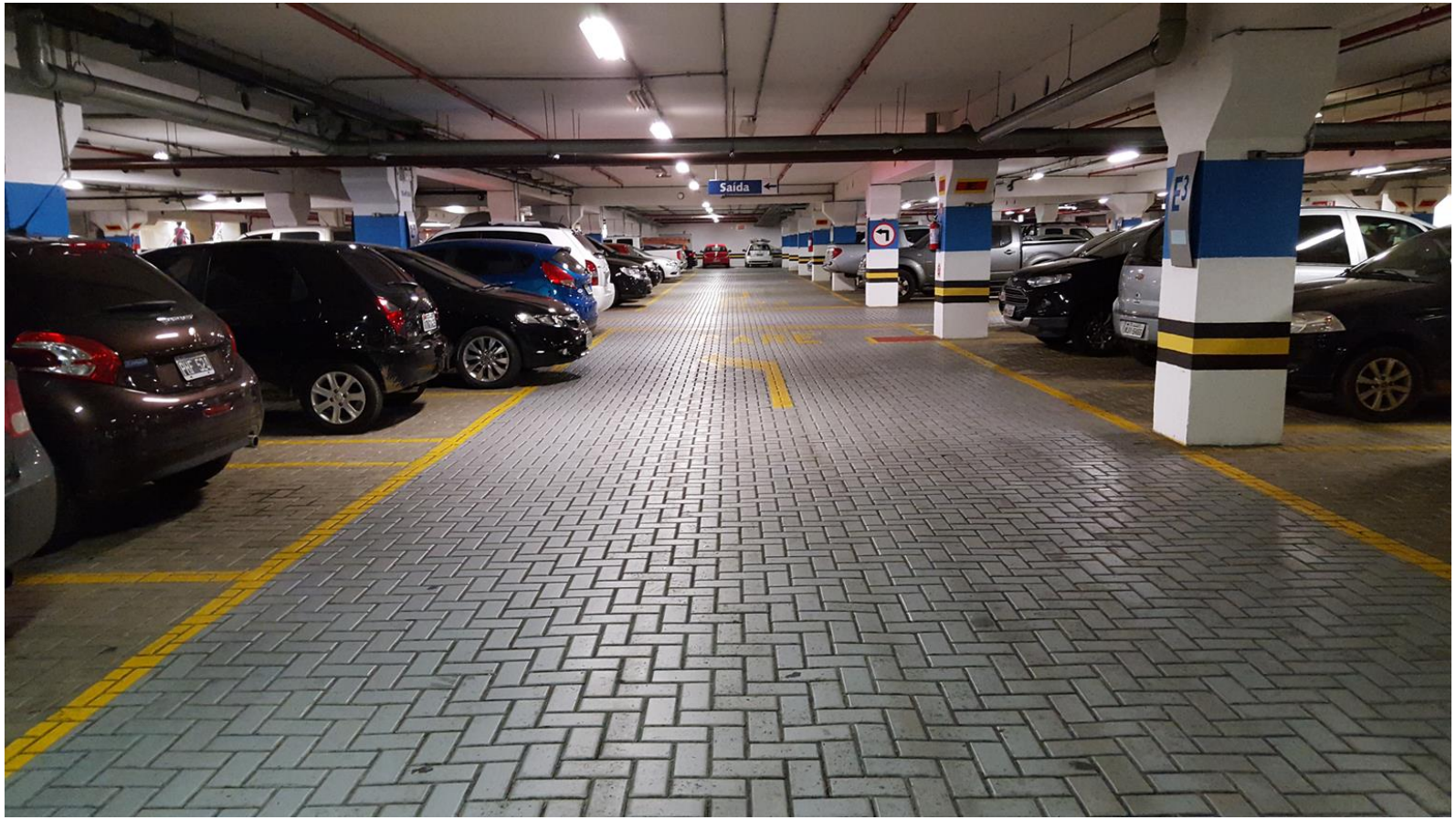
Motivation

Driving in urban environments:

Video by Mobileye

Motivation

Driving in unstructured cluttered environments:



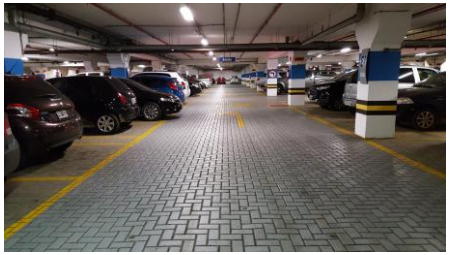
Picture by Rachmaninoff

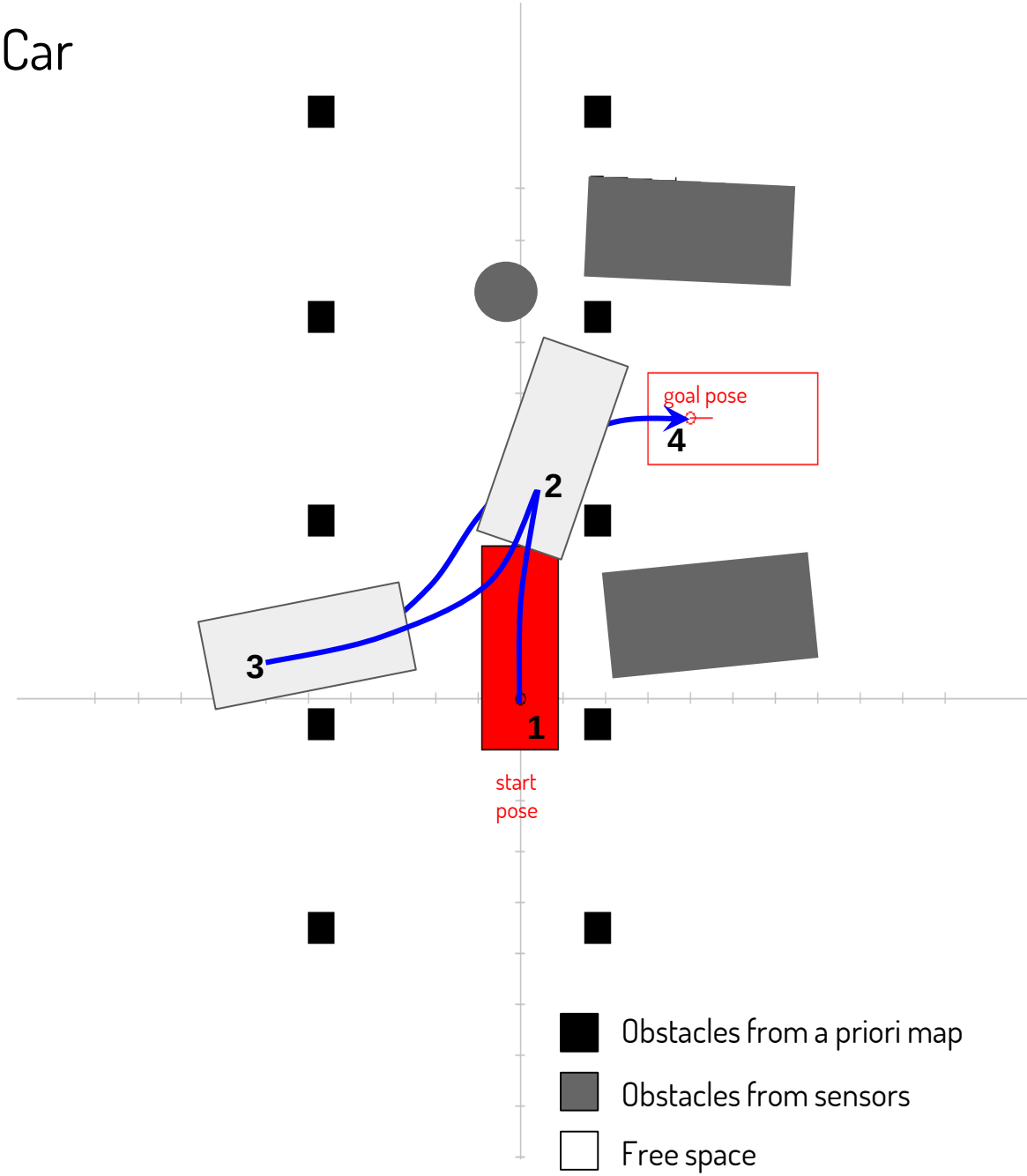
Problem Informally

Motion Planning:

The problem of finding a collision-free motion for a robot from given start pose to given destination pose.

Planning for a Conventional Car





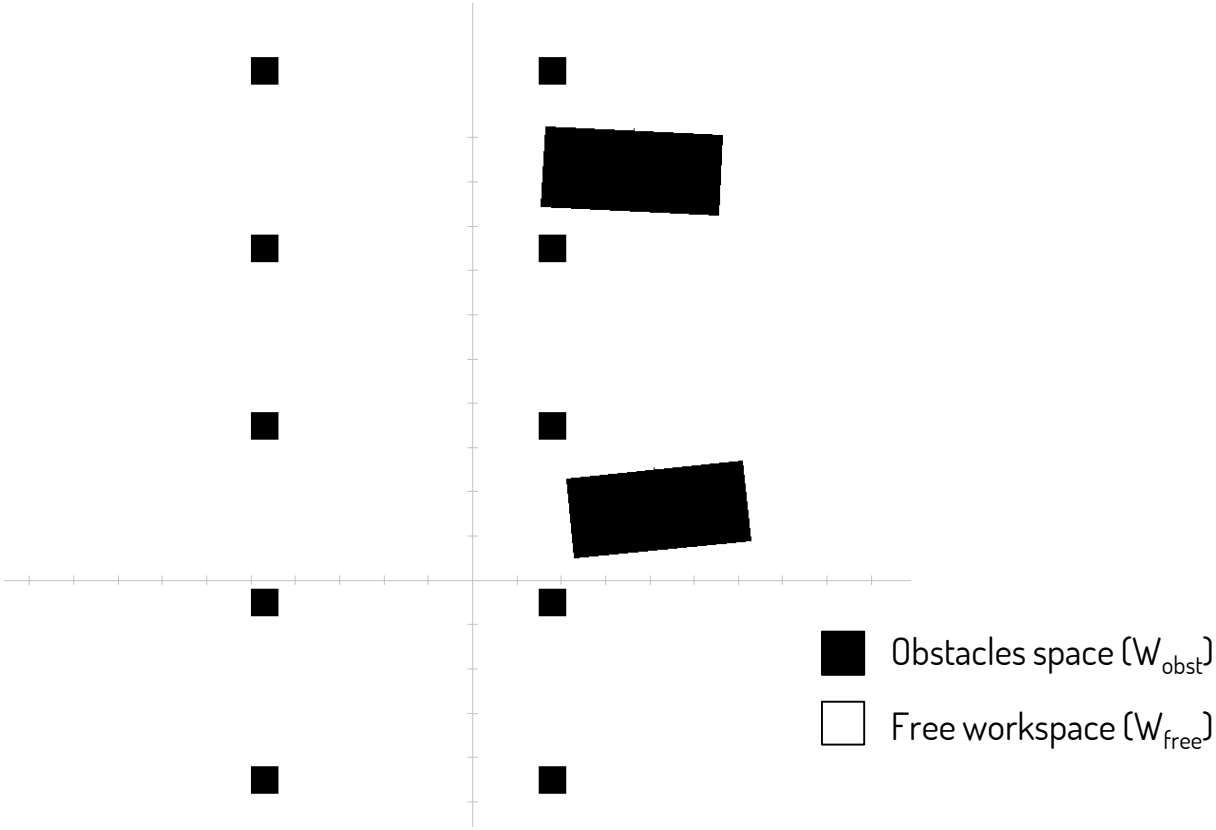
Constraints on path:

- Starts at current position
- Ends at goal position
- Robot does not collide with obstacles
- Respect limited turning-radius

Formalization

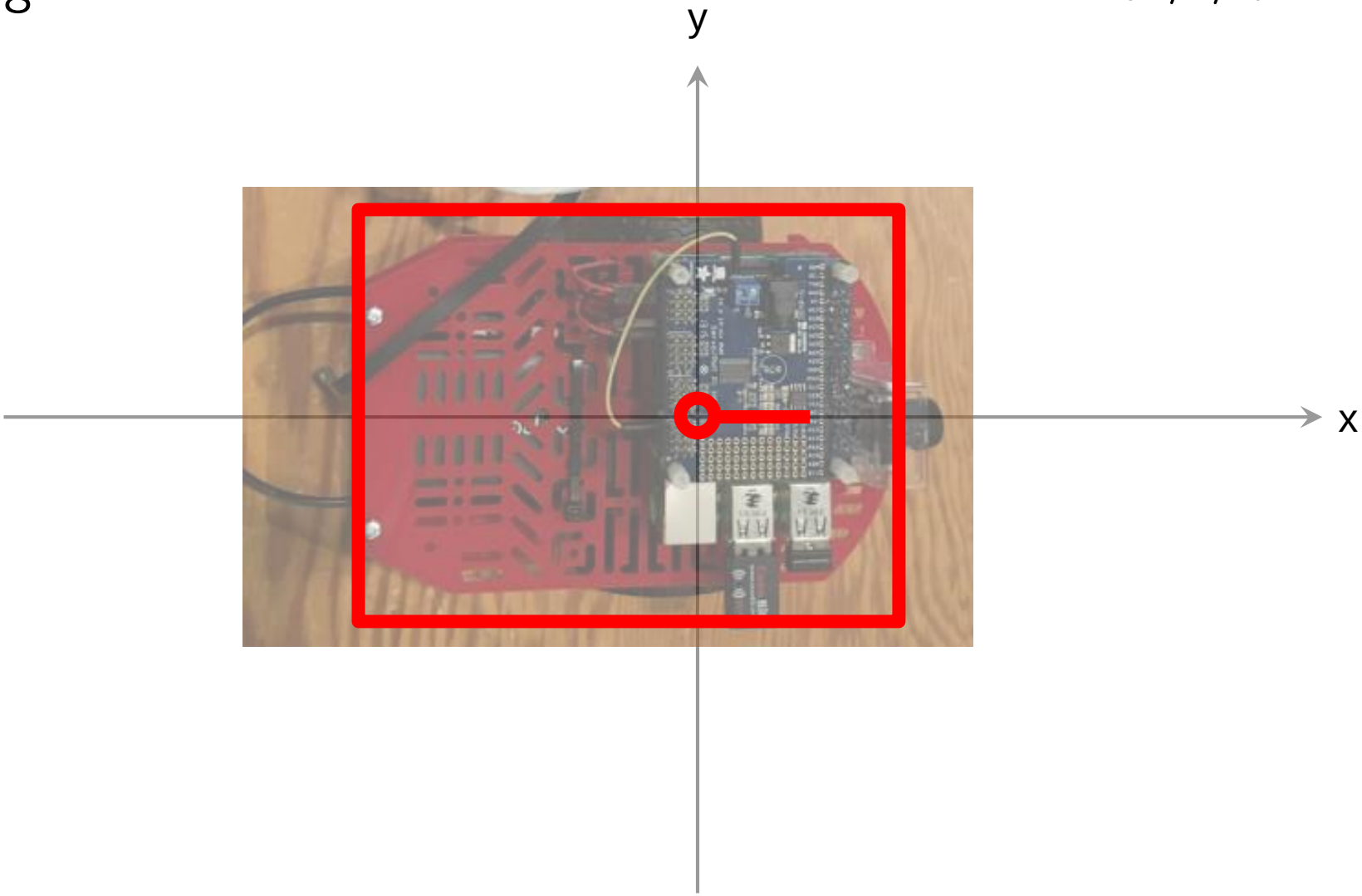
Workspace: (x,y)

$W \subset R^2$



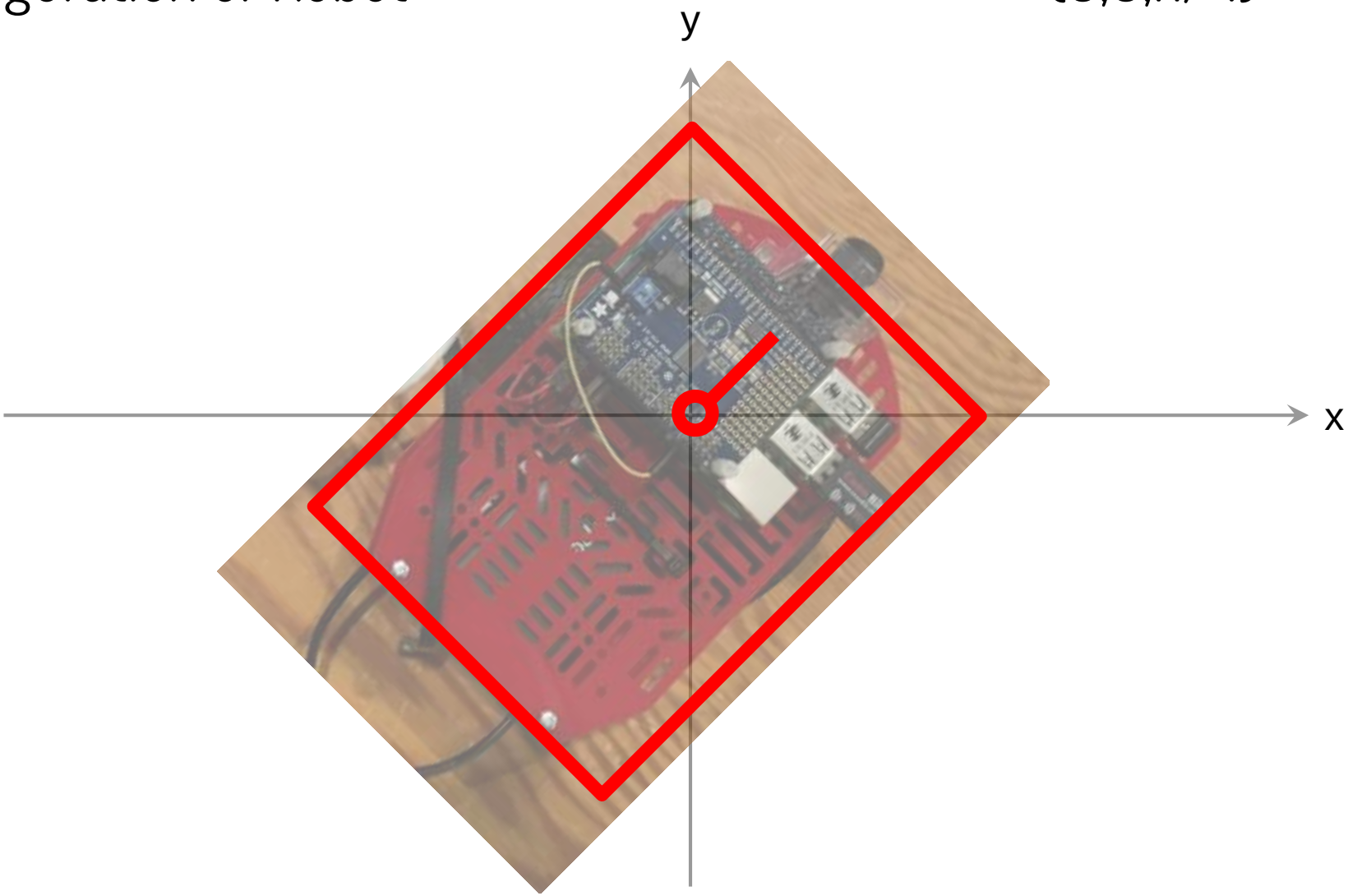
Configuration of Robot

(0,0,0)



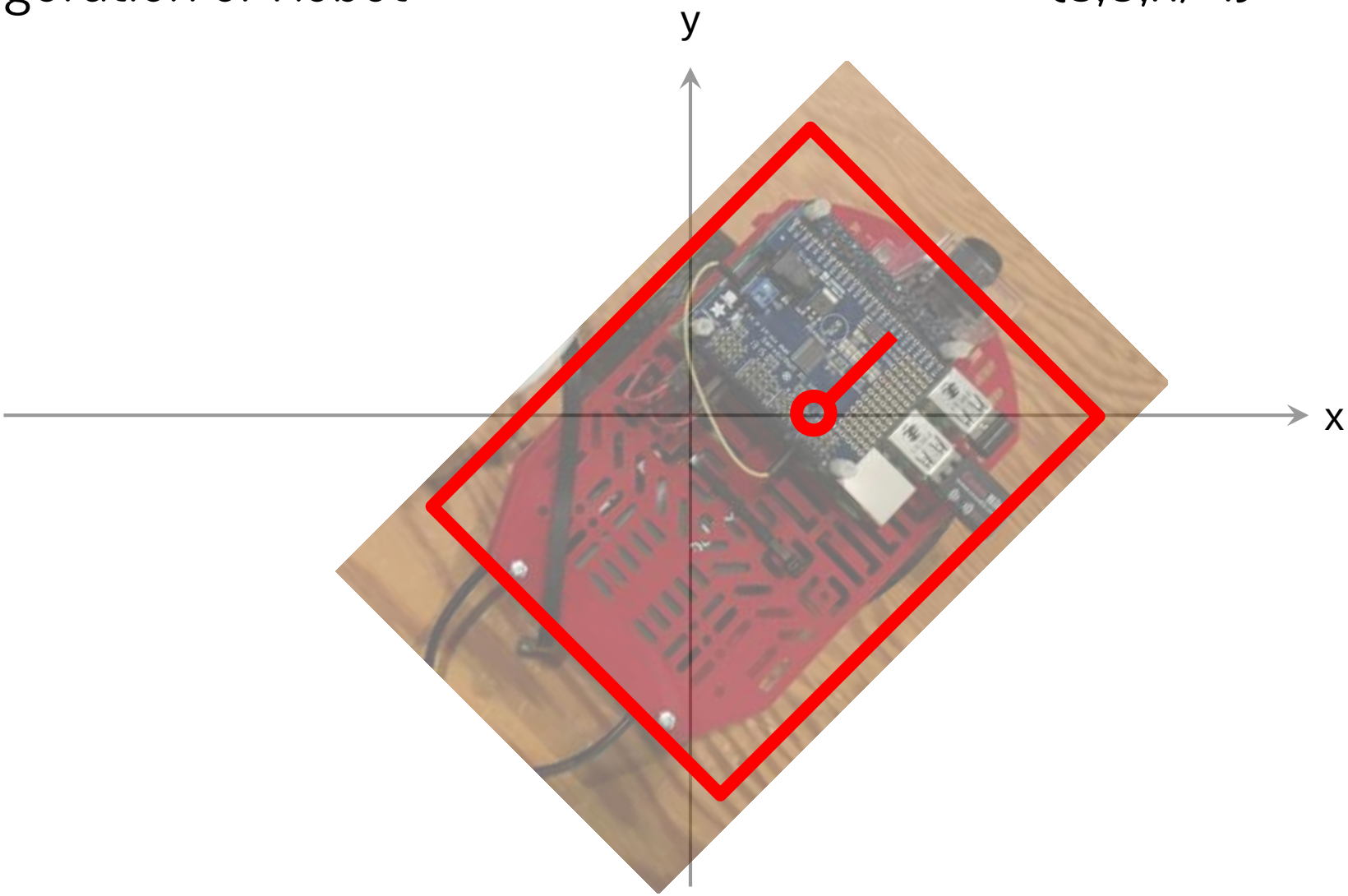
Configuration of Robot

(0,0,π/4)



