OTEVŘENÁ INFORMATIKA

Extensive-Form Games

Branislav Bošanský

AE4M36MAS, Fall 2013, Lecture 5

Game Theory

Games

- Players
- Actions
- States
- Utility
- Knowledge

Different representations

- Normal-form Games
- Extensive-form Games
- Stochastic Games

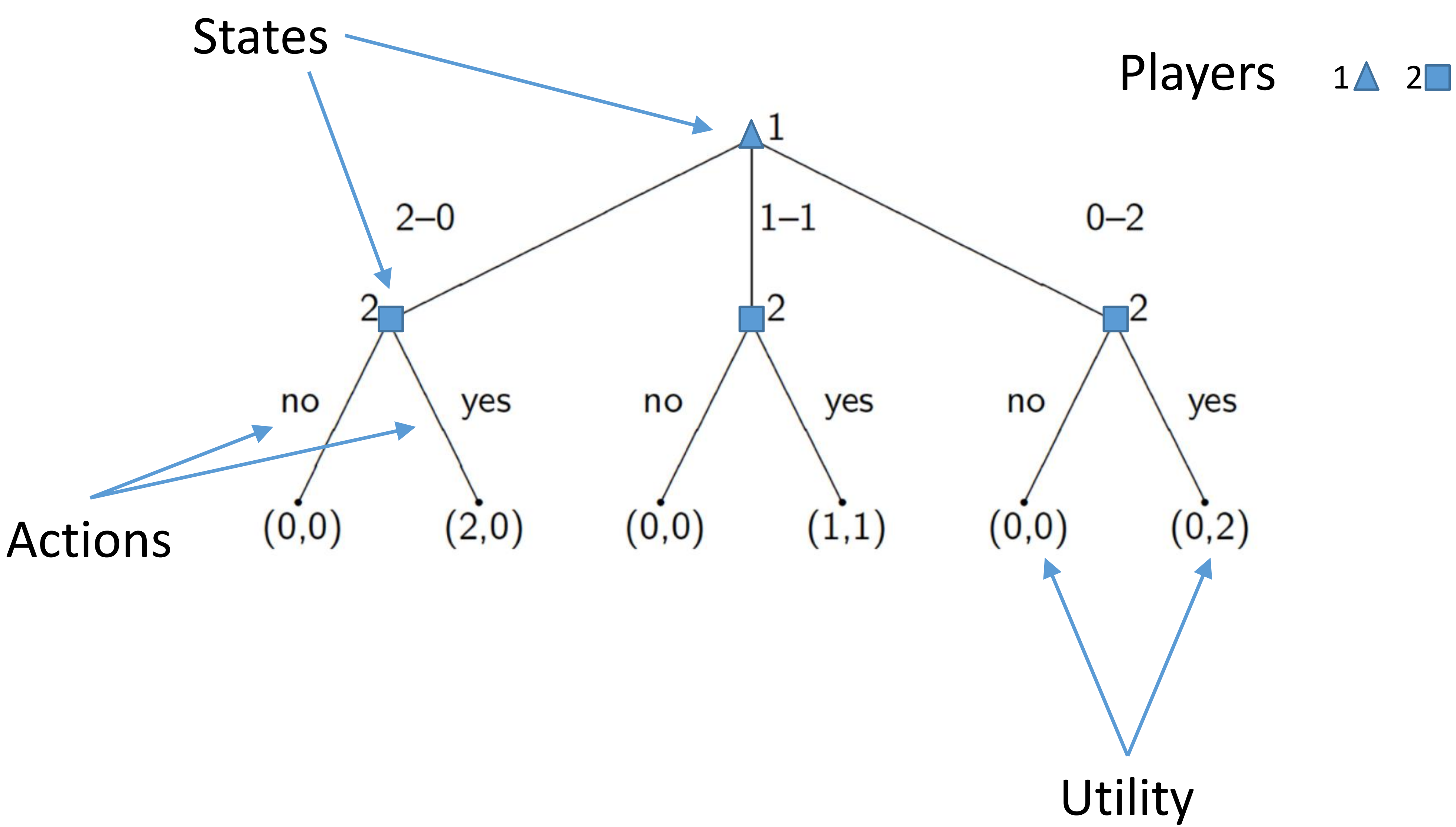
Game Theory: Extensive-Form Games

Different representations

- Normal-form Games
  - one-shot games
  - visually represented as matrix games

- Extensive-form Games
  - sequential games that evolve in time
  - the concept of time is implicitly integrated into the model
  - visually represented as trees

