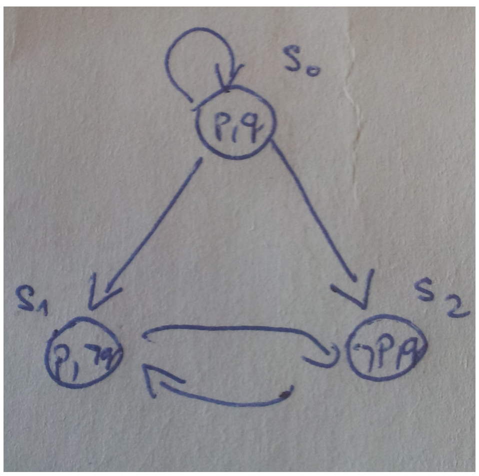
Modal logic

- informally, a logic which contains a modality, or several modalities
- a modal logic can be described with the notion of possible worlds
    - M = (W, R, V)
        - W - a set of possible worlds (states of the system)
        - R - accessibility relation (w, w’ \in R, if world w’ is reachable (or it is considered possible) from state w), we can denote this as (w R w’)
        - V - evaluation of propositions in each world of W
    - basic modality: $w \models \Box p$ -> p is necessary true if and only if for every w’, such that w R w’, it holds that $w' \models p$
        - this for example means that in every other world state that I consider possible, p must hold
    - we can pose some restrictions on the properties of relation R
        - reflexive
        - symmetric
        - transitive
        - serial
        - euclidean
    - different modal logics apply different restrictions on the relation R
        - e.g., in modal logic representing knowledge (relation R represents that agent i knows something - K_i) uses reflexive, symmetric, transitive and euclidean assumptions
        - temporal logics have serial restriction (there is always a next state)
    - different restrictions cause different axioms to hold
        - e.g., if there is no symmetric restriction, an agent can believe that *p* is true, although it could be the case that *p* does not hold in the particular state -- i.e., the agents beliefs can be incorrect
- an example with cards
    - we have 3 players (A,B,C), 8 cards = 4xA, 4x8
    - each player receives two cards and places them on her forehead without looking at them
    - each player is able to observe the cards of the other players
    - players take turns and they try to determine what cards they have on their foreheads
    - we assume all players are rational, they do not take guesses, and they do not lie
    - an example:
        - you are player C, you observe AA, 88; it is your turn (both players A and B have said that they did not know) -- can you determine what your cards are?
    - modeling the dynamic of this game using modal logic and possible worlds
        - the player C considers possible states (AA, 88, AA) - (AA, 88, 88) - (AA, 88, 8A)
        - without any other information the player C cannot determine what cards are on her forehead
        - when player A says that she does not know, player C can remove the state (AA, 88, 88) from the state of possible worlds -- if this would have been the case, the player A could have surely determined the cards on her forehead (she would have observed 88, 88 -- there is no other possibility than having AA), but A has said “I do not know”, therefore this state is no longer possible.

- example with a robot -- Where is waldo?
  - we can use temporal logic to describe behavior/dynamics of an agent or a system, but also e.g., the goals
  - we need to specify the states, sensors (r_1 … r_4 -> in which region the robot is, s_w -> whether the robot senses waldo)
  - we can specify the goal as follows: □◊(r_2 OR s_w) AND □◊(r_4 OR s_w)
    - the robot has to look for Waldo repeatedly in regions 2 and 4 until it sees Waldo
    - □◊p - this means that p will hold infinitely often in temporal logics
  - we can use model checking tools to verify whether our robot based on some simple rules can actually accomplish the goal
  - alternatively, there are approaches that could generate behavior of agent/robot based on the specification of the environment and goals in (restricted variant of) temporal logic
- temporal logic
  - modalities refer to the possible worlds in the future
- example
  - consider a following transitions (we start at state S0)
  - 

  - write down the beginning of the (infinite long) runs of the system
  - which of the following formulae hold in state S0?
    - EG(p & q)
    - AG(p & q)
    - AX p
    - AG (p v q)
    - EX p
    - EF (p v !q)
    - AF (p v !q)
    - EFG p
    - AGF p