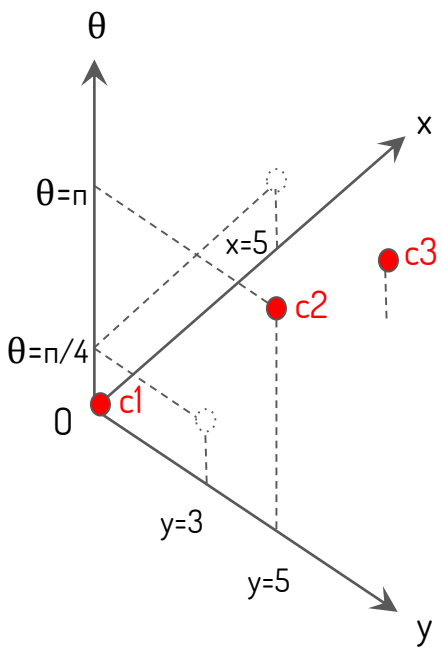
Configuration of Robot

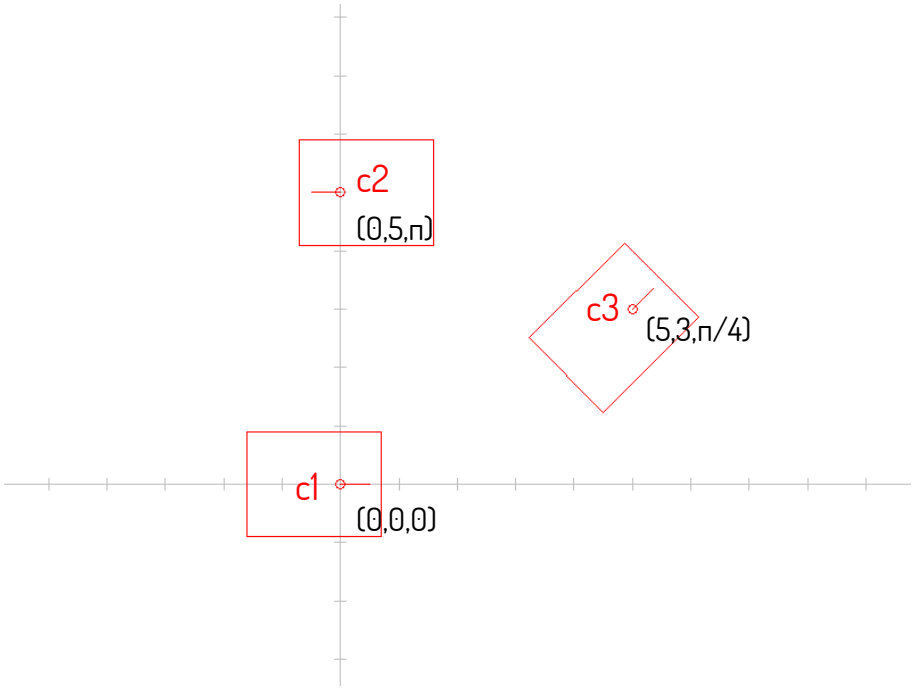
(5,0,π/4)



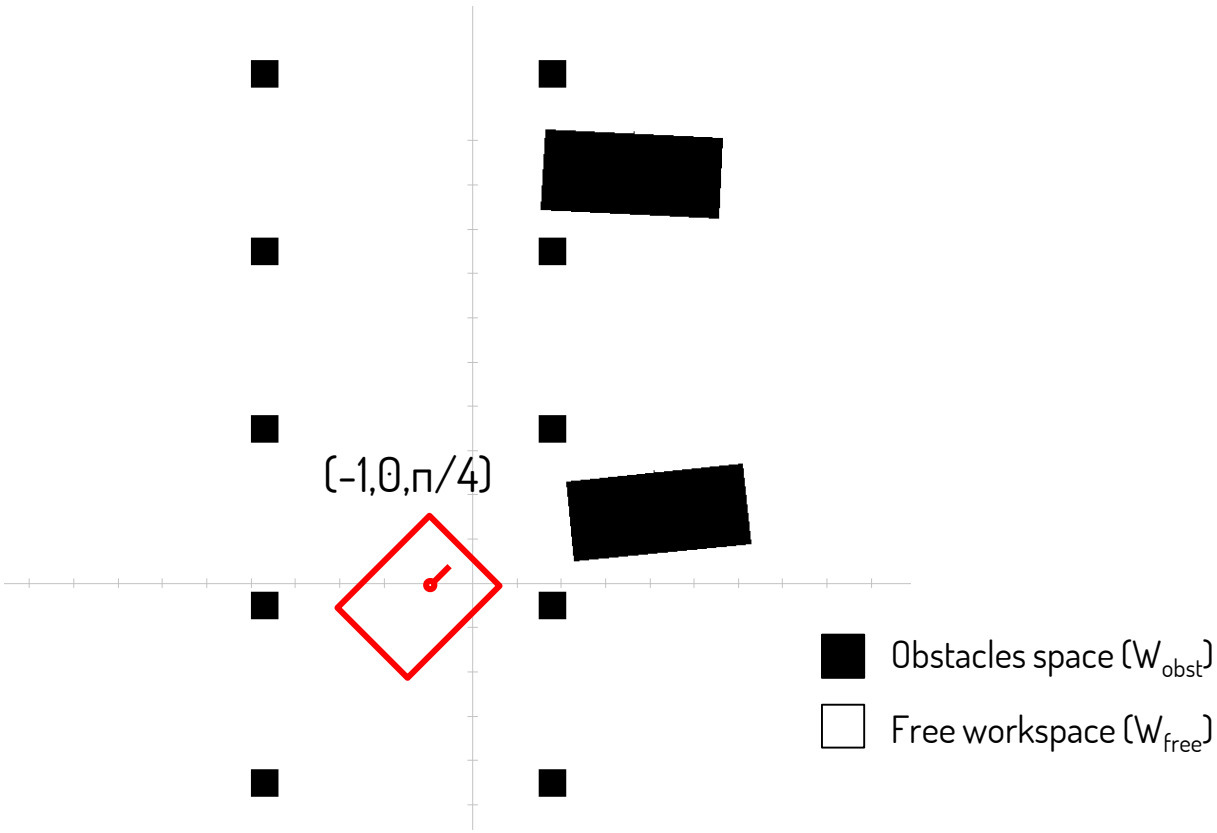
Configuration Space: (x,y,θ)

$$X \subset R^2 \times [-\pi,+\pi]$$

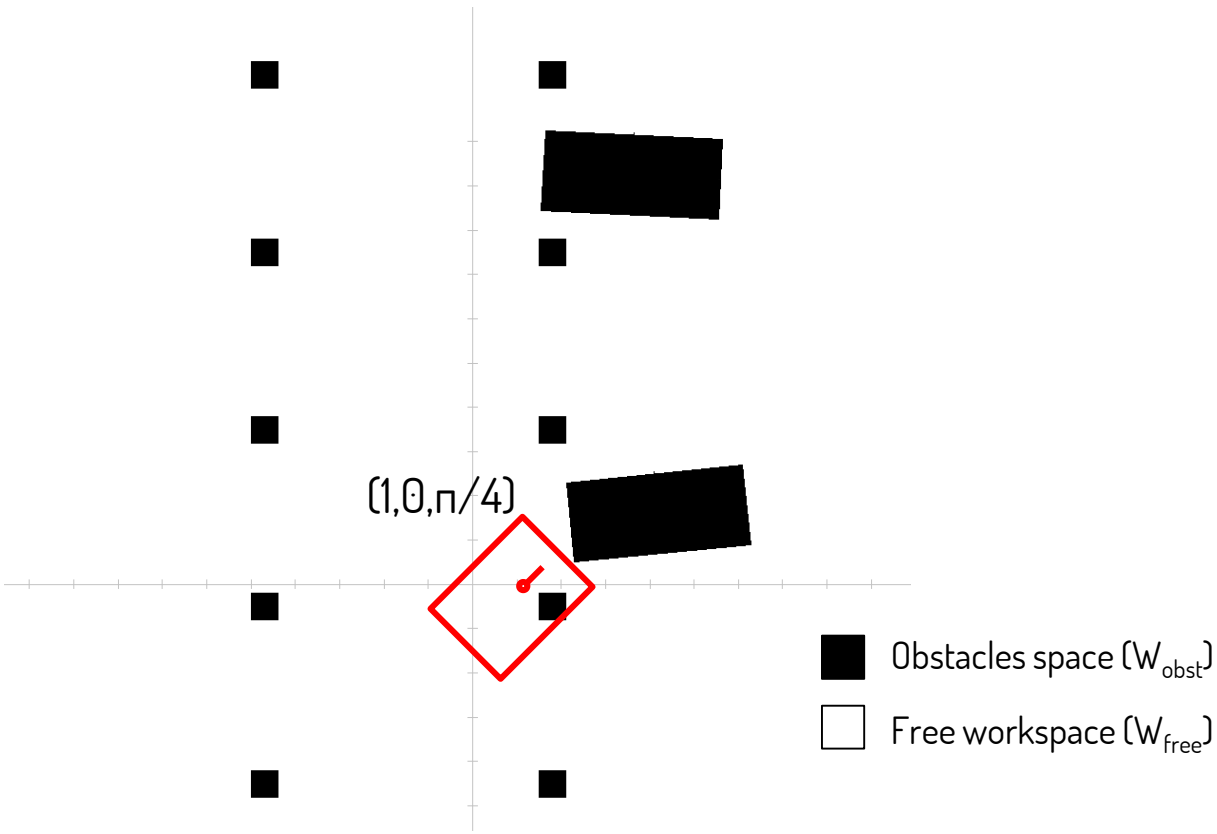




Free Configuration:



Collision Configuration:



Free Configuration Space

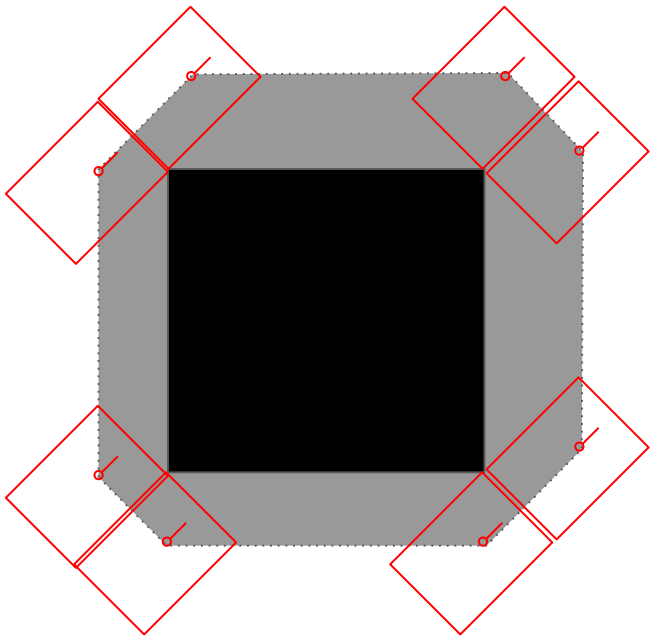
Let R(x) be the region occupied by robot at configuration $x \in X$.

The set of all collision-free configurations is

$$X_{free} = \{x : x \in X \text{ and } R(x) \subset W_{free}\}$$

Free Configuration Space

Slice of X_{free} for $\theta=\pi/4$:



Obstacle space (W_{obst})

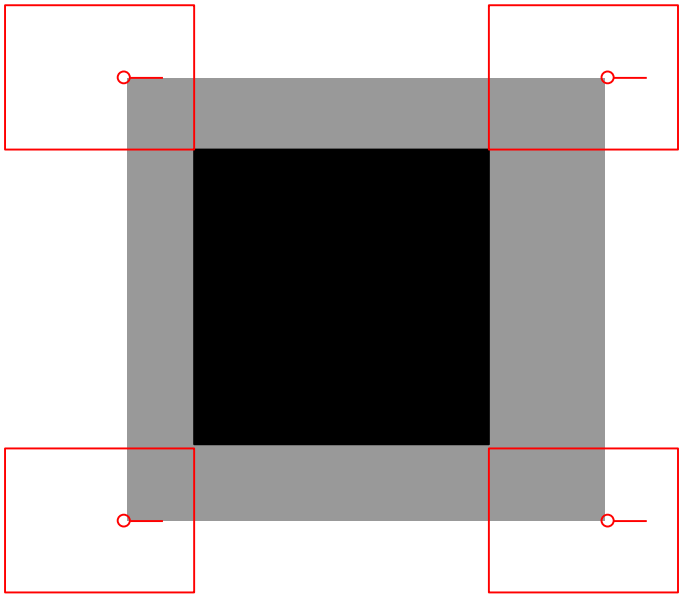
Collision configurations

Free configuration space (C_{free})

Free Configuration Space -- Illustration

Free Configuration Space

$\theta = 0$

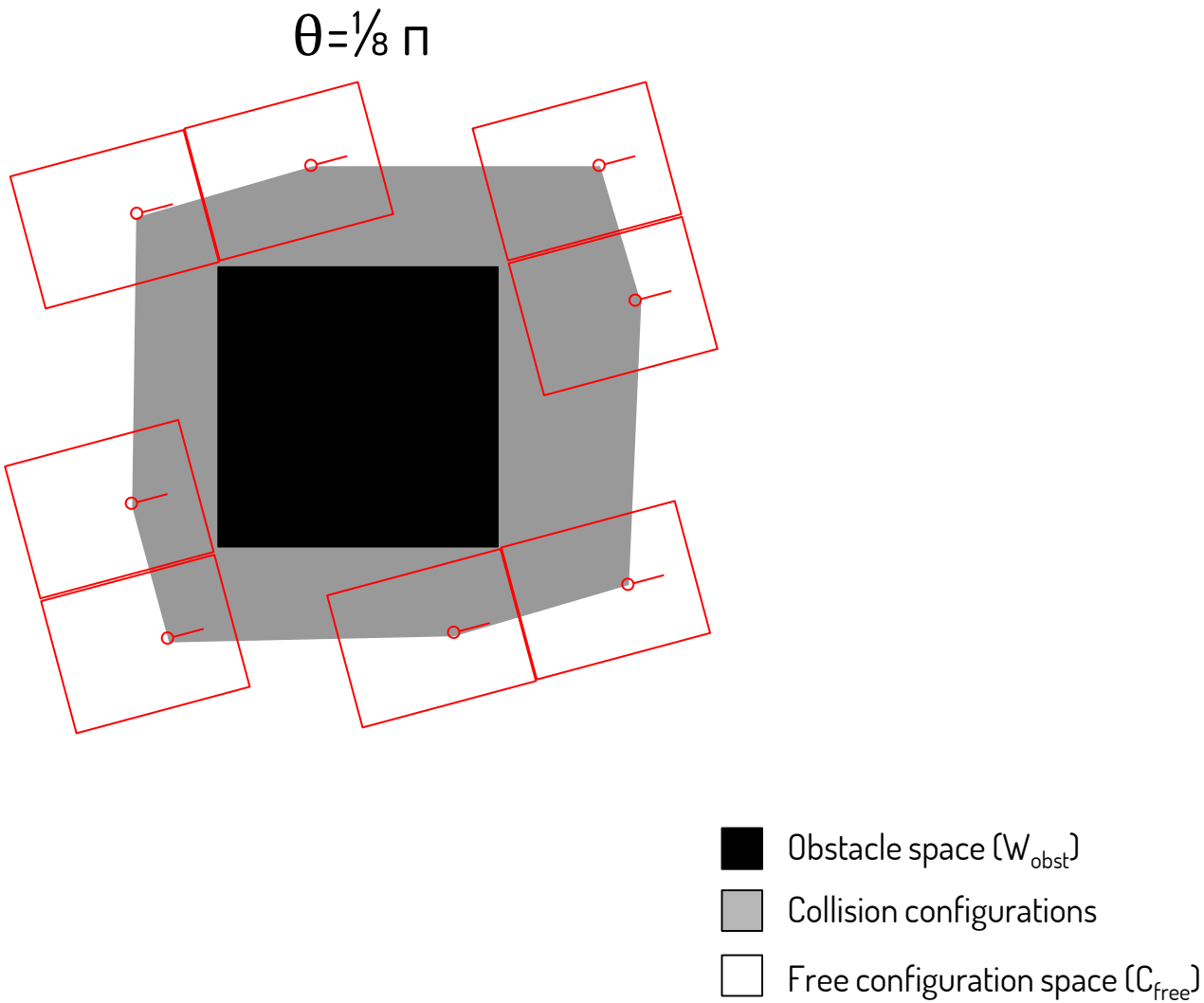


Obstacle space (W_{obst})

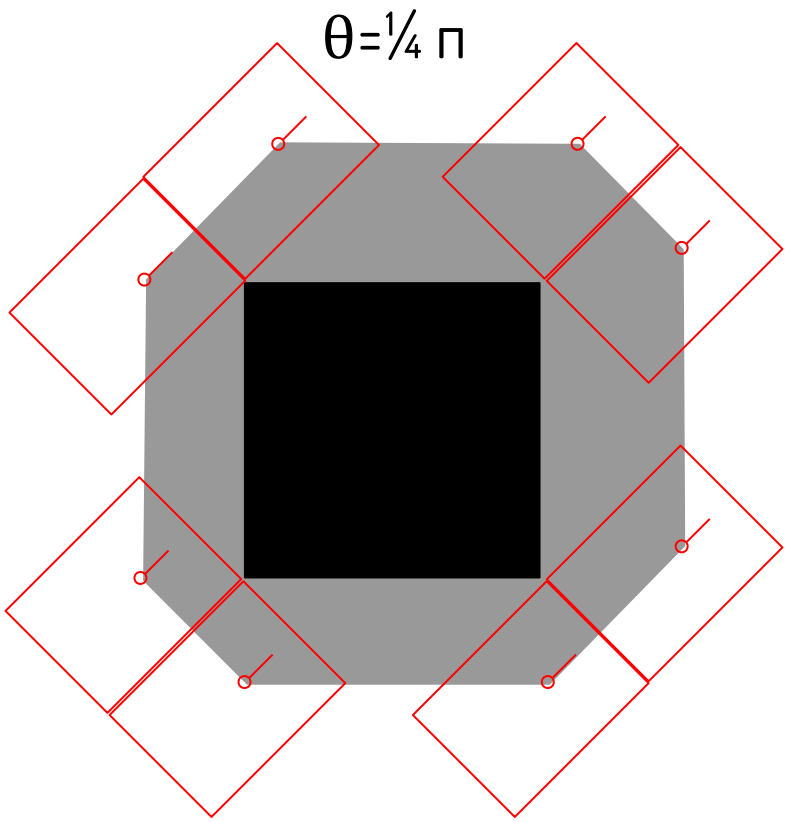
Collision configurations

Free configuration space (C_{free})