Game Theory: Extensive-Form Games



Game Theory: Extensive-Form Games

Definition:

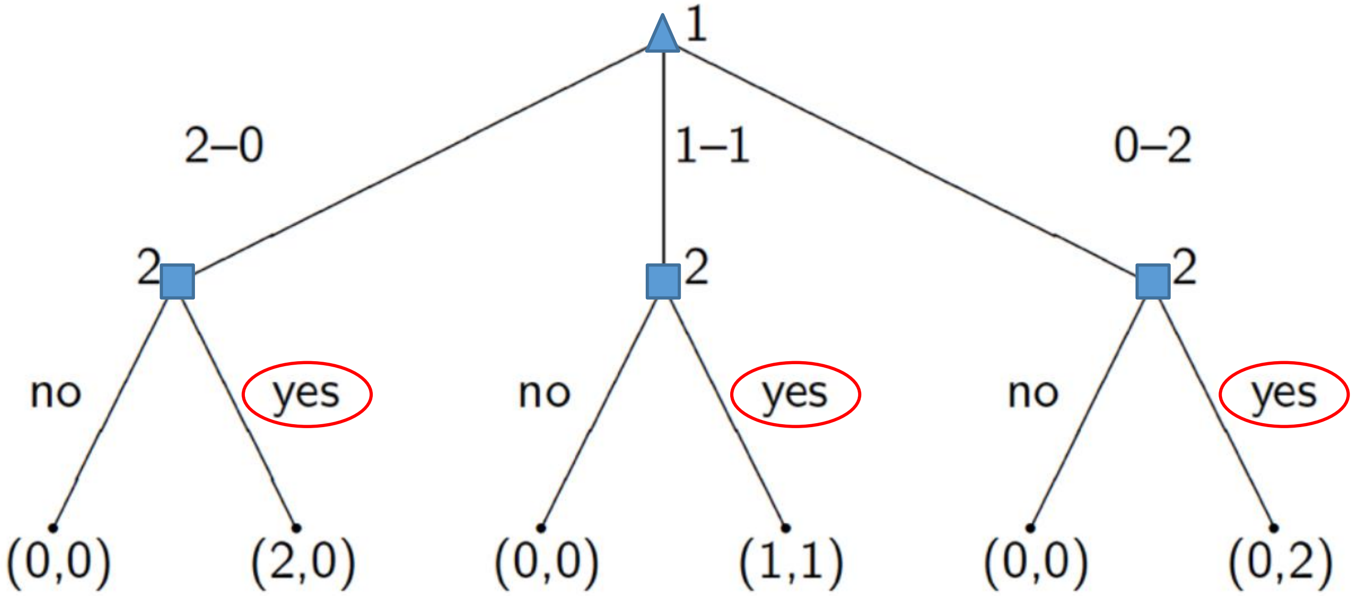
A (finite) perfect-information game in the extensive form is defined as a tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players $N = \{1, 2, \dots\}$
- Actions A
- Choice nodes and label for these nodes
  - Choice nodes: H
  - Action function: $\chi : H \to 2^A$
  - Player function: $\rho : H \to N$
- Terminal nodes Z
- Successor function $\sigma : H \times A \to H \cup Z$
- Utility function $u = (u_1, \dots, u_n)$; $u_i : Z \to \mathbb{R}$

