Solving Normal-form Games

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The story so far...

- Several examples of normal-form games
- Computation of pure strategy Nash equilibrium
- Social welfare
- Pareto optimality
- Domination
  - Prisonner's dilemma from lecture

What is next

- Computation of mixed strategy Nash equilibrium
- Minimax, Maximin
- Properties of Nash equilibria in various classes of games
- Computation of Nash equilibrium in practice

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Strategies

- A pure strategy s_i in normal-form games represents the choice of specific action $a \in A_i$ for player i
- A mixed strategy σ_i is a strategy distribution over pure strategies
- Strategy profile P is a set of pure/mixed strategies, one for every player
- Overloading of utility function $u(s_i, s_{-i})$, $u(\sigma_i, \sigma_{-i})$, $u(P)$

Nash equilibrium

- A strategy σ_i^* is the best response to strategies σ_{-i}, written as $\sigma_i^* \in BR(\sigma_{-i})$ iff

$$\forall \sigma_i \in \Sigma_i : u_i(\sigma_i^*, \sigma_{-i}) \geq u_i(\sigma_i, \sigma_{-i}) \tag{1}$$

- Nash equilibrium
  - Strategy profile $P = \{\sigma_1, ..., \sigma_n\}$ is a Nash equilibrium iff

$$\forall i \in N : \sigma_i \in BR(\sigma_{-i}) \tag{2}$$

- Stable against deviations of players as every player plays his best response to the strategies of the rest
- Assumes self-interested rational players
- Every finite game has a non-empty set of Nash equilibria

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What is next

- Computation of mixed strategy Nash equilibrium
- Minimax, Maximin
  - In general not equilibrium but prescription of behavior for one player
- Properties of Nash equilibria in various classes of games
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LP for solving zero-sum NFG

$$min\ U_{-i} \tag{3}$$

$$s.t. \sum_{a_i \in A_i} u_{-i}(a_i, a_{-i})\sigma_i(a_i) \leq U_{-i}, \quad \forall a_{-i} \in A_{-i} \tag{4}$$

$$\sum_{a_i \in A_i} \sigma_i(a_i) = 1 \tag{5}$$

$$\sigma_i(a_i) \geq 0, \quad \forall a_i \in A_i \tag{6}$$