Free Configuration Space



Free Configuration Space

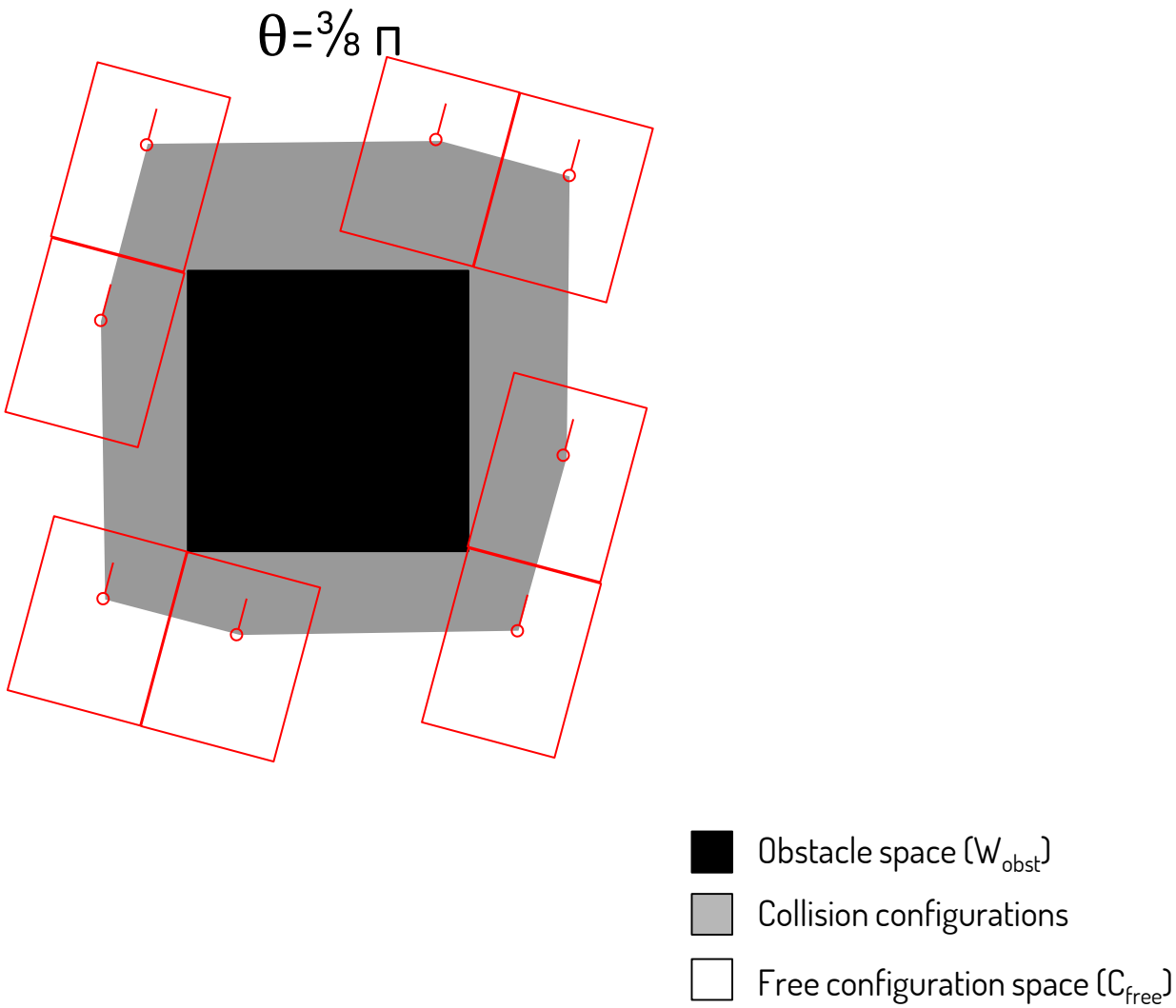


Obstacle space (W_{obst})

Collision configurations

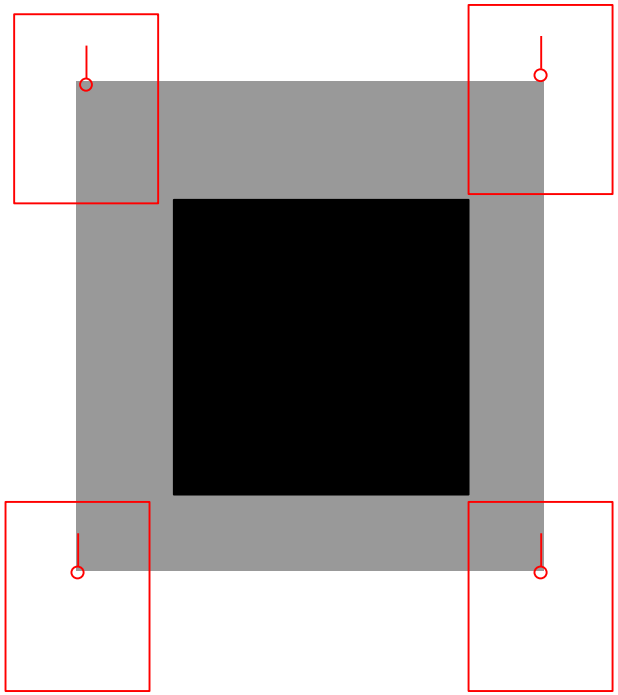
Free configuration space (C_{free})

Free Configuration Space



Free Configuration Space

$\theta = \frac{1}{2}\pi$



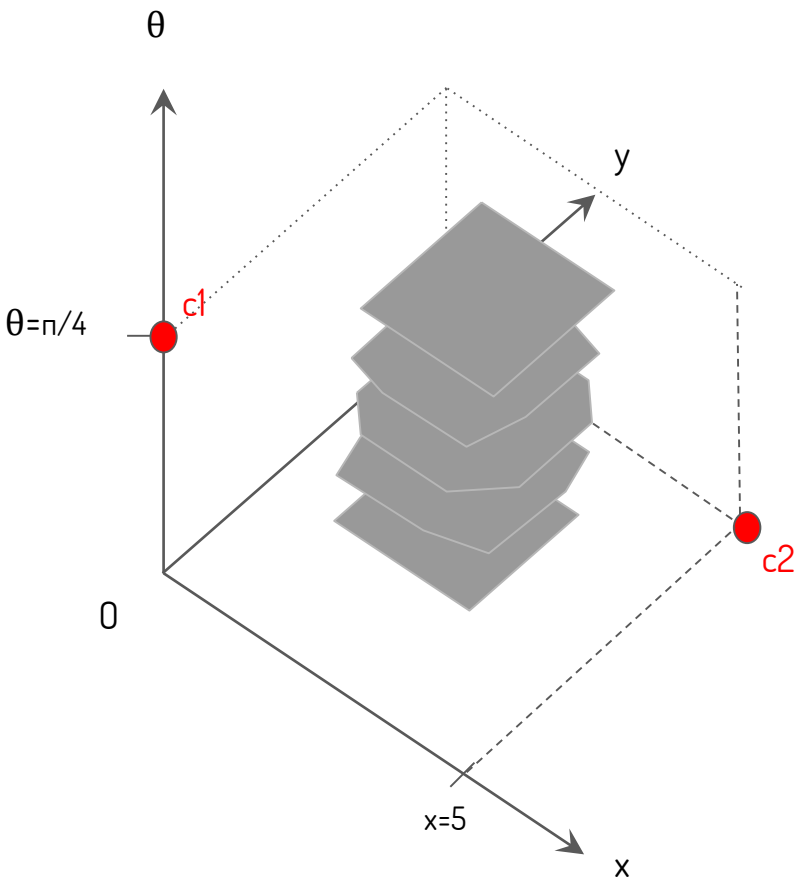
Obstacle space (W_{obst})

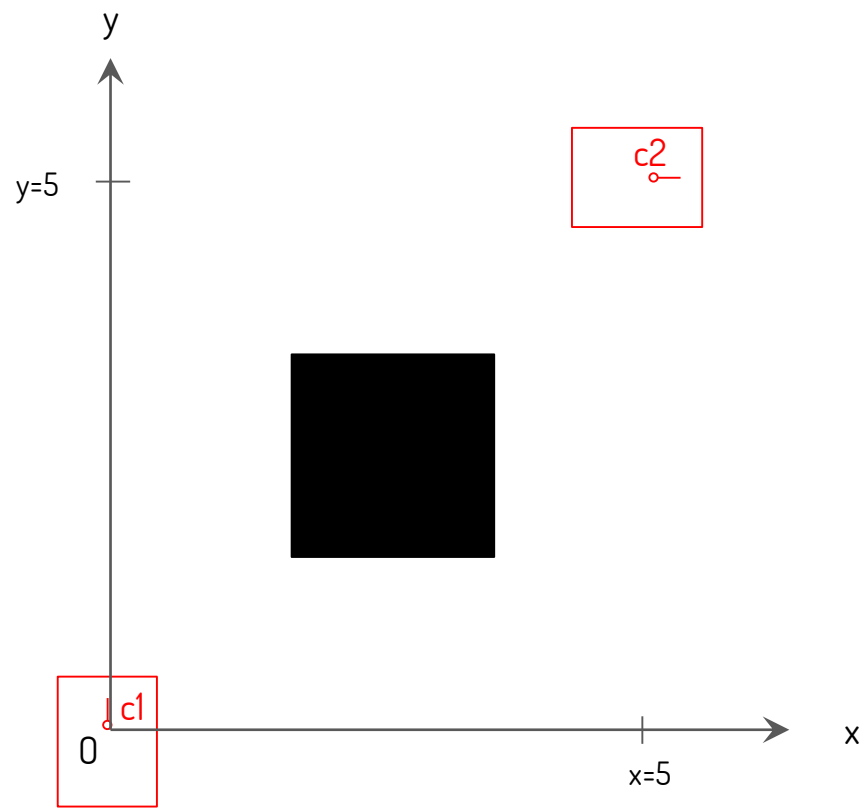
Collision configurations

Free configuration space (C_{free})

Path in Configuration Space

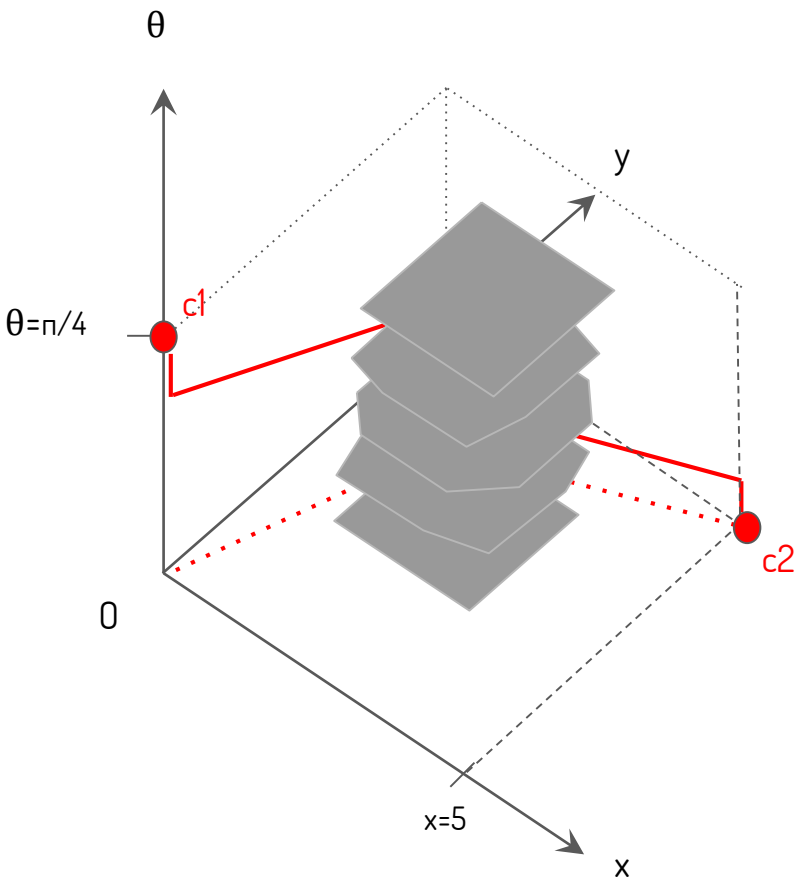
$\sigma(\alpha): [0,1] \rightarrow X$

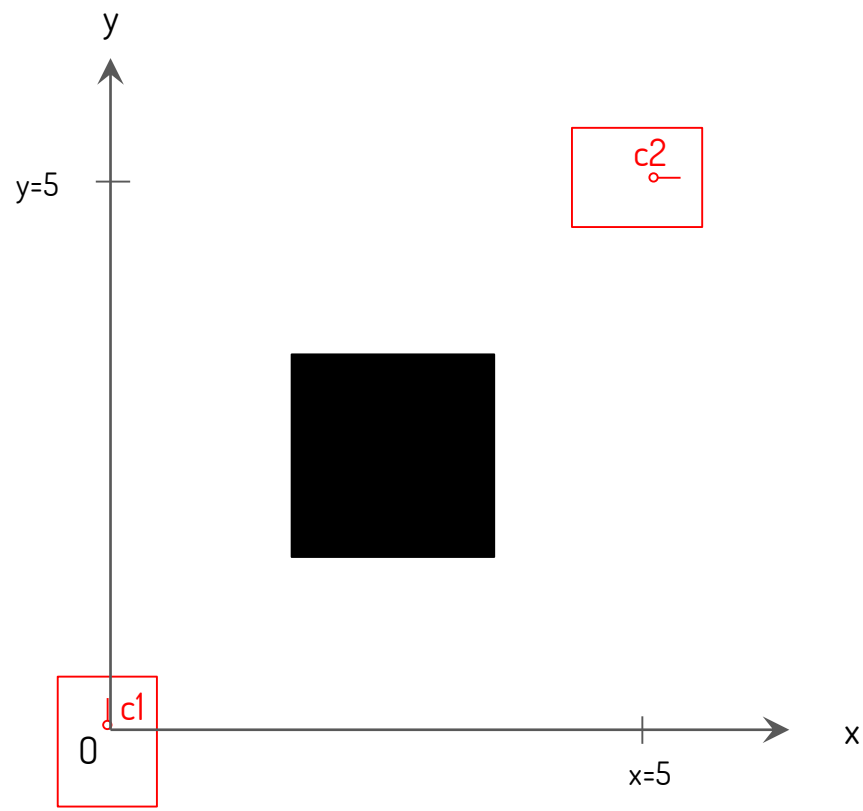




Path in Configuration Space

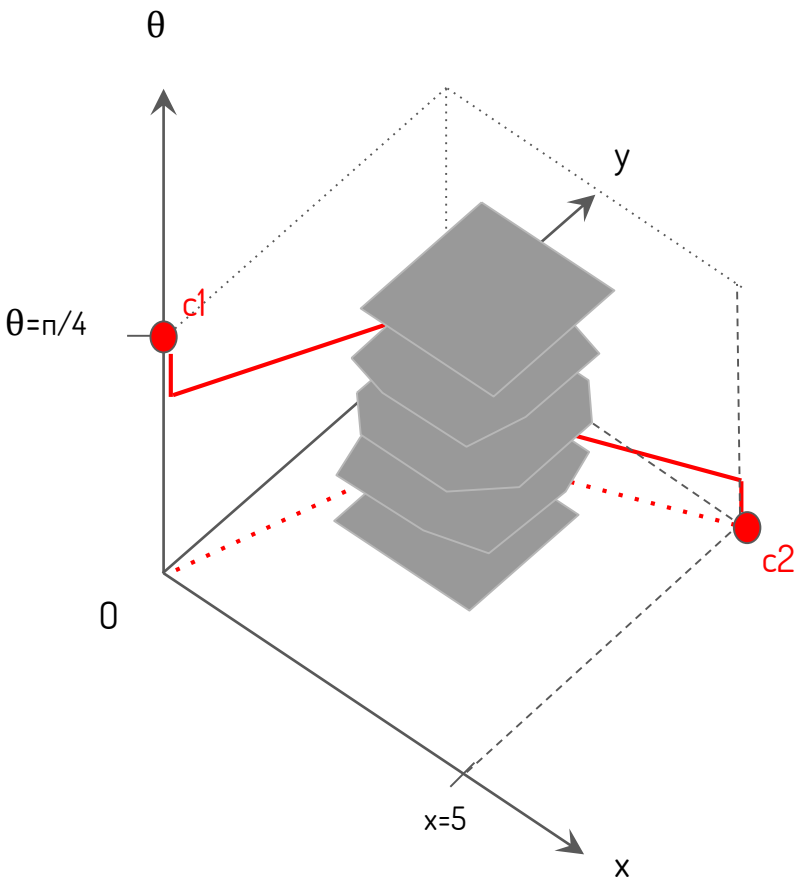
$\sigma(\alpha): [0,1] \rightarrow X$

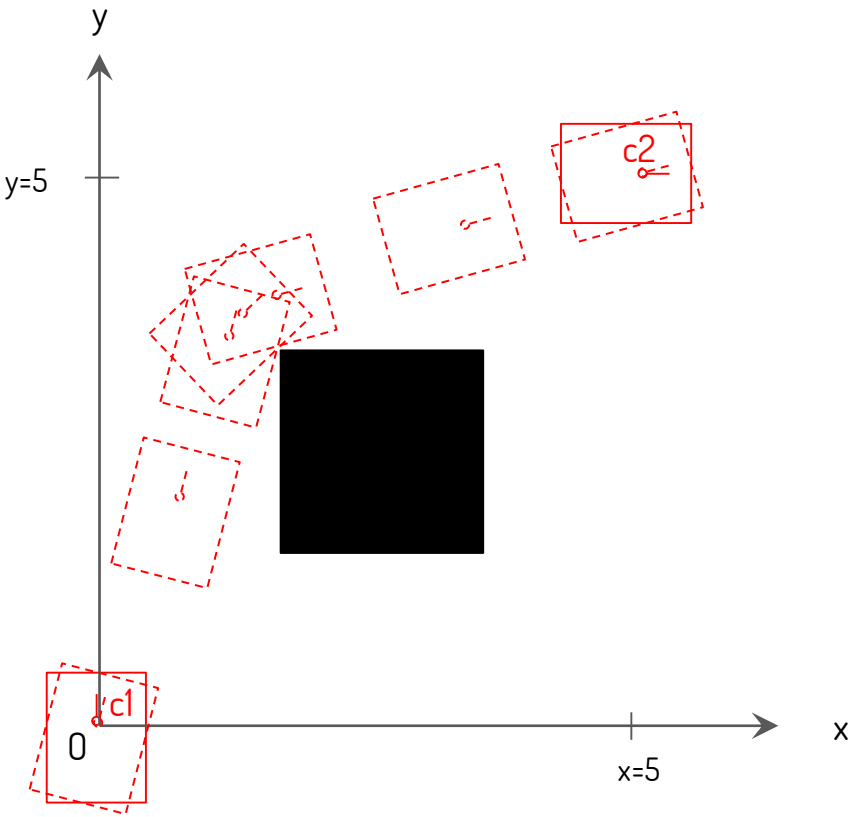




Path in Configuration Space

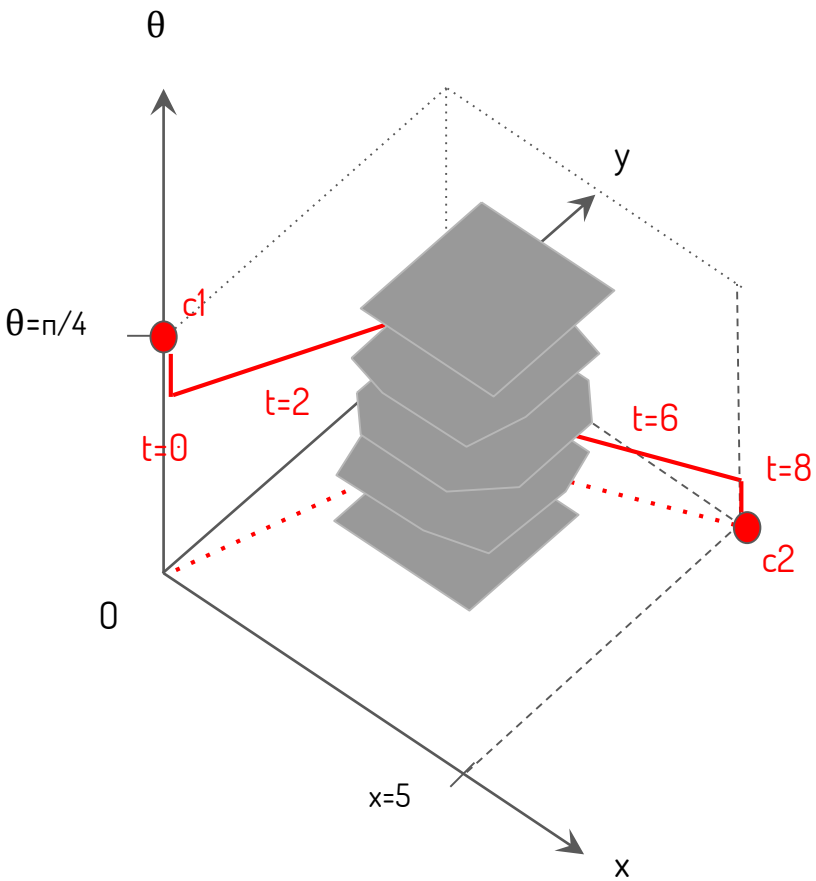
$\sigma(\alpha)$: $[0,1] \rightarrow X$

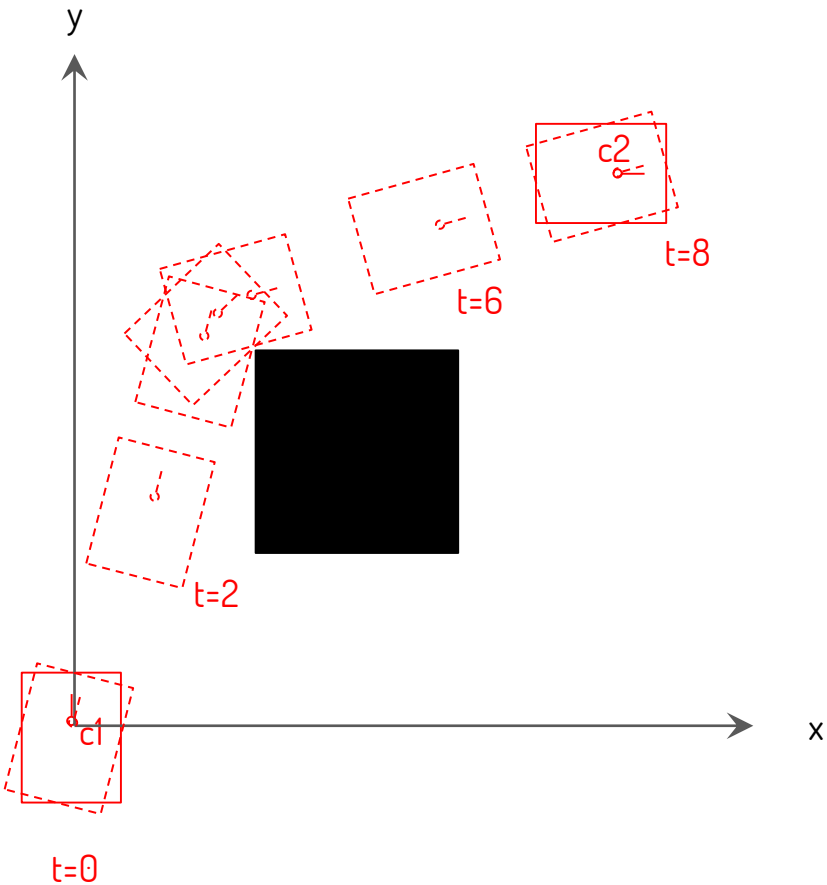




Trajectory in Configuration Space

$\pi(t): [0,T] \rightarrow X$





Path Planning Problem Formulation

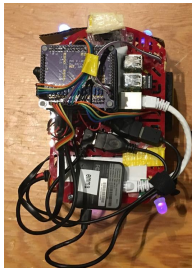
Can be formulated as an optimization problem over all paths in configuration space:

$$\arg\min \; J(\boldsymbol{\sigma}) \text{ subject to}$$
$$\boldsymbol{\sigma}(0) = x_{init}$$
$$\boldsymbol{\sigma}(1) \in X_{goal}$$
$$\boldsymbol{\sigma}(\alpha) \in X_{free} \qquad \forall \alpha \in [0,1]$$