EFGs: Actions and Strategies

Pure strategy: an assignment of an action for each state

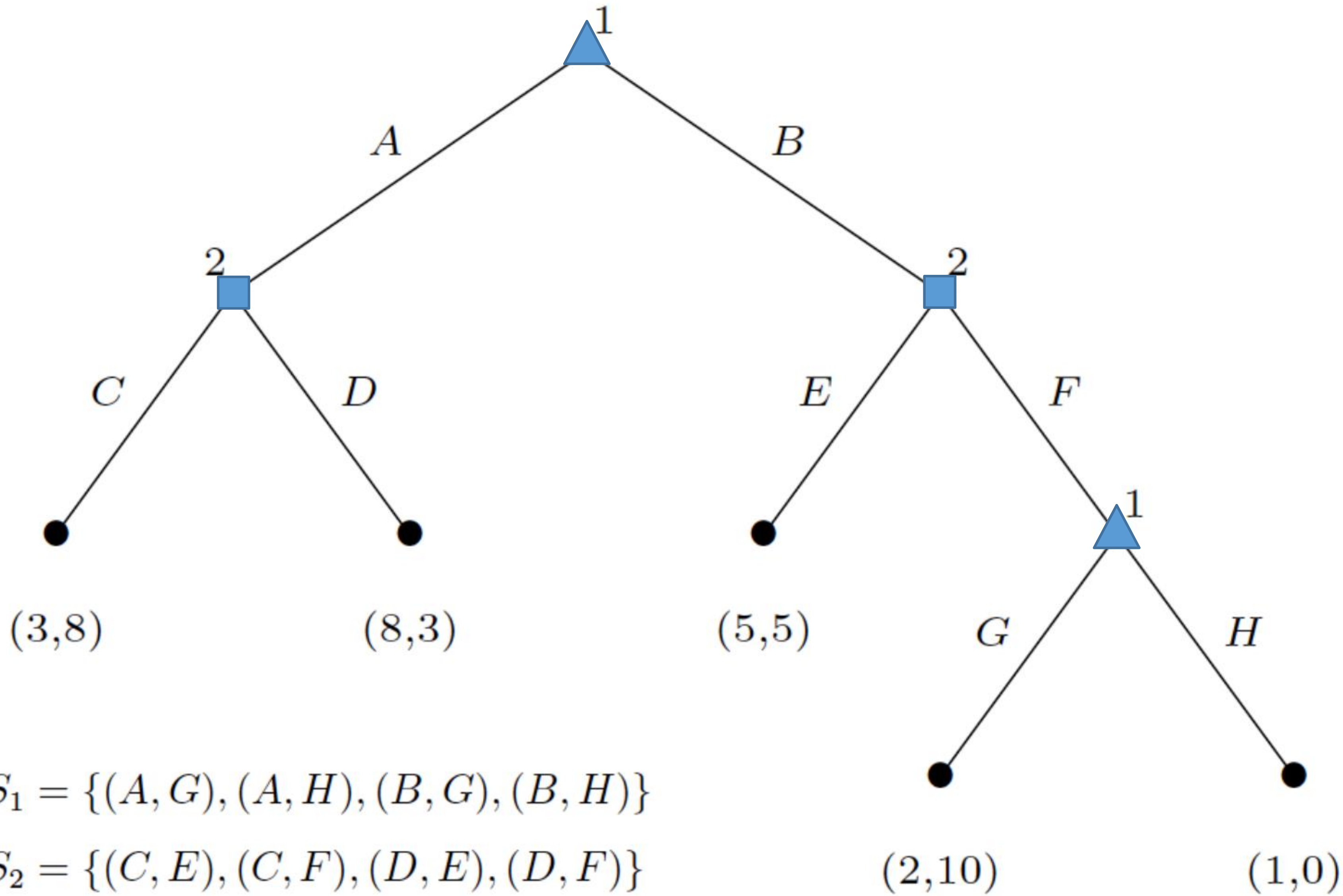
Action is uniquely identified by the state, in which it is taken.



$A_1 = \{2-0, 1-1, 0-2\}$ $A_2 = \{no_{\{2-0\}}, yes_{\{2-0\}}, no_{\{1-1\}}, yes_{\{1-1\}}, no_{\{0-2\}}, yes_{\{0-2\}}\}$

$S_1 = \{2-0, 1-1, 0-2\}$ $S_2 = \{\{no_{\{2-0\}}, no_{\{1-1\}}, no_{\{0-2\}}\}, \{no_{\{2-0\}}, no_{\{1-1\}}, yes_{\{0-2\}}\}, \ldots\}$

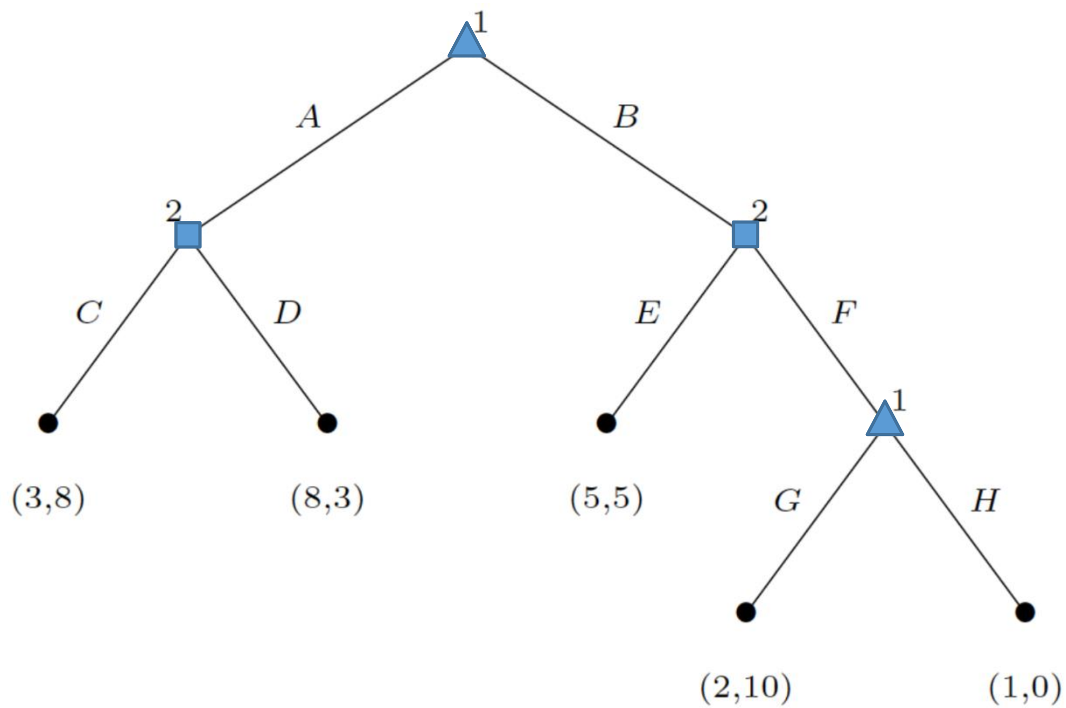
Pure Strategies in EFGs



$S_1 = \{(A, G), (A, H), (B, G), (B, H)\}$

$S_2 = \{(C, E), (C, F), (D, E), (D, F)\}$

Induced Normal Form

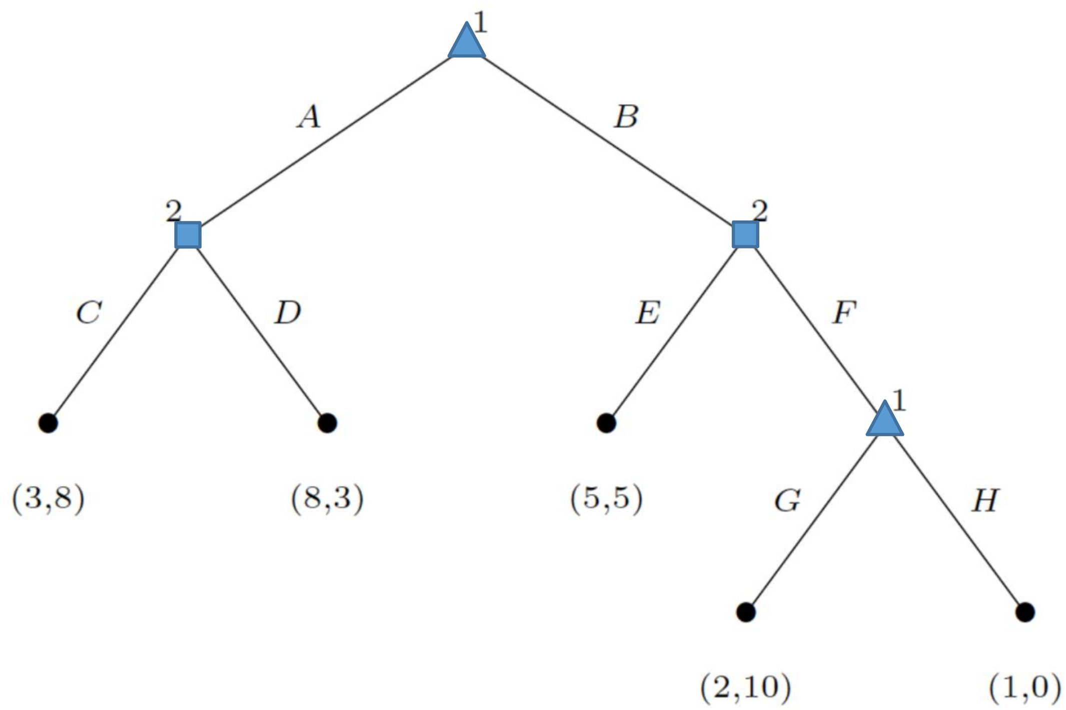


$S_1 = \{(A,G),(A,H),(B,G),(B,H)\}$