- $\boldsymbol{\sigma}(\alpha)$ is a continuous function $[0,1] \rightarrow X$
- $J(\boldsymbol{\sigma})$ is a cost functional
- x_{init} is the initial configuration of the robot
- X_{goal} is the set of goal configurations
- X_{free} is the free configuration space

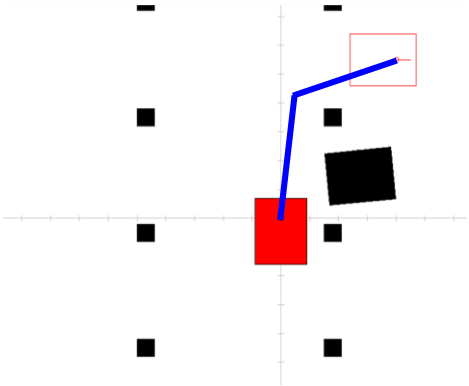
Holonomic System

Holonomic system:
(no differential constraints)



arg min $J(\boldsymbol{\sigma})$ subject to

$$\boldsymbol{\sigma}(0) = x_{init}$$
$$\boldsymbol{\sigma}(1) = X_{goal}$$
$$\boldsymbol{\sigma}(\alpha) = X_{free} \quad \forall \alpha \in [0,1]$$



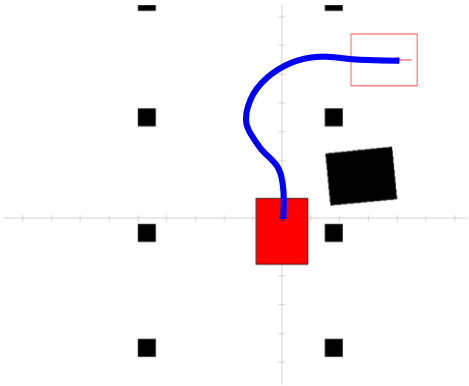
Nonholonomic System

Nonholonomic system:
(differential constraints)

arg min $J(\boldsymbol{\sigma})$ subject to

$$\boldsymbol{\sigma}(0) = x_{init}$$
$$\boldsymbol{\sigma}(1) = X_{goal}$$
$$\boldsymbol{\sigma}(\alpha) = X_{free} \quad \forall \alpha \in [0,1]$$
$$D(\boldsymbol{\sigma}(\alpha), \boldsymbol{\sigma}'(\alpha), \boldsymbol{\sigma}''(\alpha), \ldots) \quad \forall \alpha \in [0,1]$$

E.g., bound on path curvature k can be enforced as $|\boldsymbol{\sigma}'(\alpha)\ \boldsymbol{\sigma}''(\alpha)|/|\boldsymbol{\sigma}'(\alpha)|^3 < k$.



Complexity of Path Planning

- Path planning “Piano Movers problem” is PSPACE-hard [Reif ‘79]
- Complete (non-optimal) algorithms for exist [Canny ‘88] but have running time exponential in dimension of configuration space.

Solution Techniques for Path Planning Problem

- Variational Methods
- Graph-search Methods
  - Cell decomposition
  - Visibility graph
  - Sampling-based roadmap construction
  - Tree of motion primitives
- Incremental Search Methods
  - RRT: Rapidly-exploring Random Trees
  - RRT*: Optimal Rapidly-exploring Random Trees

Properties of Path Planning Methods

An algorithm is

- **Complete:** if it finds valid path or detect non-existence of thereof in finite time.
- **Optimal:** if it find optimal path in finite time.

- **Anytime:** if it can be terminated at any point of the execution, but the path quality improves with computation time

- **Probabilistically Complete:** if the probability that the algorithm finds valid solution goes to 1 with running time.
- **Asymptotic Optimal:** if it returns a sequence of solutions converging to an optimal solution.

Trajectory Planning Problem Formulation

Useful for

- Dynamic constraints
- Dynamic obstacles

Can be formulated as optimization in the space of trajectories over time interval [0,T] in configuration space:

$$
\begin{aligned}
\arg\min \; & J(\pi) \text{ subject to} \\
& \pi(0) = x_{init} \\
& \pi(T) \in X_{goal} \\
& \pi(t) \in X_{free}(t) \qquad \forall t \in [0,T] \\
& D(\pi(t), \pi'(t), \pi''(t), \ldots) \qquad \forall t \in [0,T]
\end{aligned}
$$

Solution Techniques for Trajectory Planning Problem

- Variational Methods
- Convert to Path Planning in Space-Time:

trajectory planning in (x,y,θ)

=>

path planning in (x,y,θ,t) + diff. constraints

Solution Techniques for Path Planning Problem

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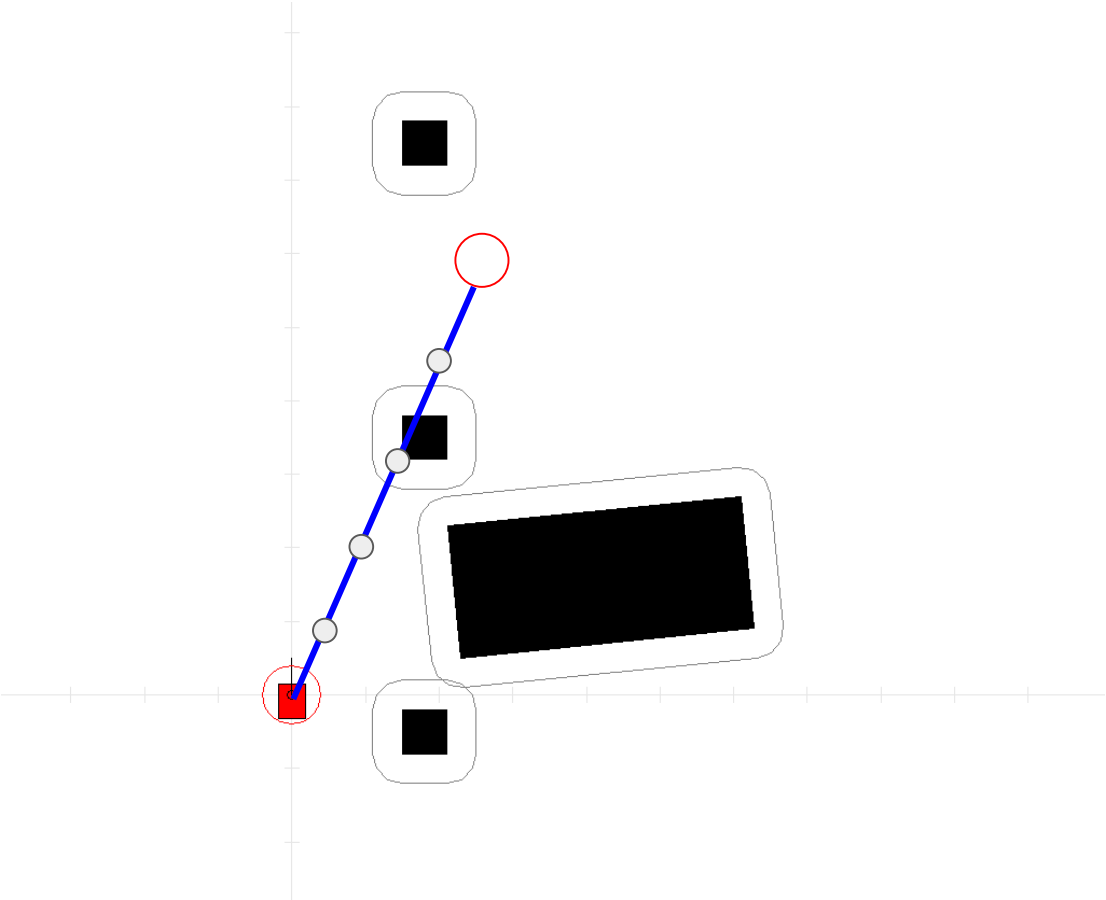
Variational Techniques

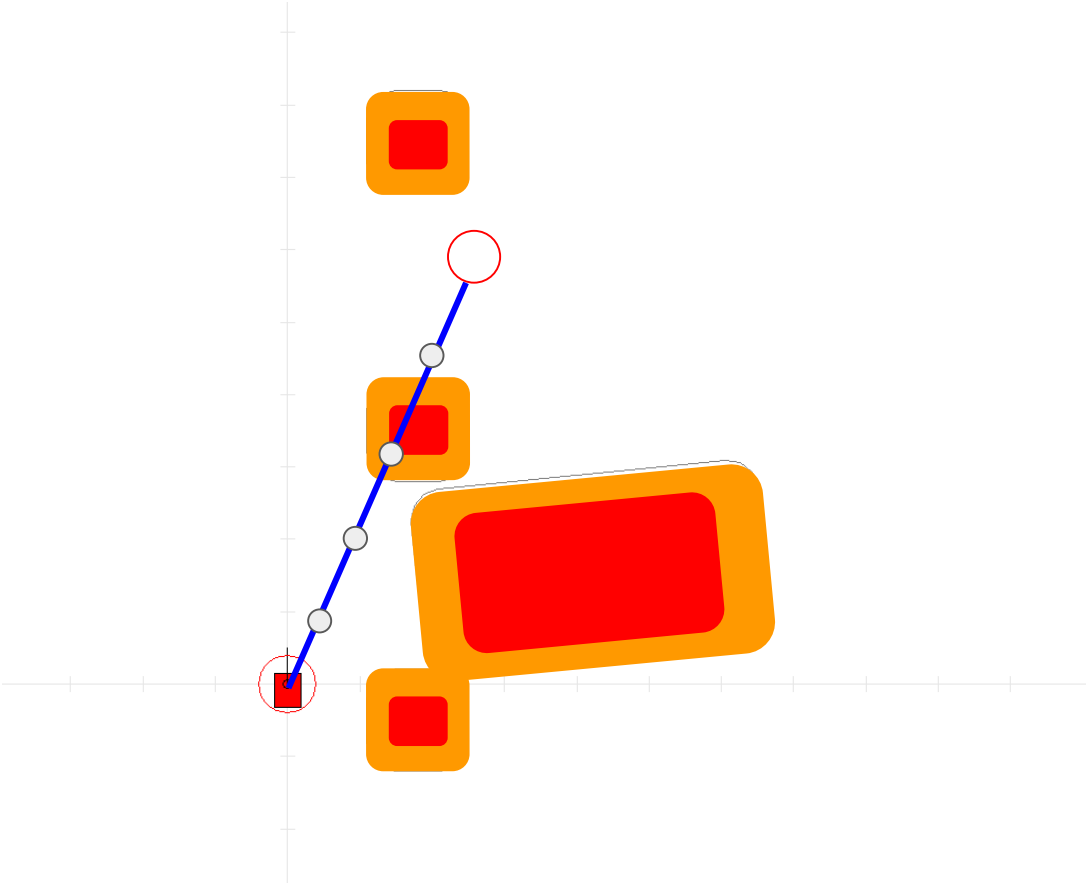
Aka Trajectory optimization, Optimal control, etc.

- Non-linear optimization
- Path is represented as a spline
- Position of control points is optimized



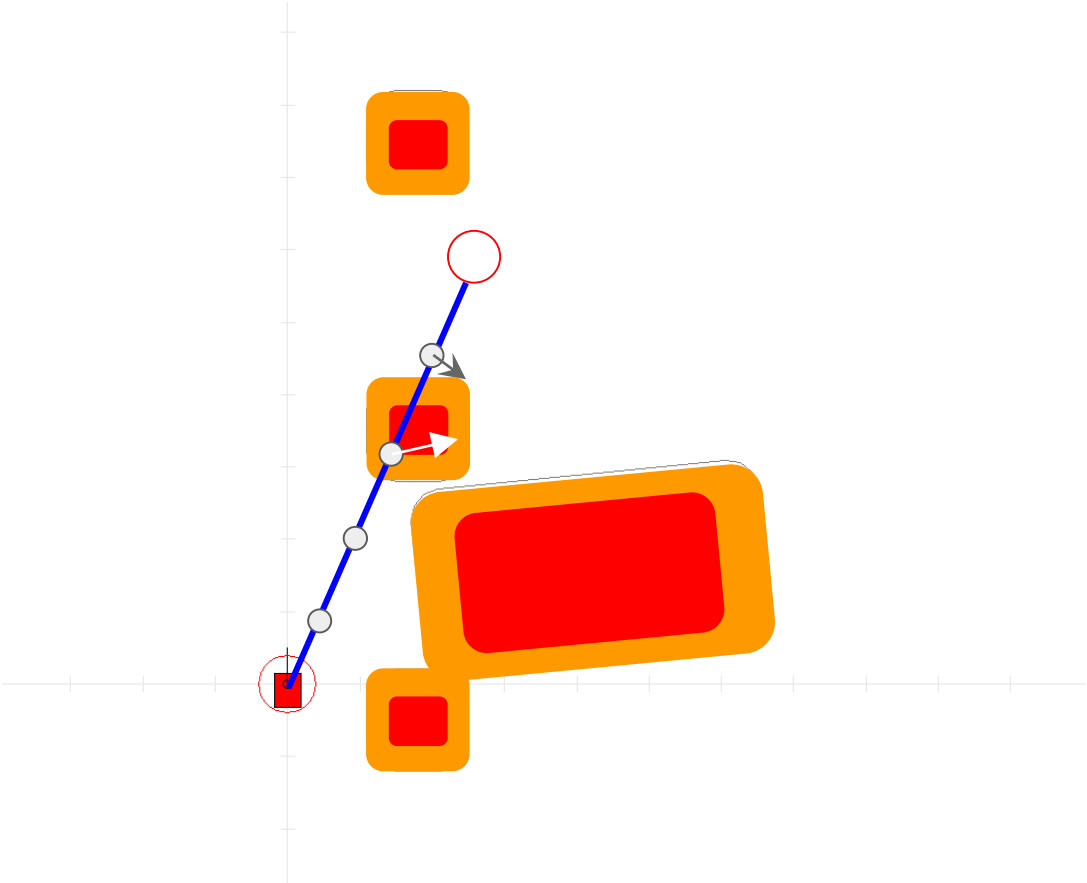
Variational Techniques

Obstacles are modelled as high-cost regions



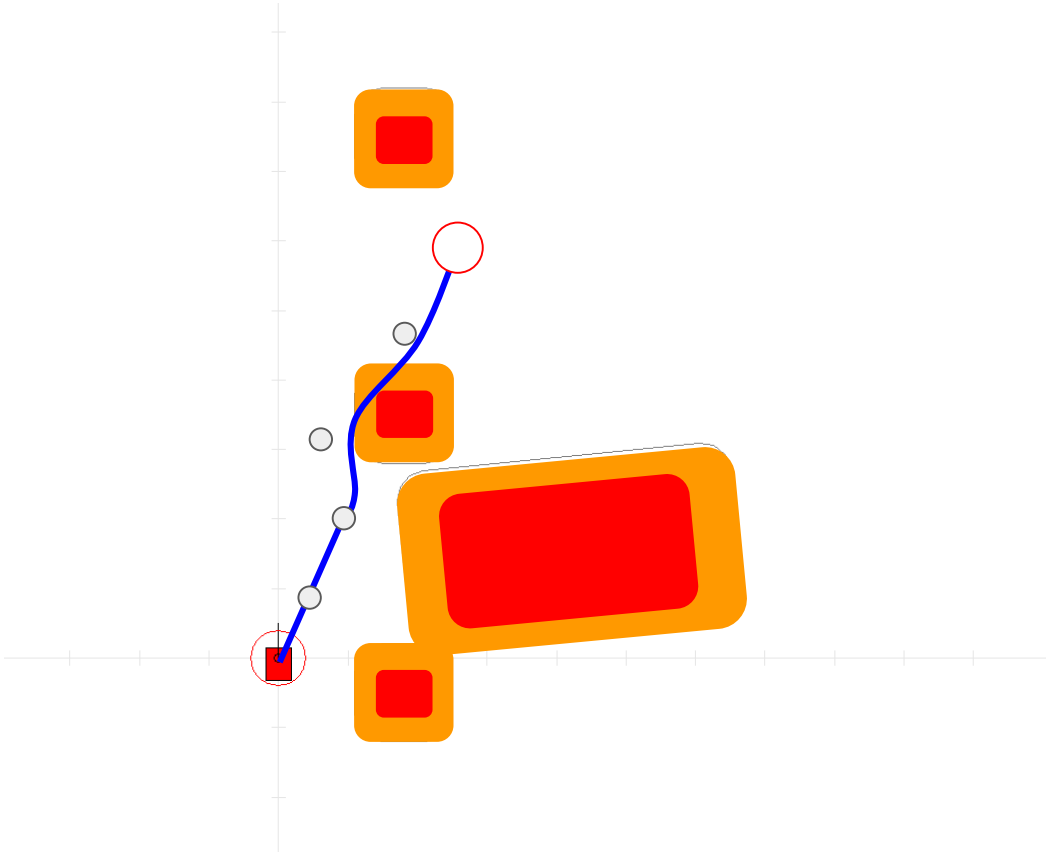
Variational Techniques

Find gradient



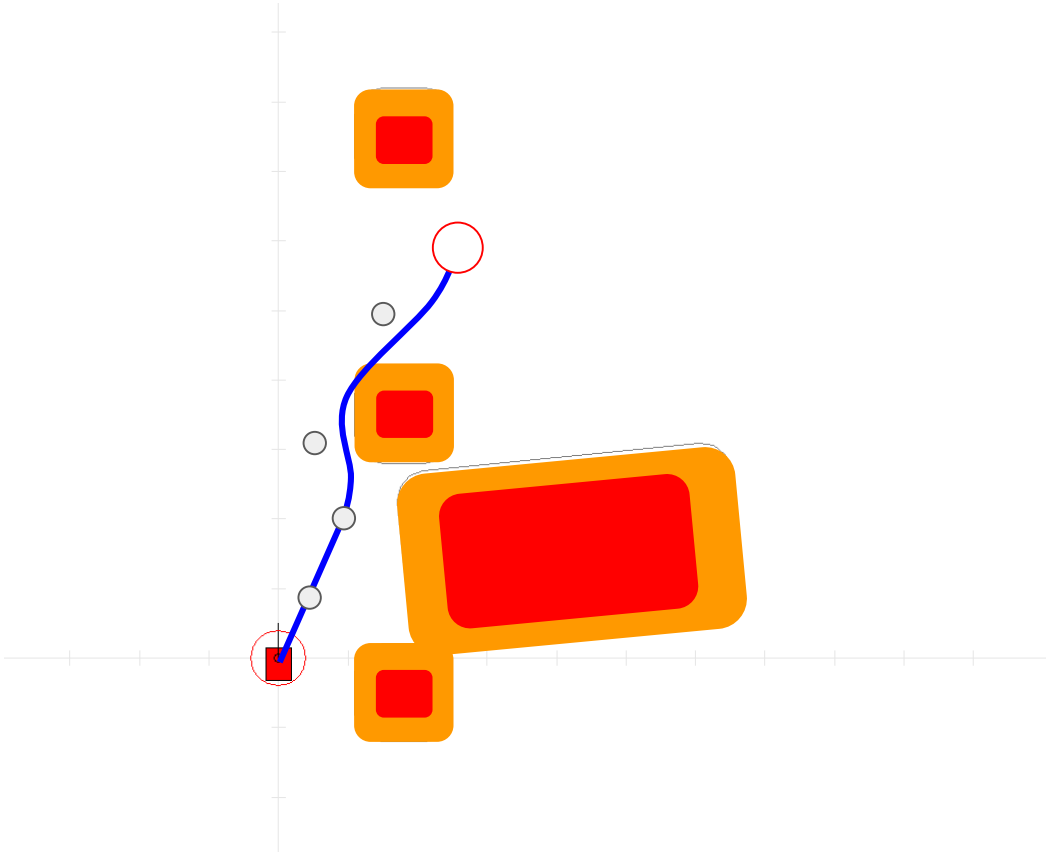
Variational Techniques

Move the control points in the direction of negative gradient

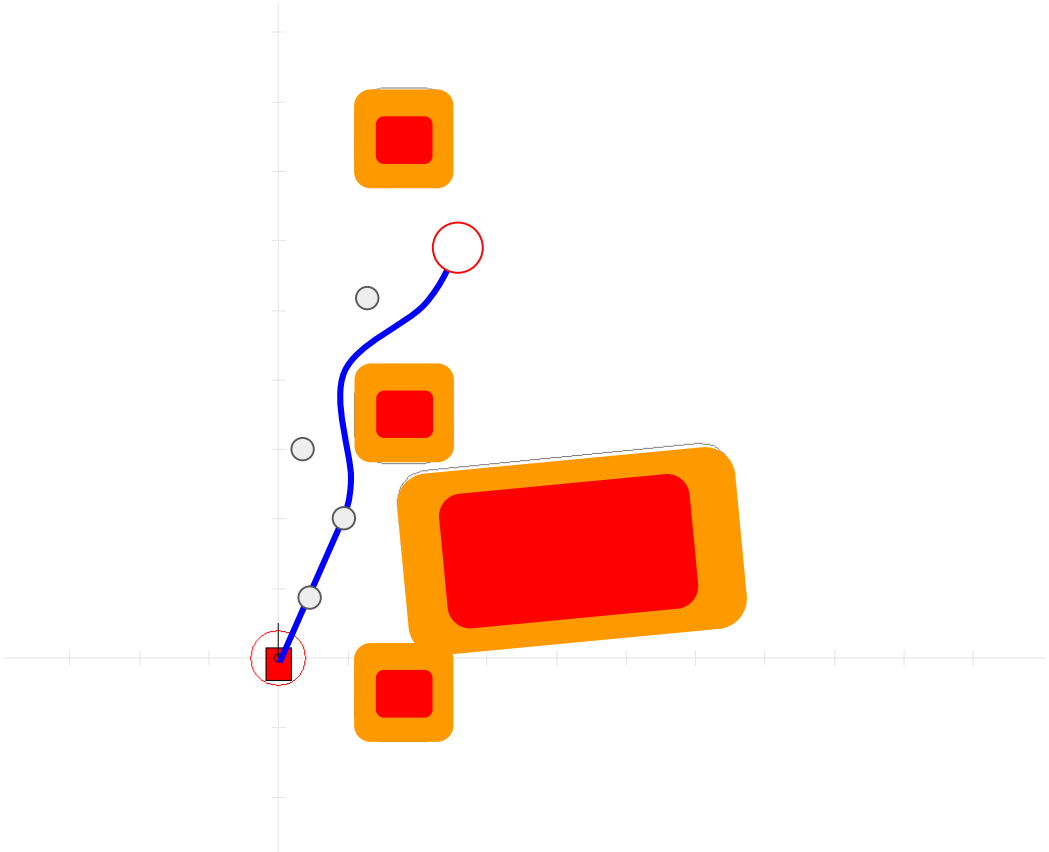


Variational Techniques

Repeat until convergence



Variational Techniques



Variational Techniques

Pros:

- Efficient
- Widely applicable

Cons:

- Only local convergence
    - Incomplete
    - Locally optimal

Notes:

- Used in for local path optimization within CMU's car 'Boss'.