$S_2 = \{(C,E),(C,F),(D,E),(D,F)\}$

	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3, 8	3, 8	8, 3	8, 3
(A,H)	3, 8	3, 8	8, 3	8, 3
(B,G)	5, 5	2, 10	5, 5	2, 10
(B,H)	5, 5	1, 0	5, 5	1, 0

Nash Equilibrium in EFGs



$$S_1 = \{(A, G), (A, H), (B, G), (B, H)\}$$

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	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3, 8	3, 8	8, 3	8, 3
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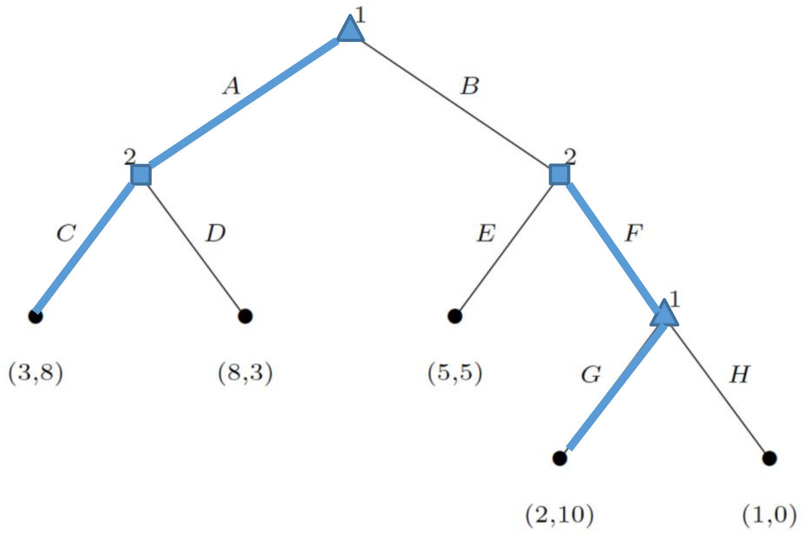

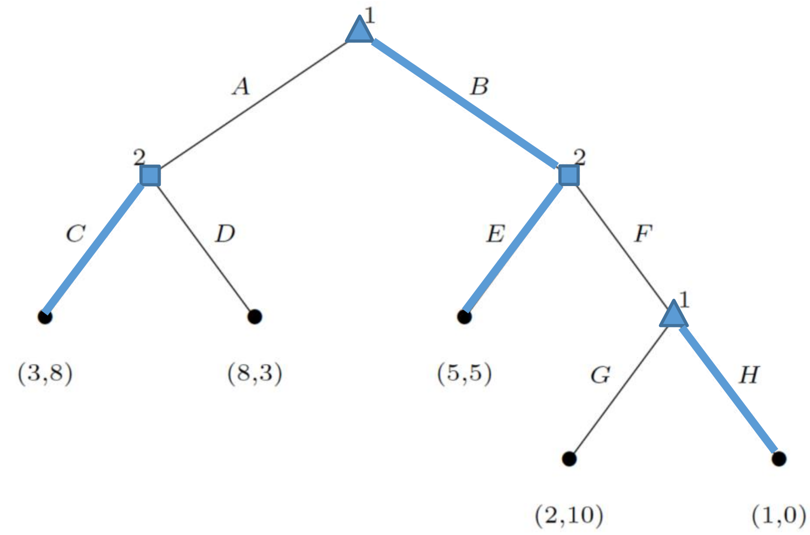

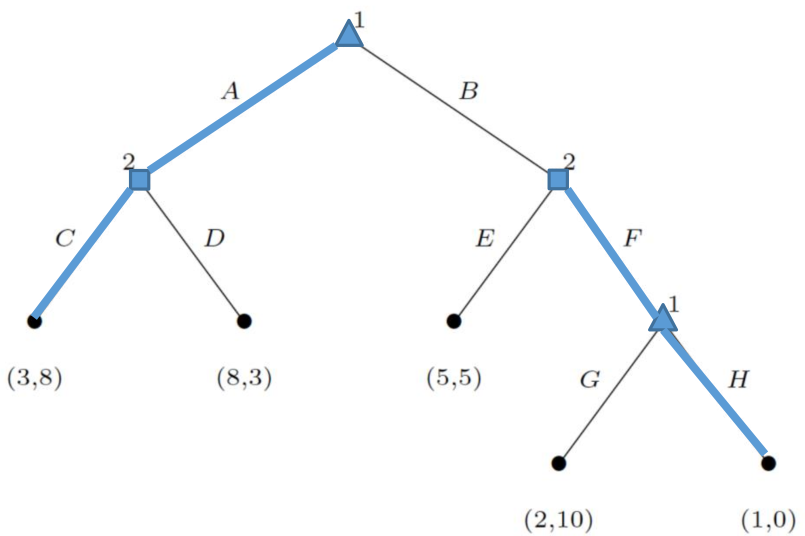