Solution Techniques for Path Planning Problem

- Variational Methods
- <u>Graph-search Methods</u>
  - Cell decomposition
  - Visibility graph
  - Sampling-based roadmap construction
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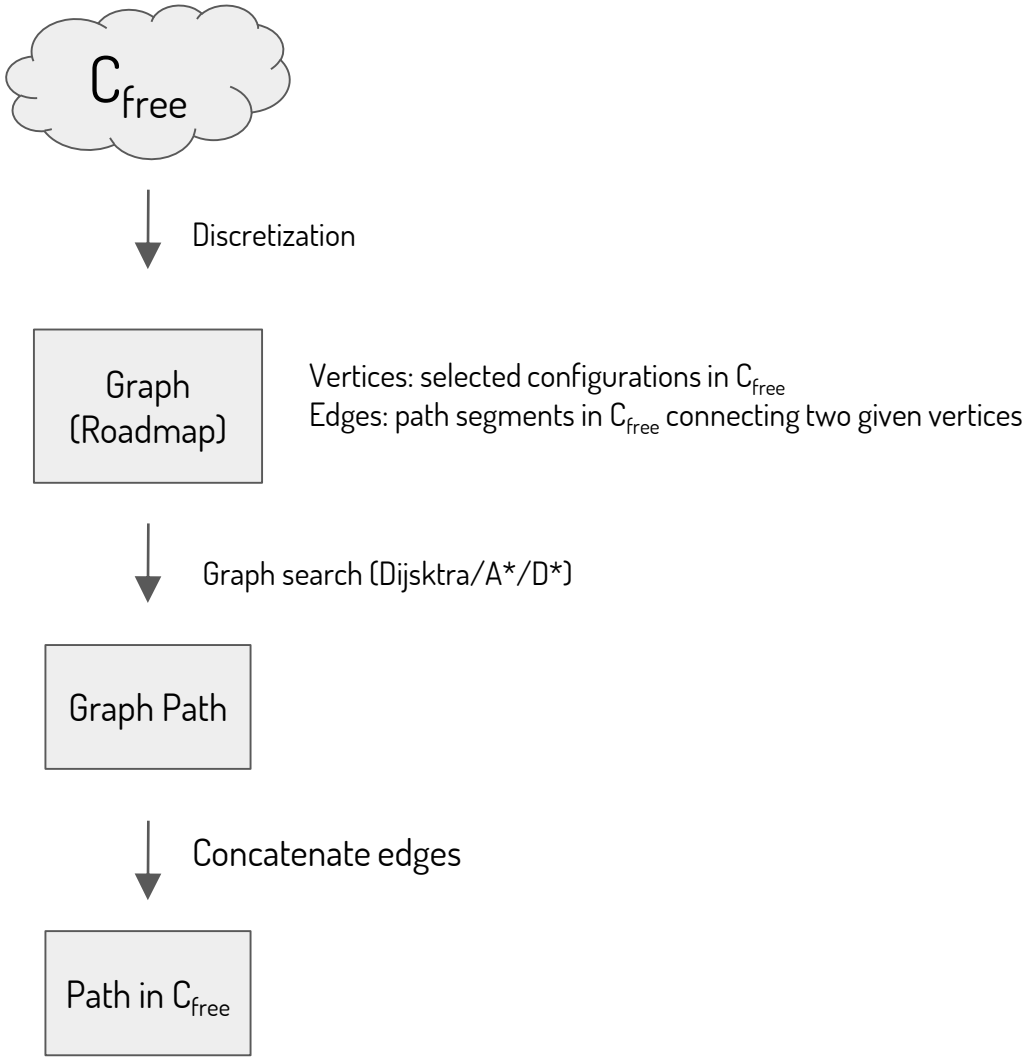
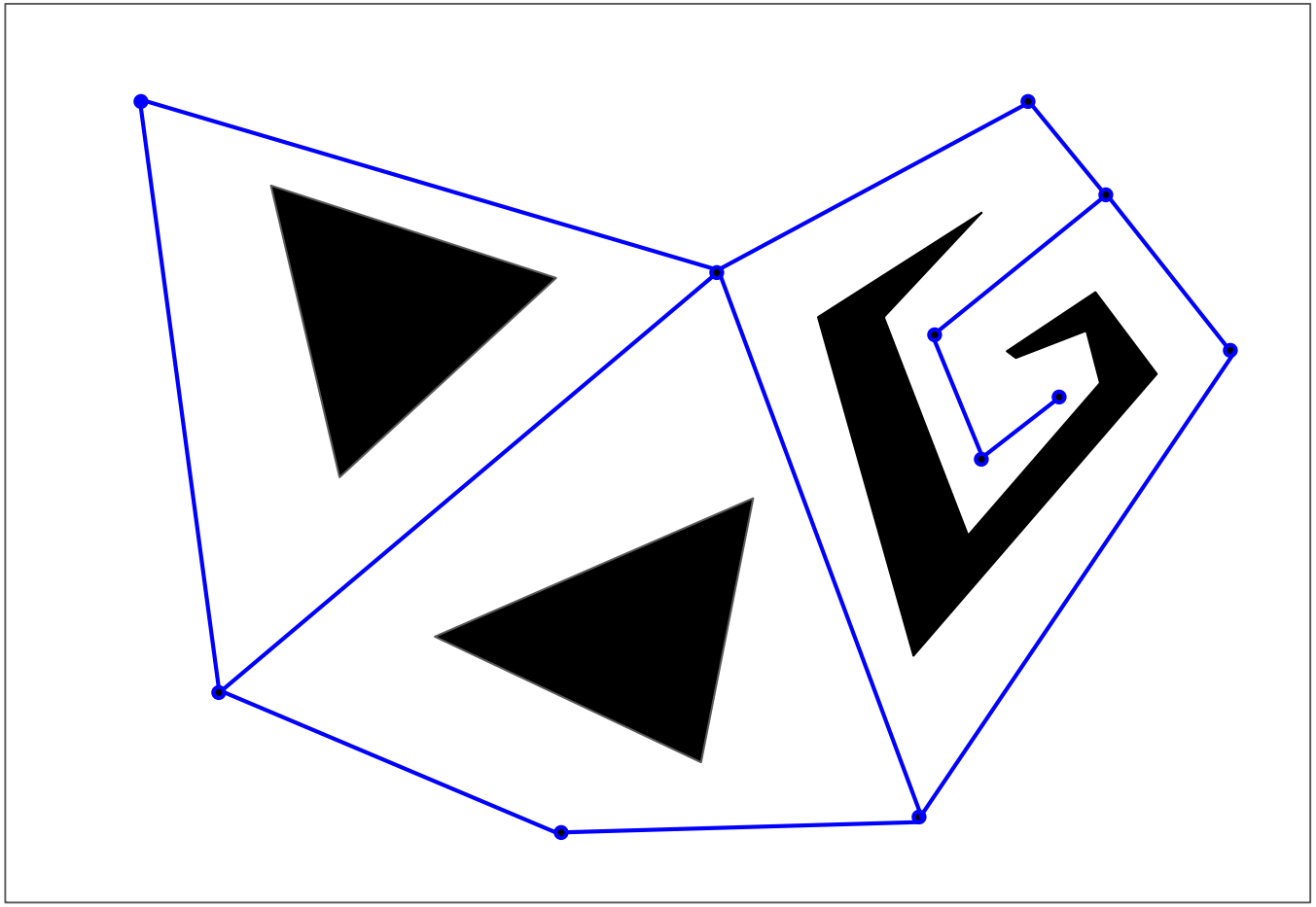
Graph-based Methods



Roadmap



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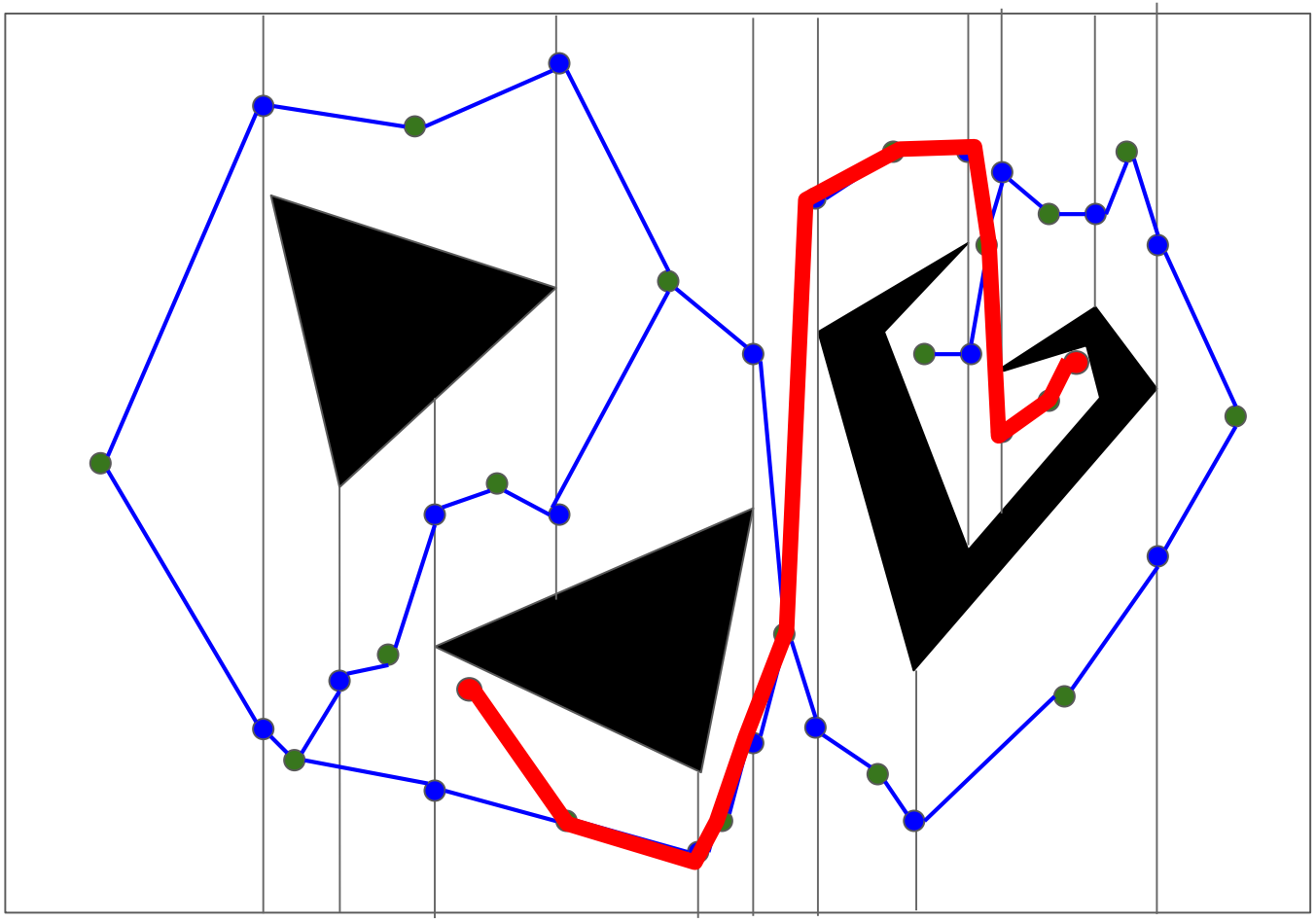
Cell Decomposition

- A method for building roadmaps in 2d polygonal environments



Cell Decomposition

Pros:

- Complete
- Generalizes to higher dimensions and beyond polygonal models

Cons:

- Only holonomic systems
- Suboptimal

Solution Techniques for Path Planning Problem

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  - <u>Visibility graph</u>
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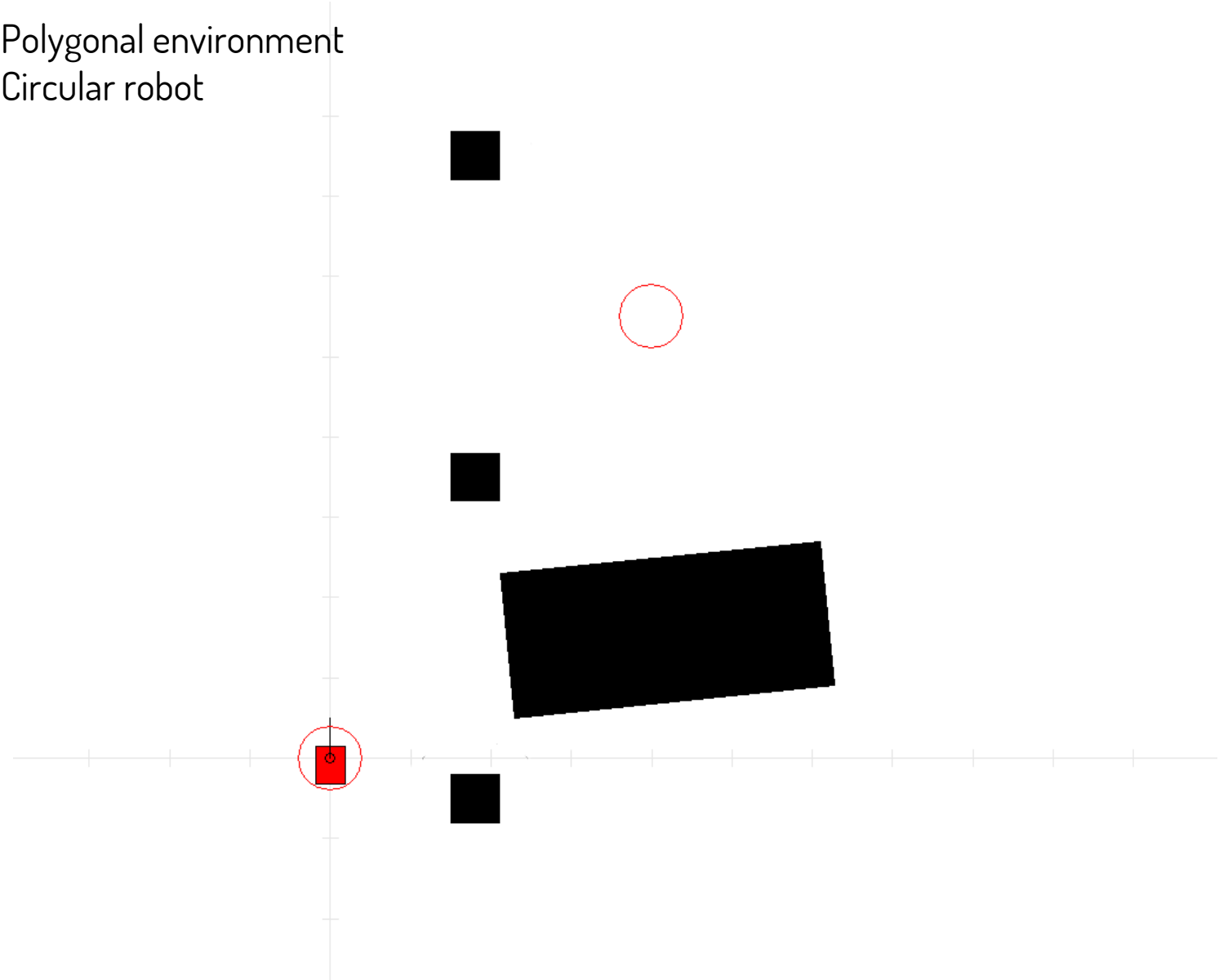
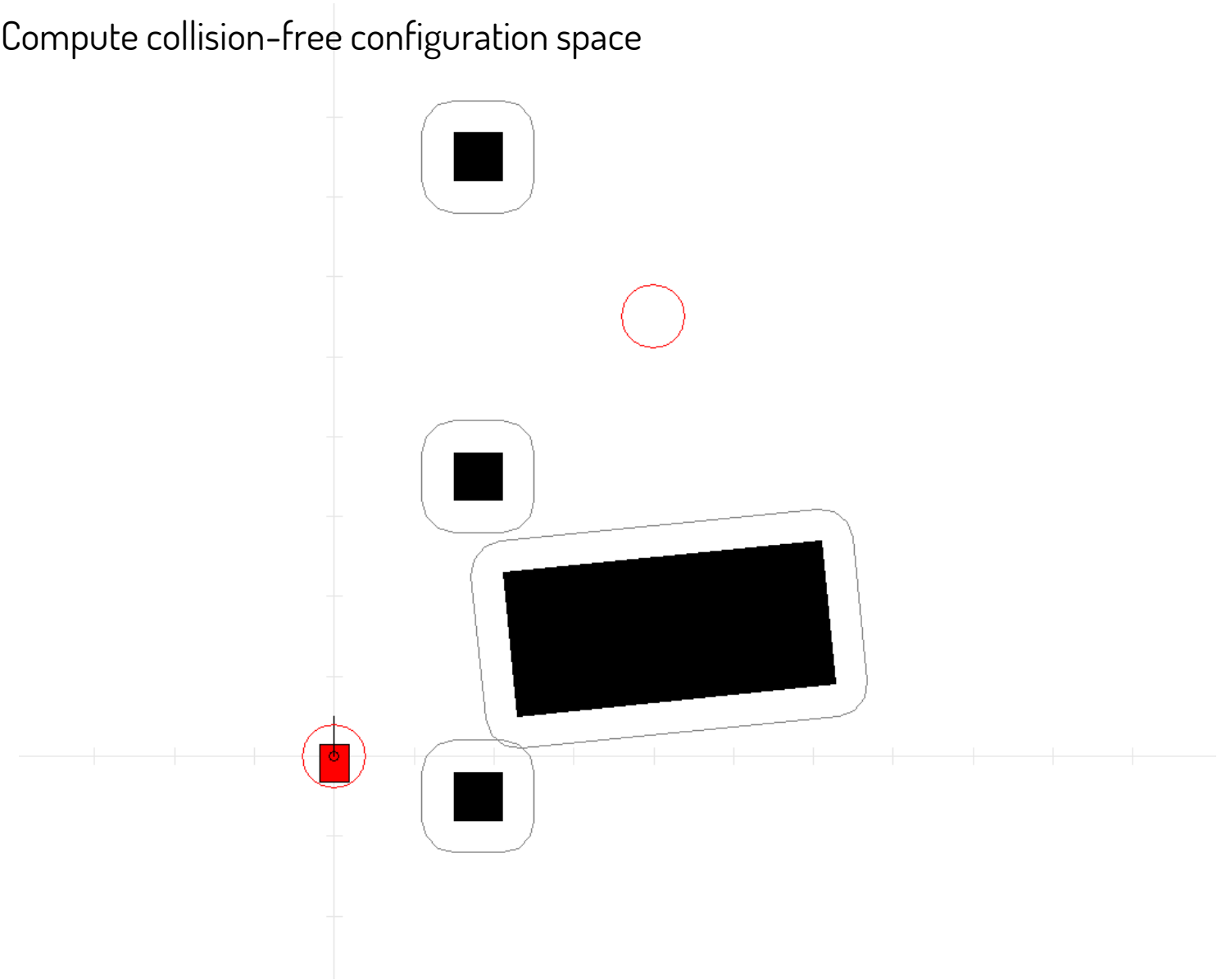
Visibility Graph