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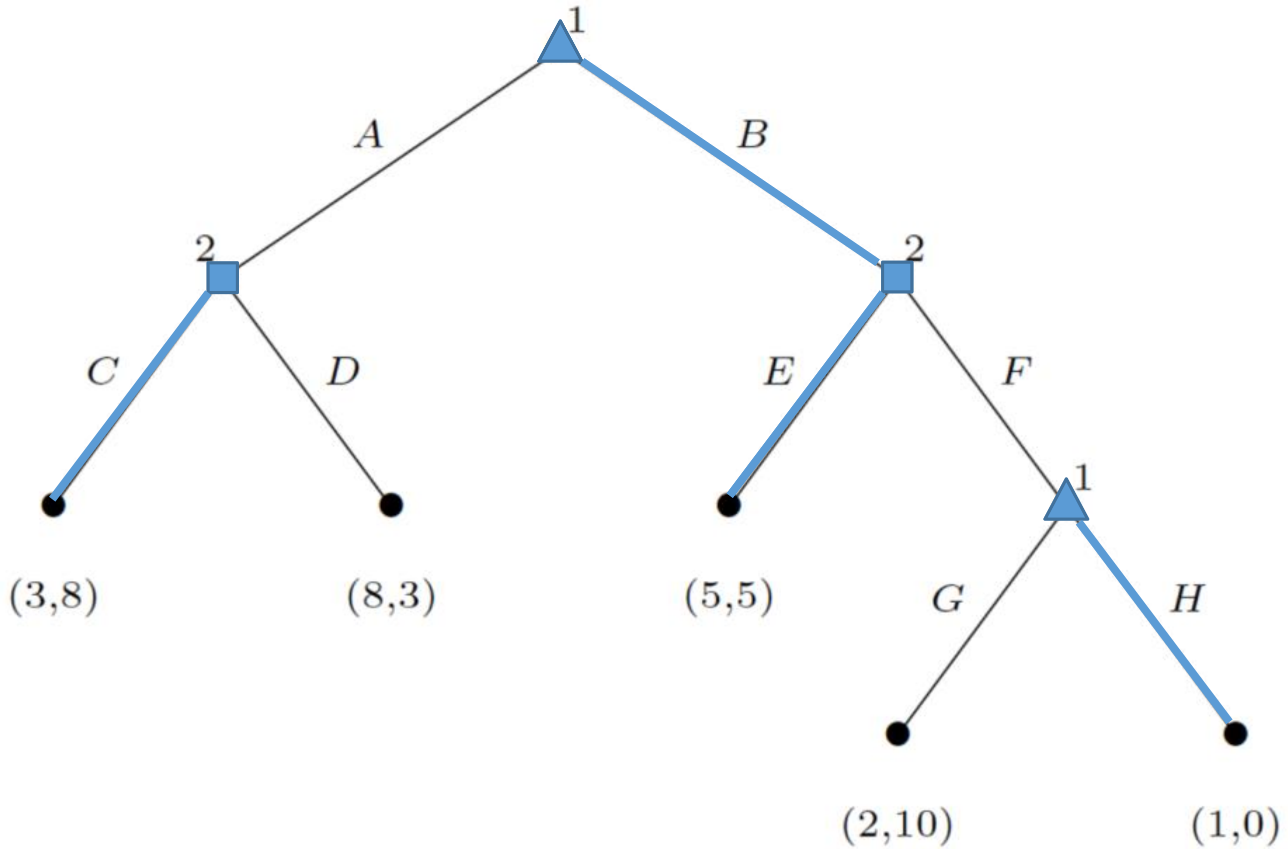
Nash Equilibrium in EFGs - threats