- Polygonal environment
- Circular robot



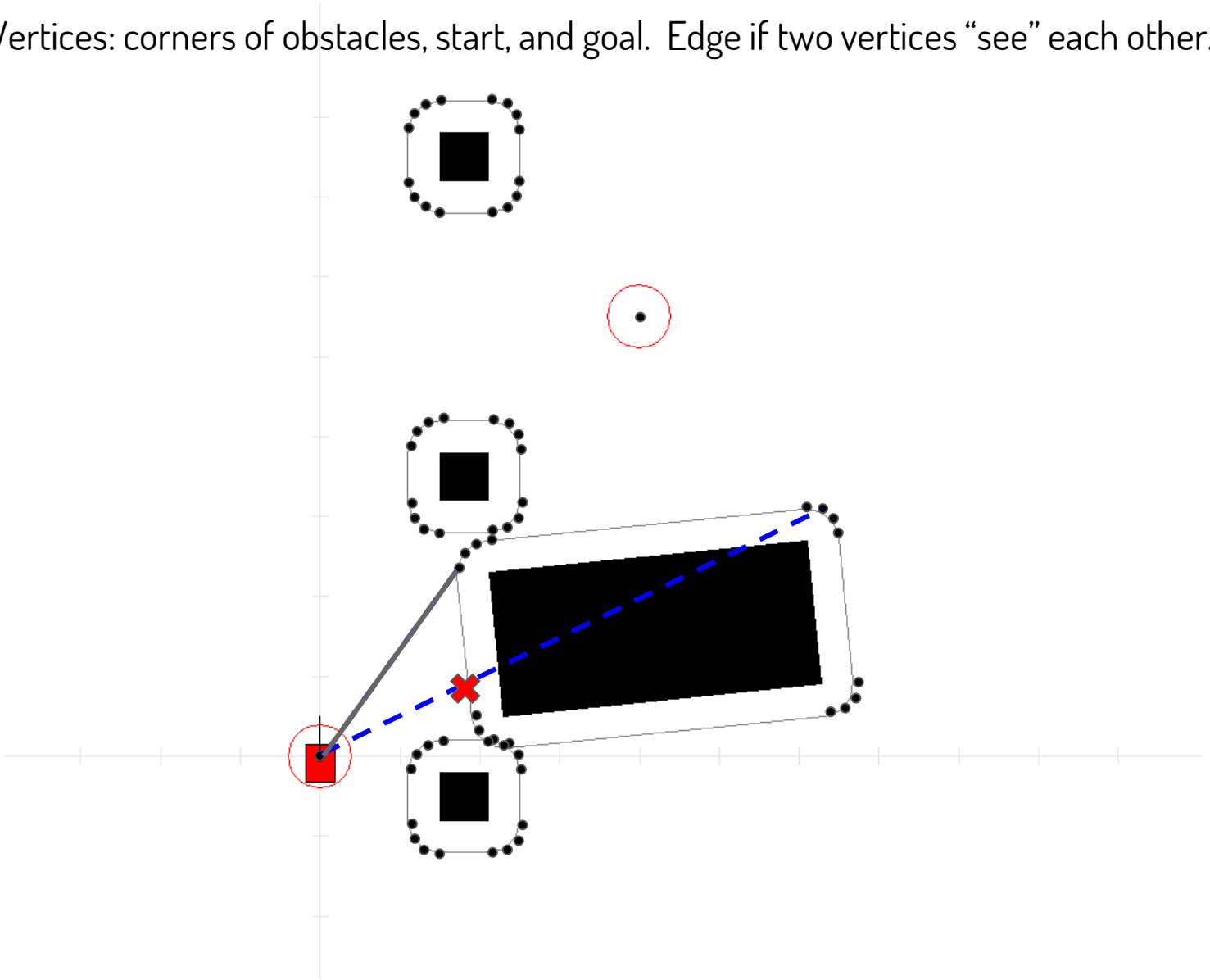
Visibility Graph

- Compute collision-free configuration space



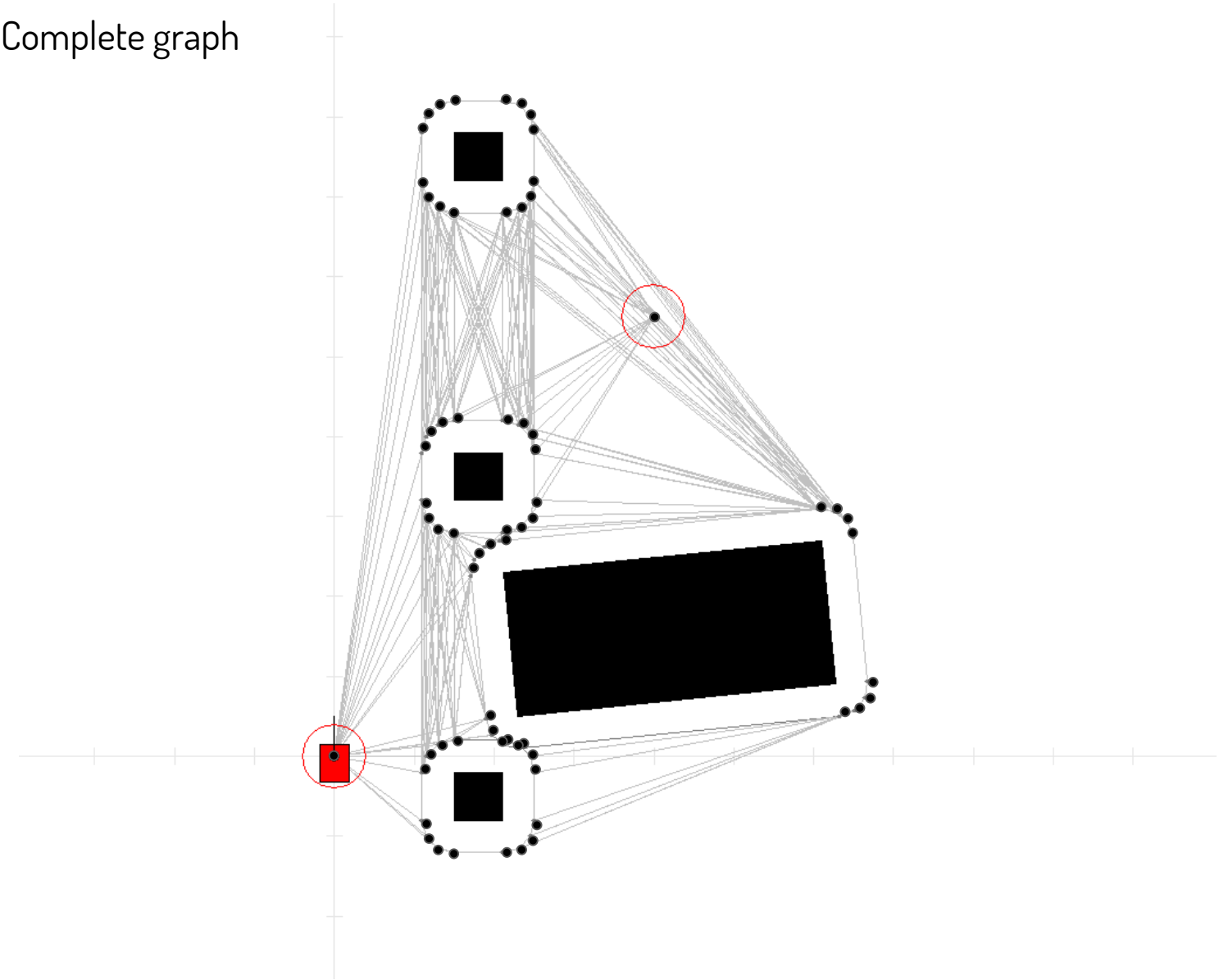
Visibility Graph

- Vertices: corners of obstacles, start, and goal. Edge if two vertices “see” each other.



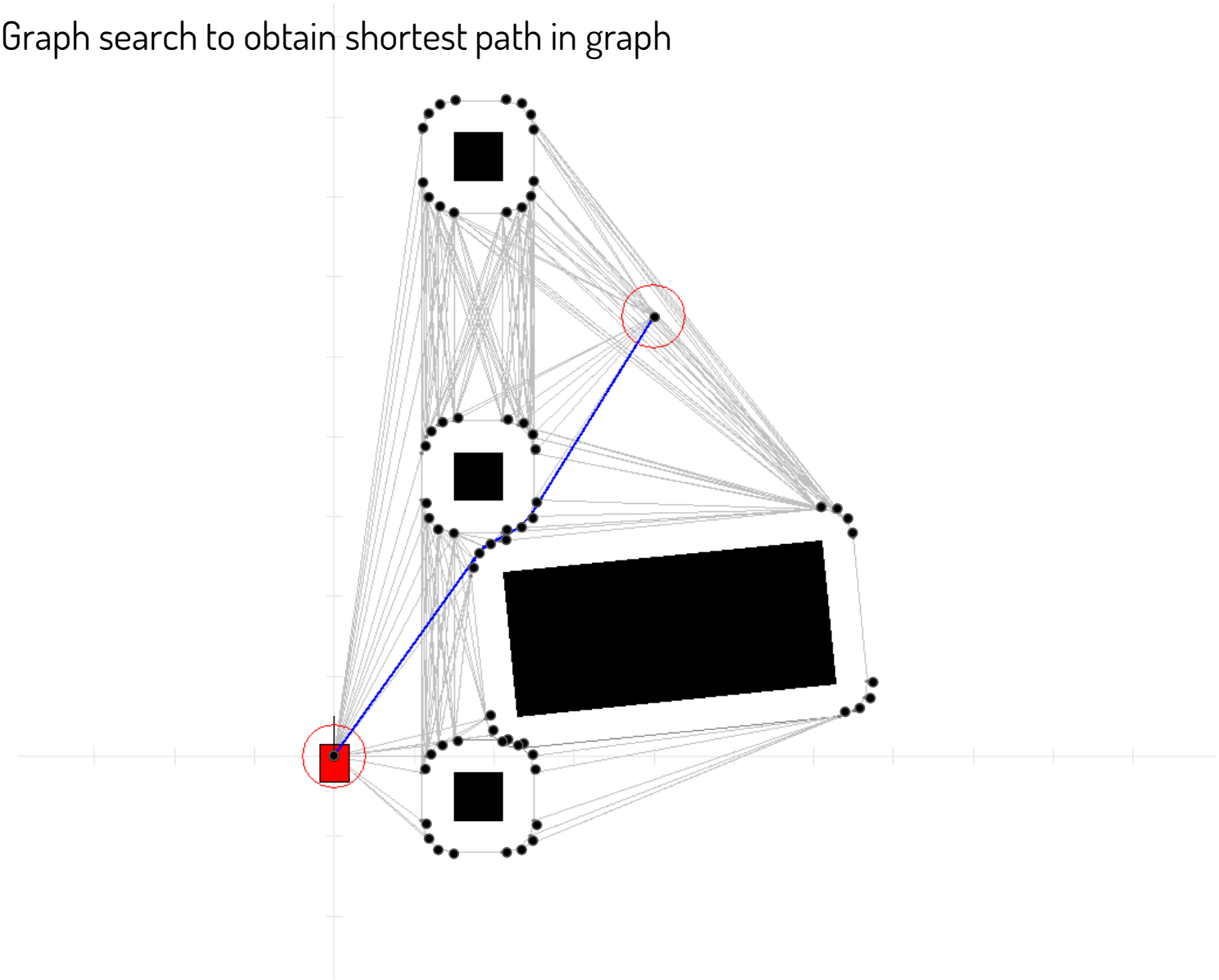
Visibility Graph

- Complete graph



Visibility Graph

- Graph search to obtain shortest path in graph



Visibility Graph

Pros:

- Efficient: $O(n^2)$
- Exact optimal

Cons:

- Optimality guarantee only for 2-d environments and circular robot
- Only for holonomic systems and polygonal models

Solution Techniques for Path Planning Problem

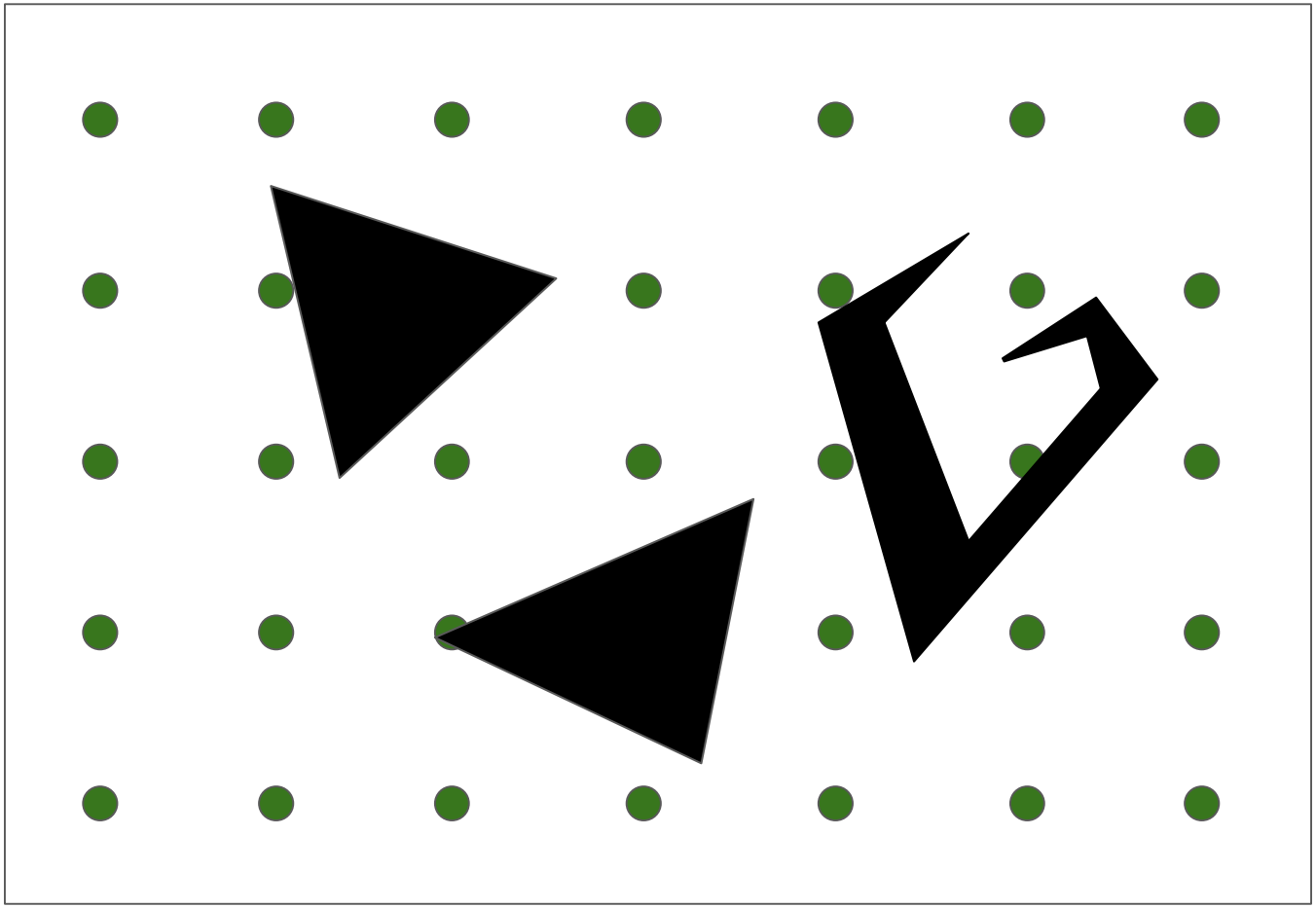
- Variational Methods
- Graph-search Methods
  - Cell decomposition
  - Visibility graph
  - <u>Sampling-based roadmap construction</u>
  - Tree of motion primitives
- Incremental Search Methods
  - RRT: Rapidly-exploring Random Trees
  - RRT*: Optimal Rapidly-exploring Random Trees

Sampling-based Roadmap Construction

Generate roadmap by:

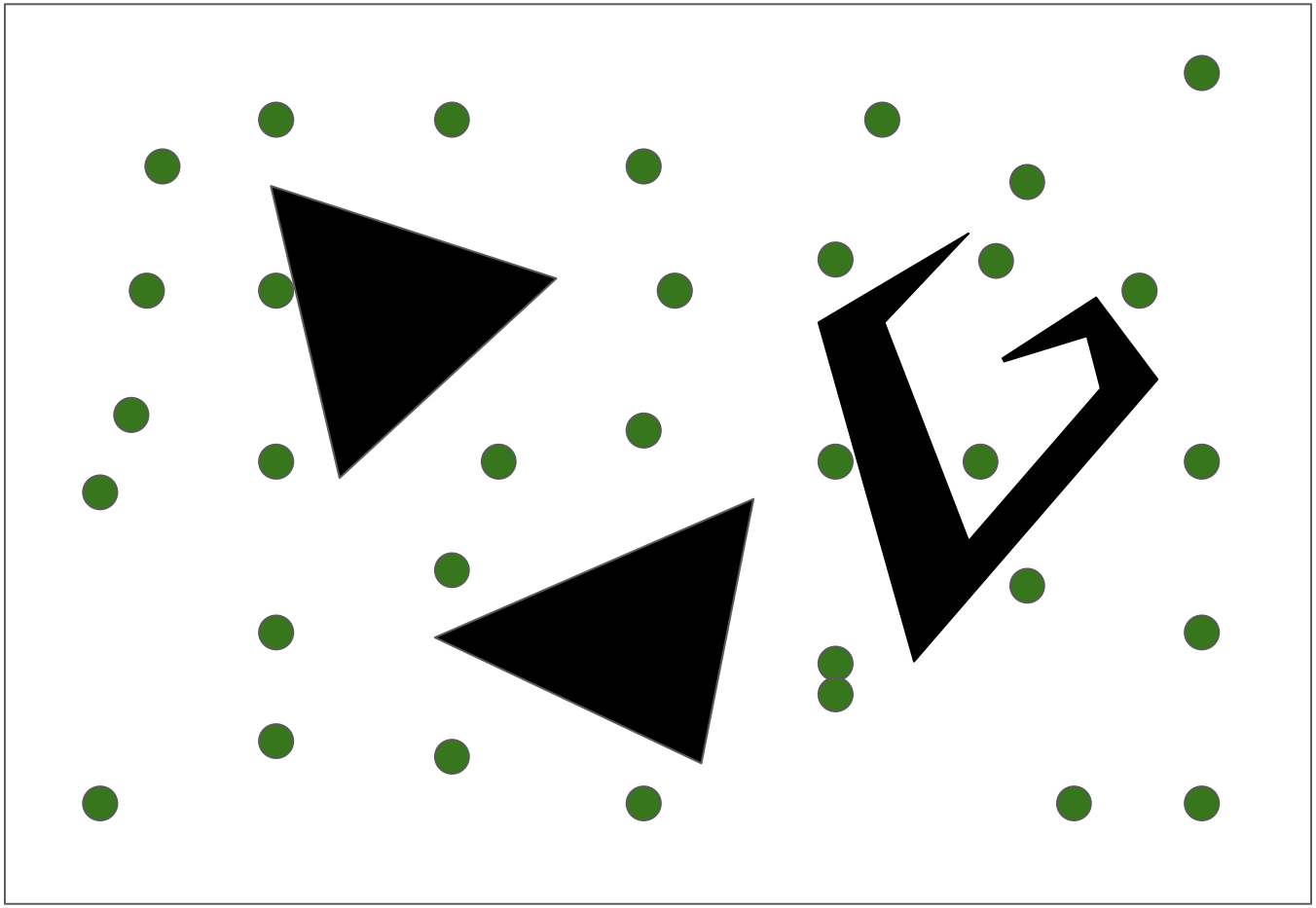
- 1. Sampling the free configuration space
  - Deterministically
  - Randomly
- 2. Connecting nearby samples
  - All neighbors closer than distance r
  - K-nearest neighbors

Sampling-based Roadmap Construction



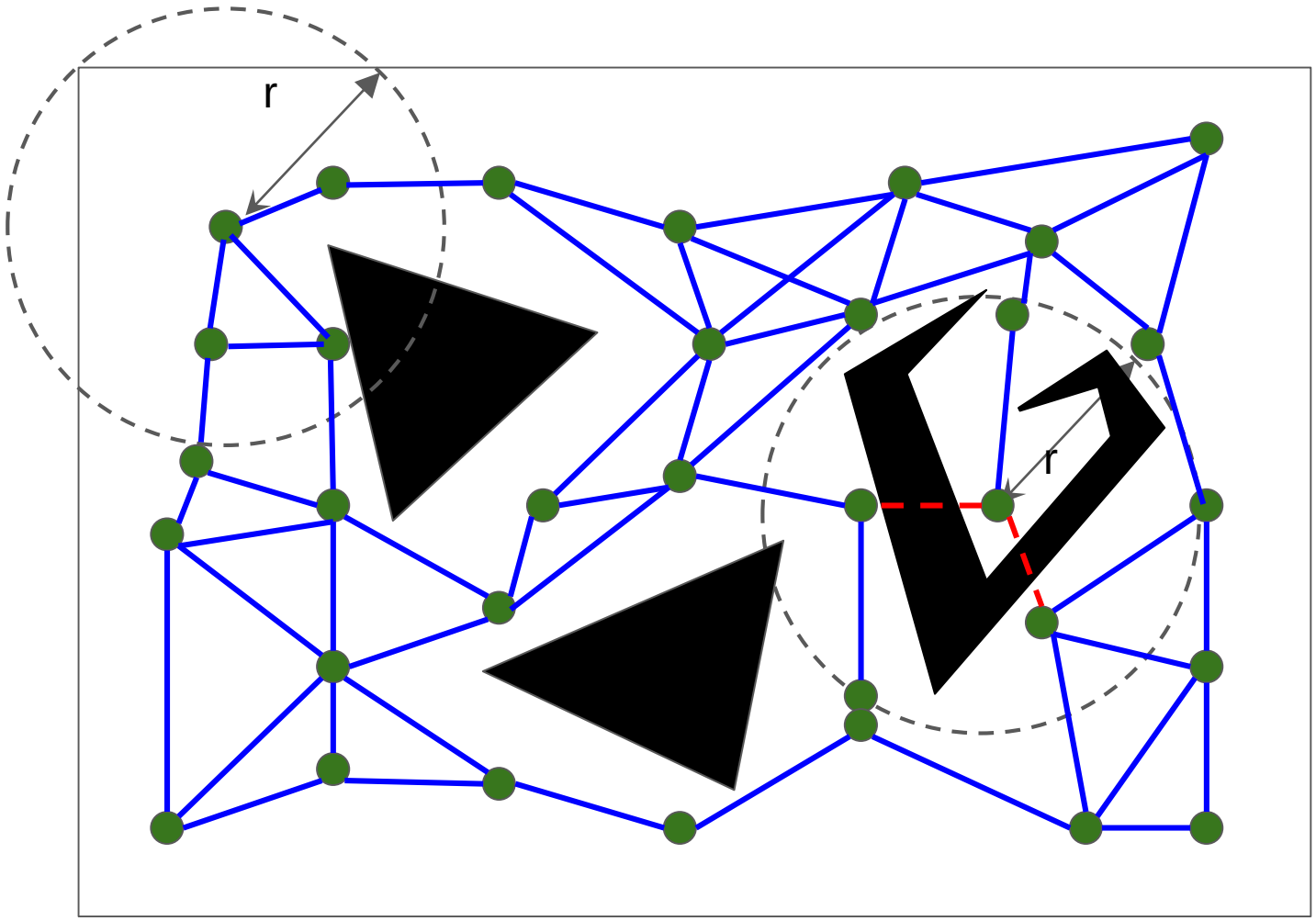
Deterministic Sampling (Sukharev Grid)

Sampling-based Roadmap Construction



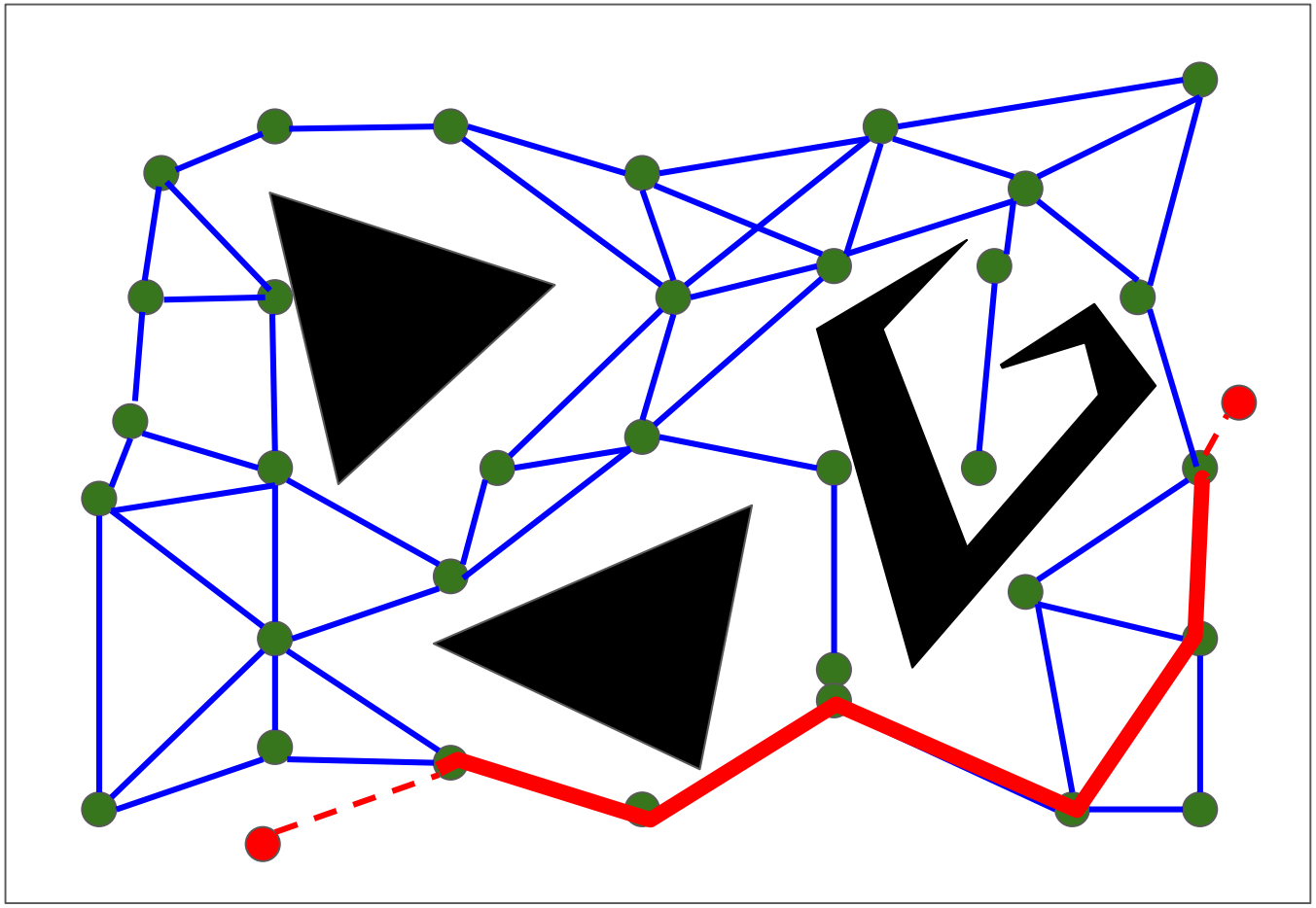
Probabilistic Roadmap

Sampling-based Roadmap Construction



Connecting Samples

Sampling-based Roadmap Construction



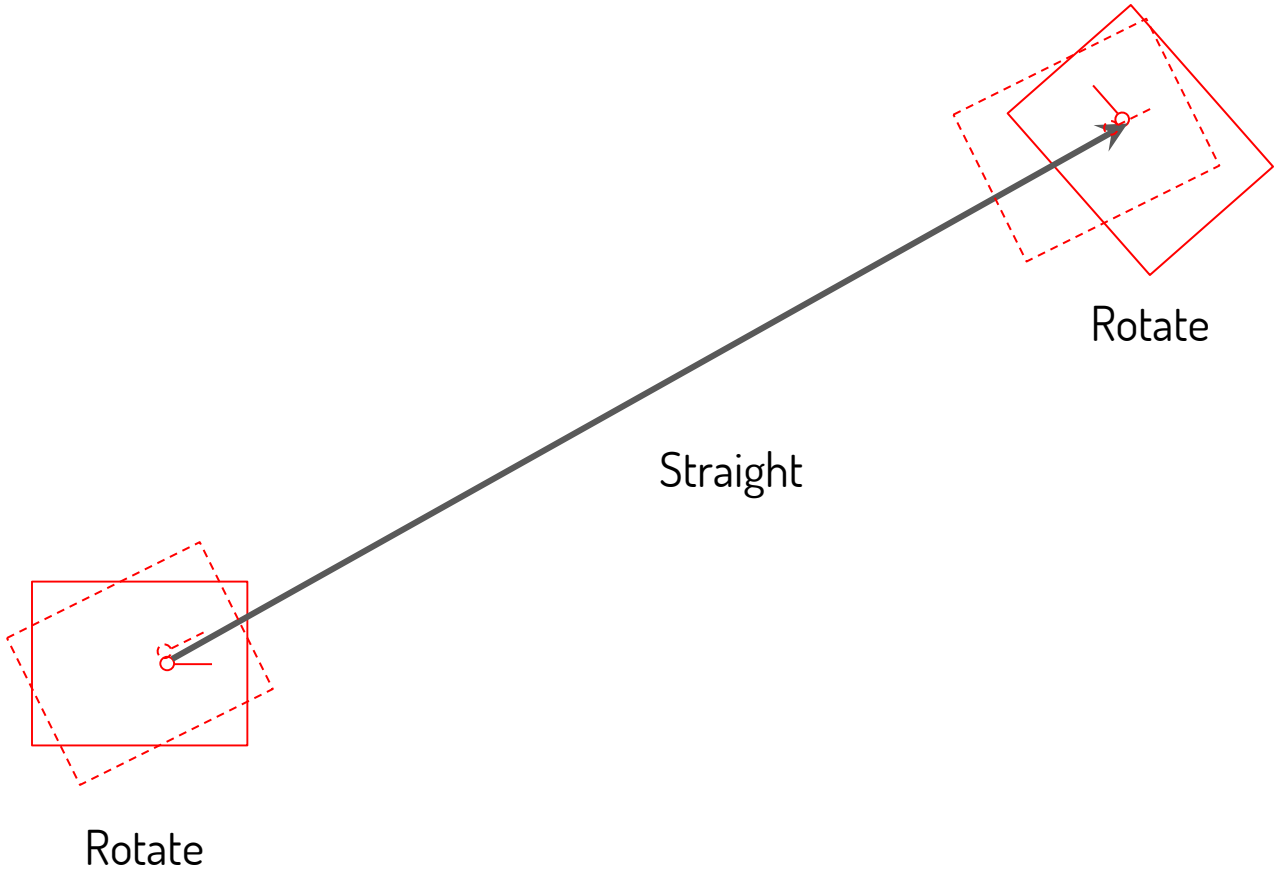
Finding Path

Steering: Connecting the Samples

Sampling-based methods rely on steering function.

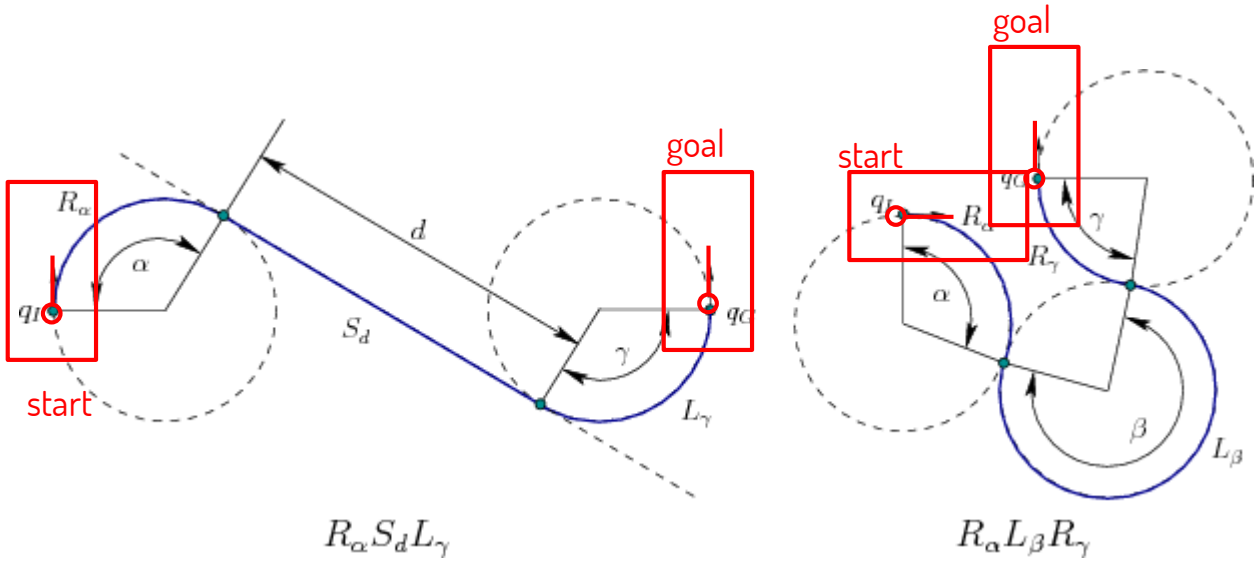
- Steer(**x**,**y**) returns a feasible path segment between configuration **x** and **y**.
- Steering function respects kinematic and dynamic constraints, but **does not consider obstacles.**
- Often obtained by simulating a dynamic model of the vehicle.

Steering for Duckiebot



Dubins Path: Steering for vehicle moving forward

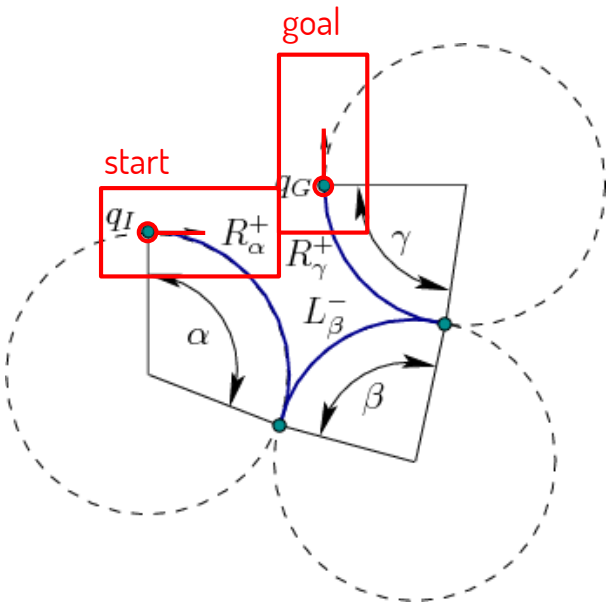
- Car that moves only forward.
- The shortest path for can be computed analytically
- It consist of three path segments: sharpest-possible turn left (L), right (R) or straight (S).
- Total six templates: {LRL, RLR, LSL, LSR, RSL, RSR}



Picture by Steven LaValle.

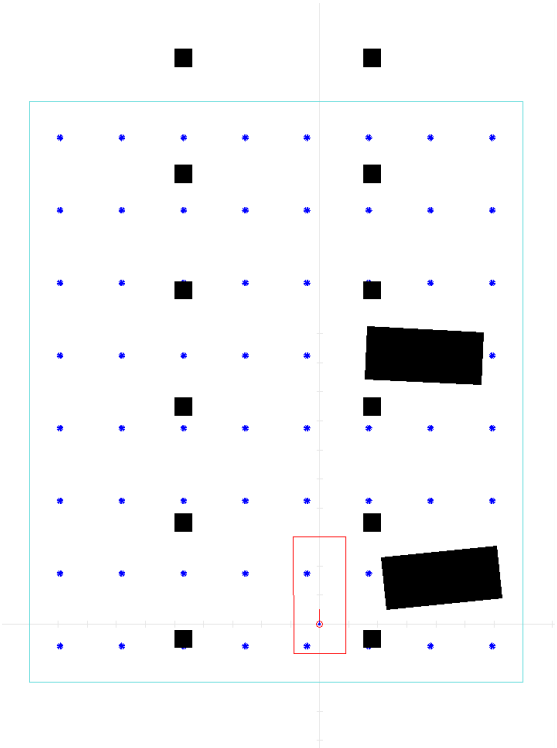
Reeds-Shepp Path: Steering for Car Moving Forwards and Backwards.

- Car that can move both forward and backwards.
- Up to 5 segments: {R+,R-, L+, L-, S+, S-}.
- 46 templates.

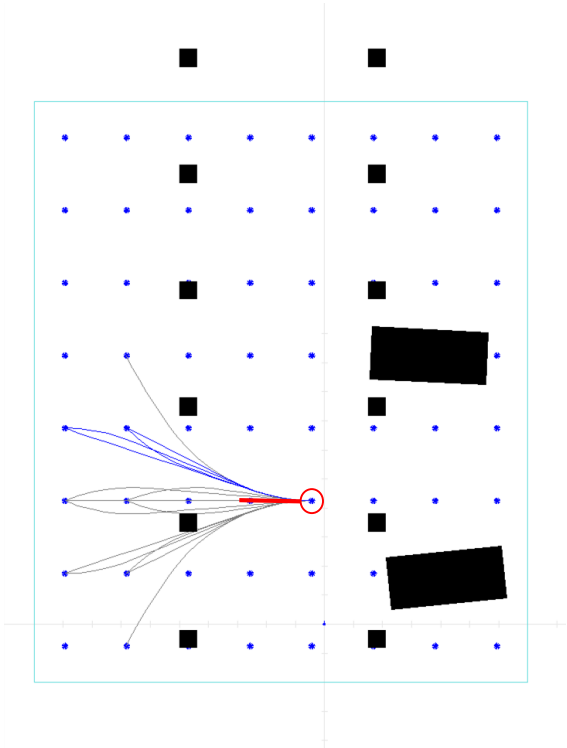


Picture by Steven LaValle.

Sampling-based Roadmap with Dubins Path

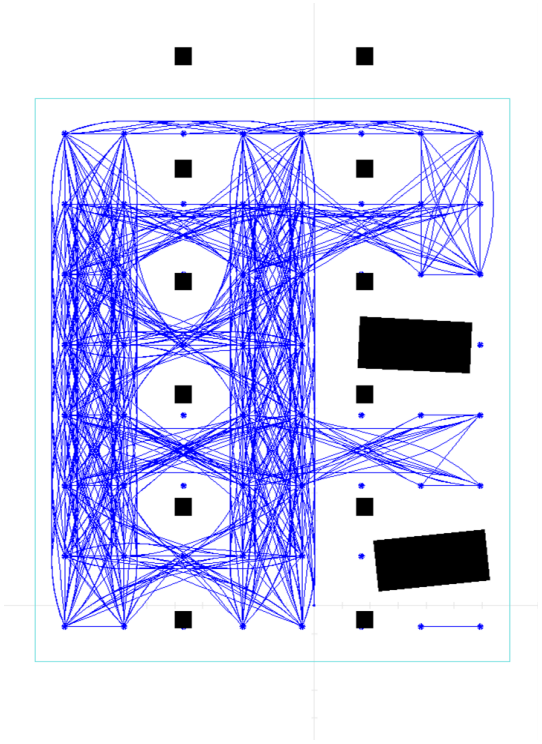


Sample the configuration space.
Here 8 x 8 x 16 = 1024 regular samples.

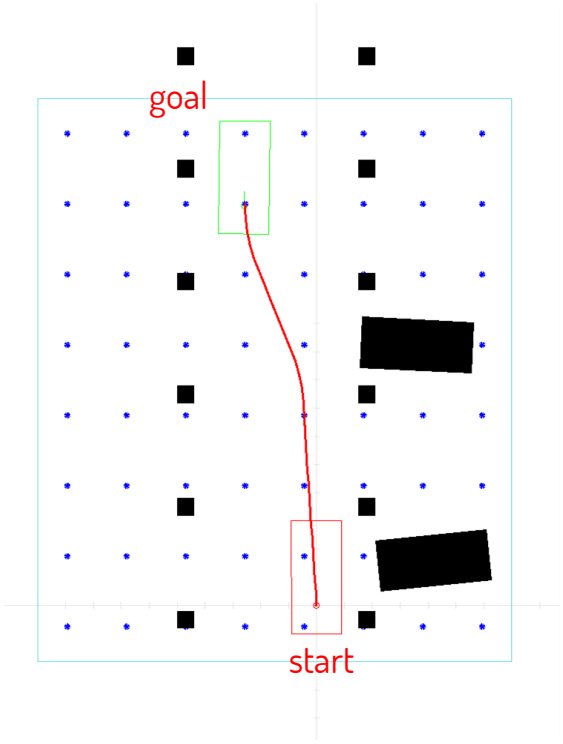


For each sampled configuration, connect neighbors closer than 6m in Dubins distance.
Blue are collision-free connections.

Sampling-based Roadmap Construction



Resulting roadmap



Graph searched using Dijkstra/A*, we obtain a feasible path for the vehicle.

Choosing Connection Radius

- How to choose connection radius?
  - too small: roadmap will be disconnected
  - Too large: to many computationally intensive

- PRM* [Karaman 2011]: for asymptotic optimality, chose connection radius as a function of number of samples of graph:

$$r = \gamma \sqrt[d]{(\log n)/n}$$

  - O(log n) connections attempted at each iteration
  - Maintains asymptotic optimality with O(n log n) complexity

Sampling-based Roadmap Construction

Pros:

- Handle differential constraints
- Model agnostic
- Multiquery
- PRM/PRM* - asymptotic optimality guarantee

Cons:

- Completeness and optimality achieved only up to discretization resolution
- Need exact steering

Solution Techniques for Path Planning Problem

- Variational Methods
- Graph-search Methods
  - Cell decomposition
  - Visibility graph
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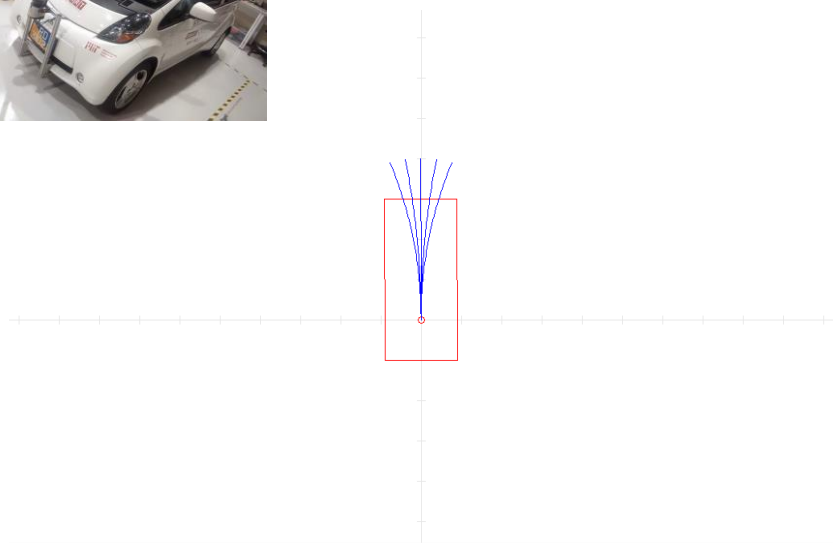
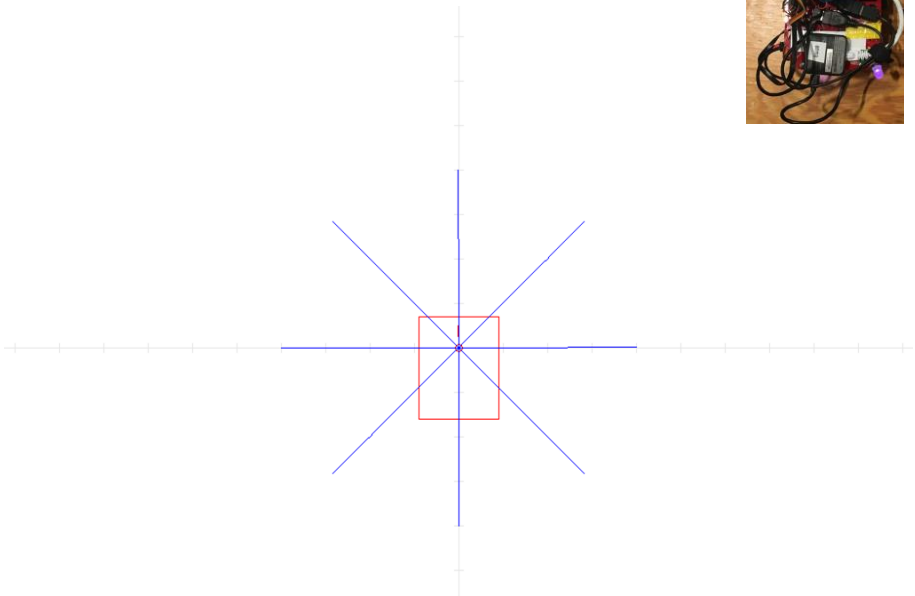
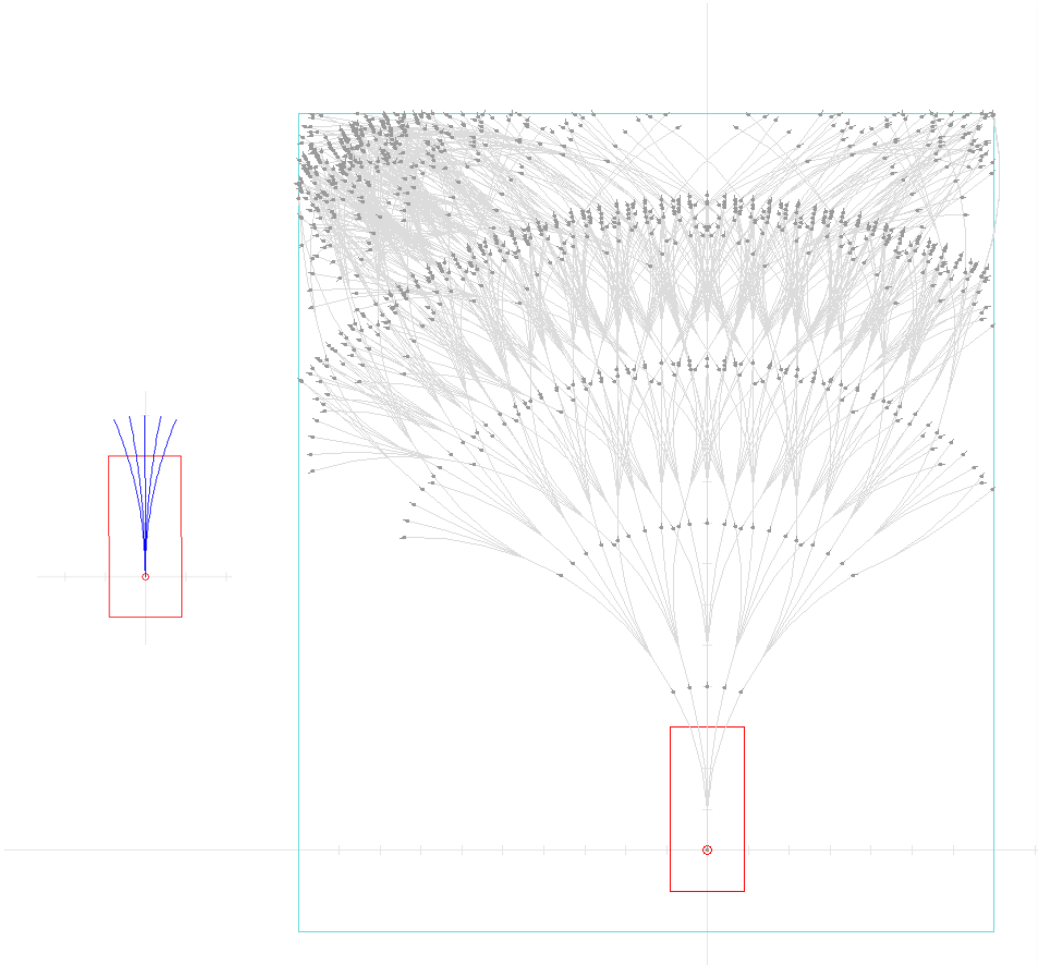
Motion Primitives

A discrete set of of maneuvers that the vehicle can execute from each configuration:

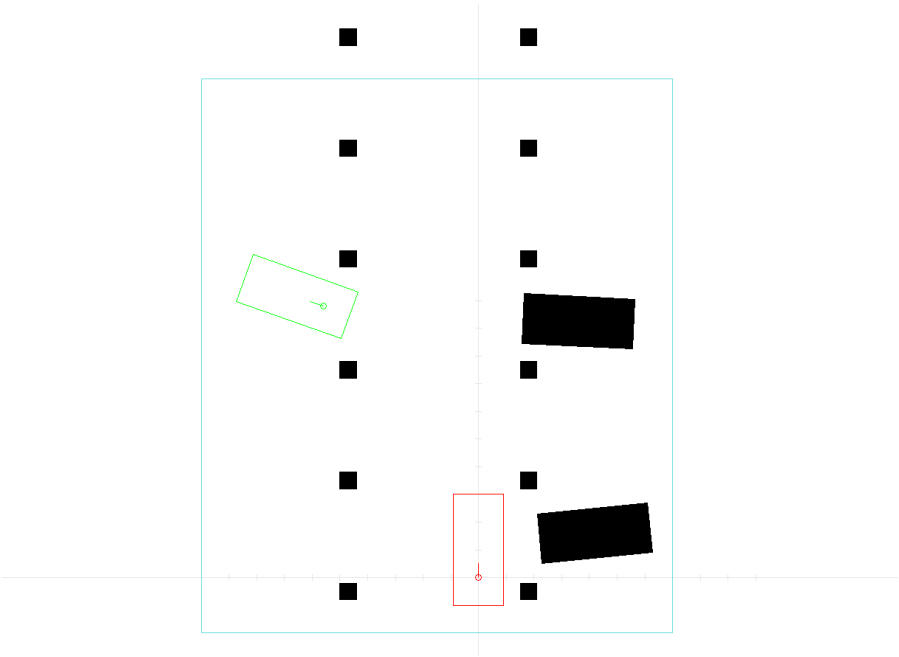




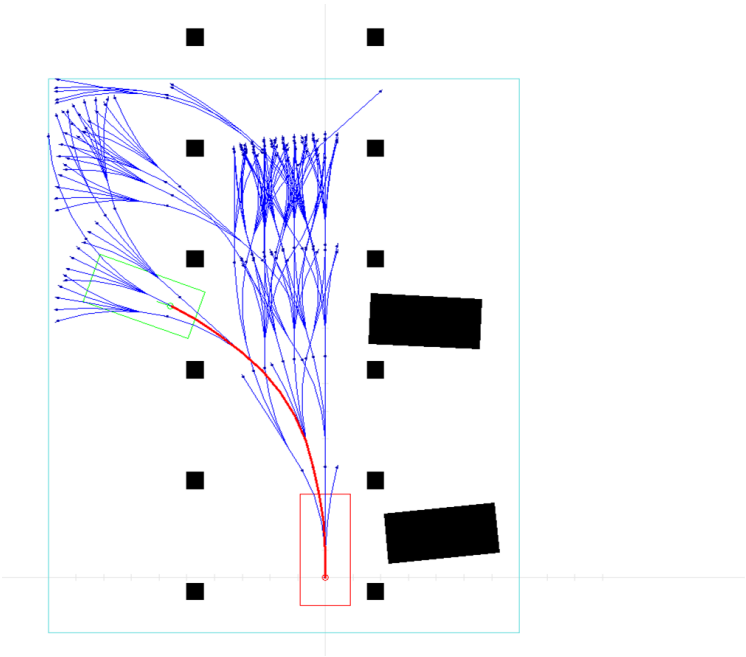
Recursive Application of Motion Primitives



Recursive Application of Motion Primitives



Start expanding motion primitives from current configuration.

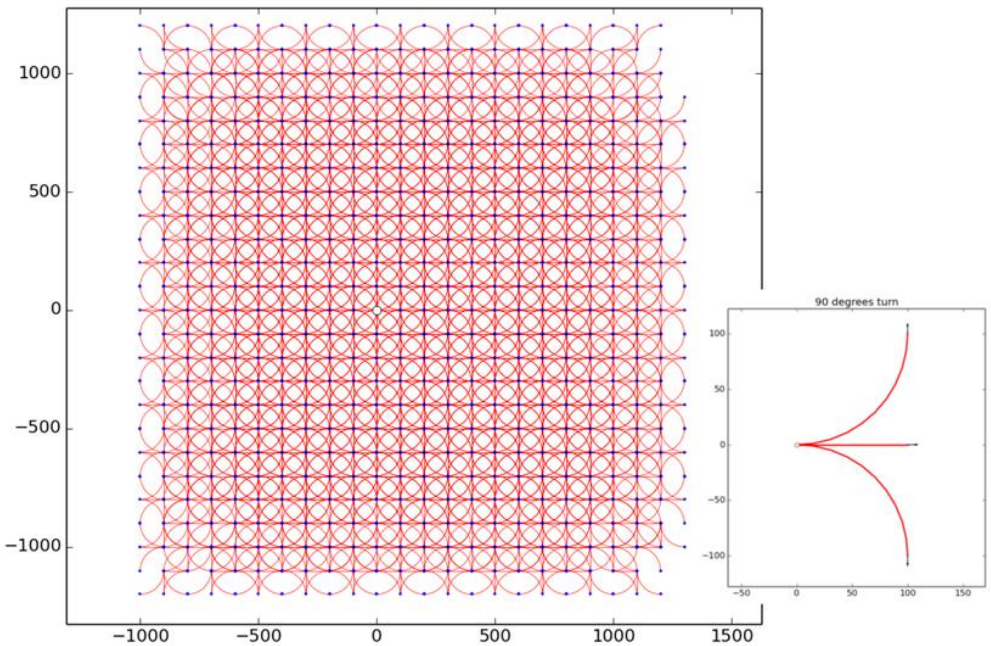


Use Dijkstra/A* to find the shortest path to the desired region in the tree.

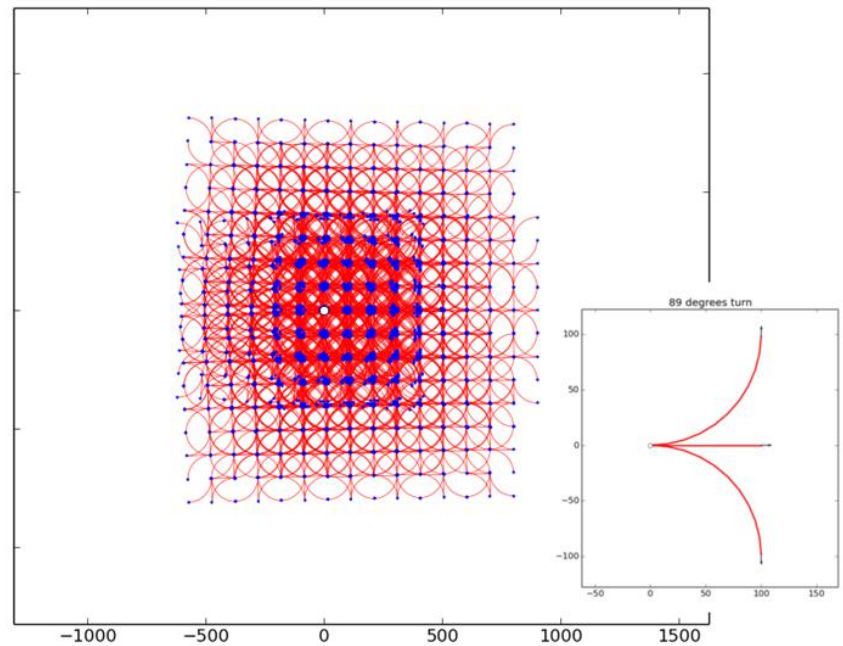
- Can be computed “lazily” during A* search.

Lattice generating motion primitives

- Some motion primitives generate regular lattice.



90-deg turns generate lattice



89-deg turns do not generate lattice.

Motion Primitives

Pros:

- No need for exact steering function
- Can handle differential constraints
- Model agnostic

Cons:

- Completeness and optimality achieved only up to discretization resolution
- Single-query

Notes:

- Used in CMU's Boss and Stanford's Junior during DARPA Urban Challenge

Solution Techniques for Path Planning Problem

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Incremental Search

- Graph-based methods plan on a fixed resolution.
  - => Path might be suboptimal
  - => They may fail to find solution

- Main idea:
  - Incrementally grow a tree rooted at initial configuration to explore the reachable region of the configuration space.
  - Once first branch reaches goal region, return the branch as the first solution.
  - Keep reporting the shortest branch found so far

- Anytime

Solution Techniques for Path Planning Problem

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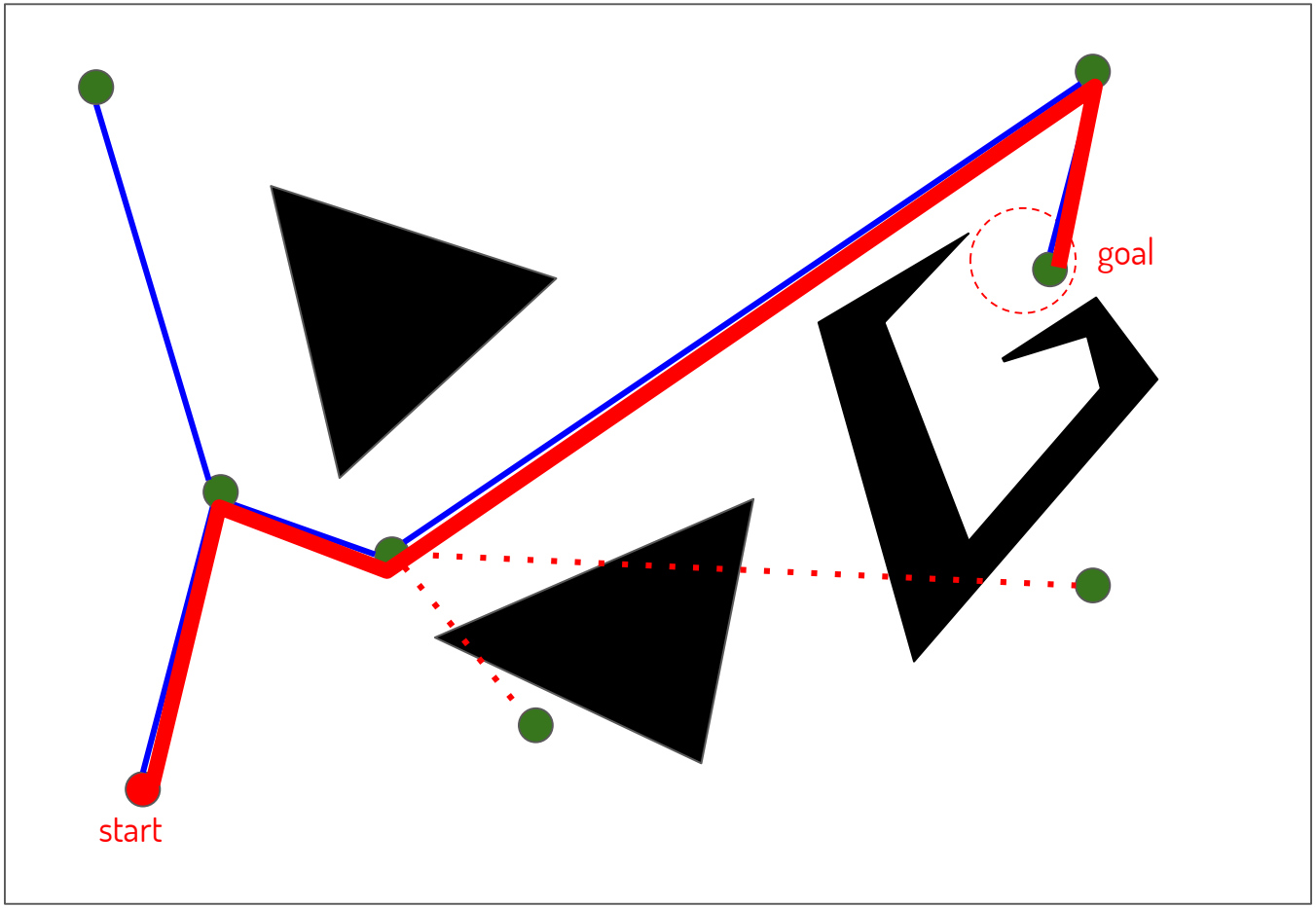
Rapidly-exploring Random Tree (RRT)



Rapidly-exploring Random Tree (RRT)

Pros:

- Anytime
- Handles differential constraints
- Does not need exact steering
- Probabilistic completeness guarantee (shown for some variants of the algorithm)
- Demonstrated good performance in high-dimensional systems

Cons:

- Suboptimal
- Single-query

Notes:

- Used in MIT Talos Urban Challenge Vehicle

Solution Techniques for Path Planning Problem

- Variational Methods
- Graph-search Methods
  - Cell decomposition
  - Visibility graph
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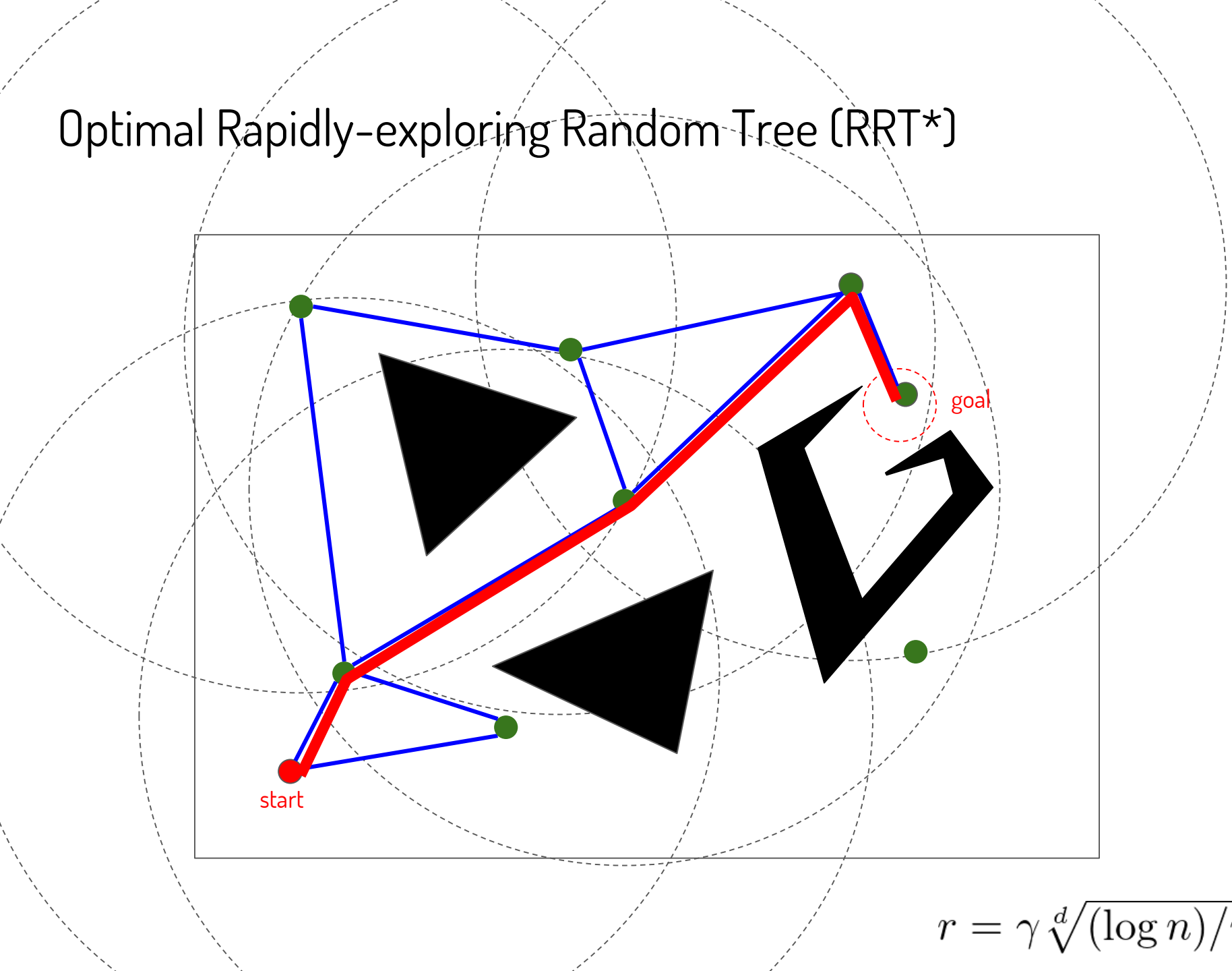
- Incremental Search Methods
  - RRT: Rapidly-exploring Random Trees
  - RRT*: Optimal Rapidly-exploring Random Trees

Optimal Rapidly-exploring Random Tree (RRT*)



$$r = \gamma \sqrt[d]{(\log n)/n}$$

Optimal Rapidly-exploring Random Tree (RRT*)

Pros:

- Anytime
- Asymptotic optimality/Probabilistic completeness guarantee
- Can handle differential constraints

Cons:

- Requires exact steering
- Single-query

Summary

- Motion planning is needed in complex driving situations
- Path Planning vs. Trajectory Planning
- Different solution approaches
  - Variational
  - Graph-based
  - Incremental