	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3, 8	3, 8	8, 3	8, 3
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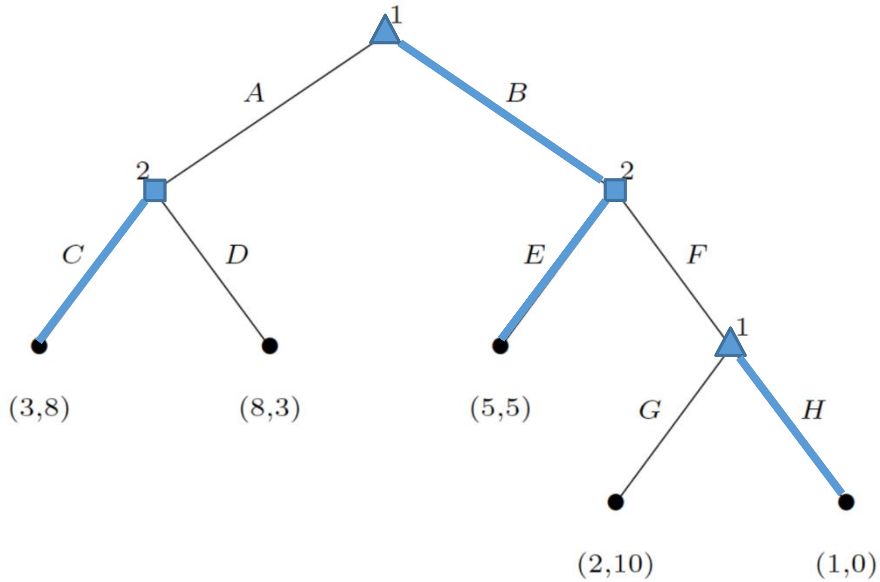
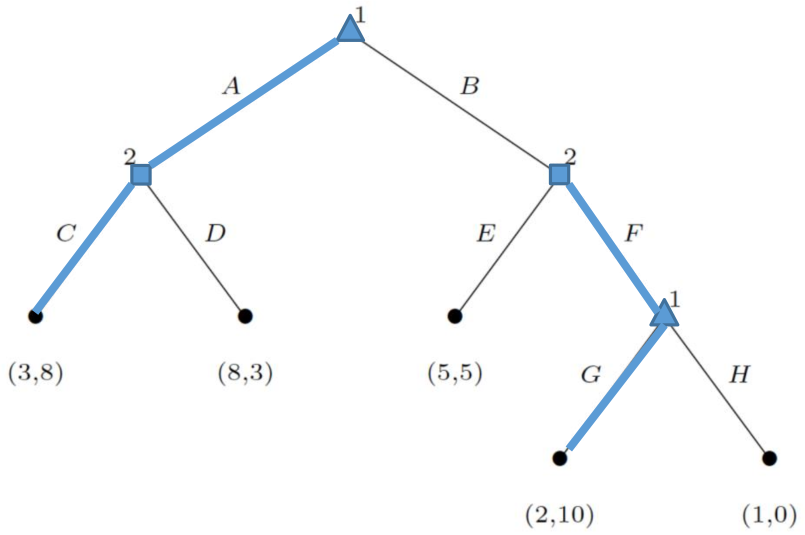
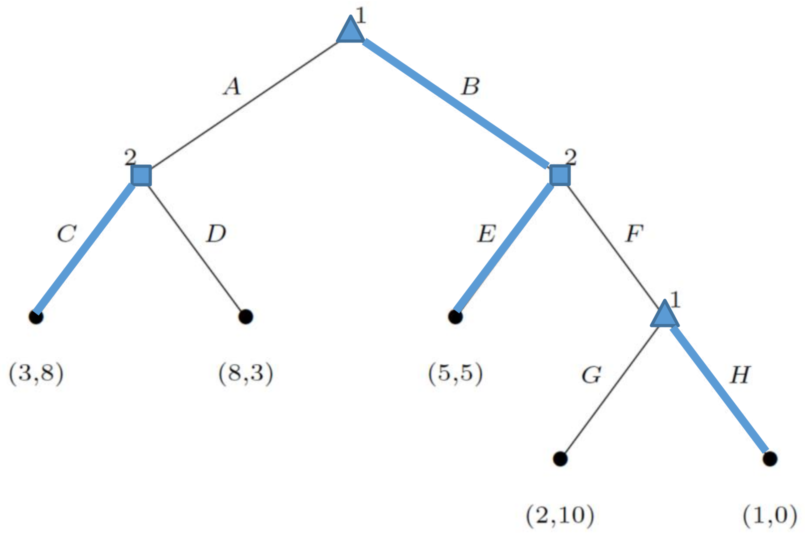
Solution Concepts in EFGs

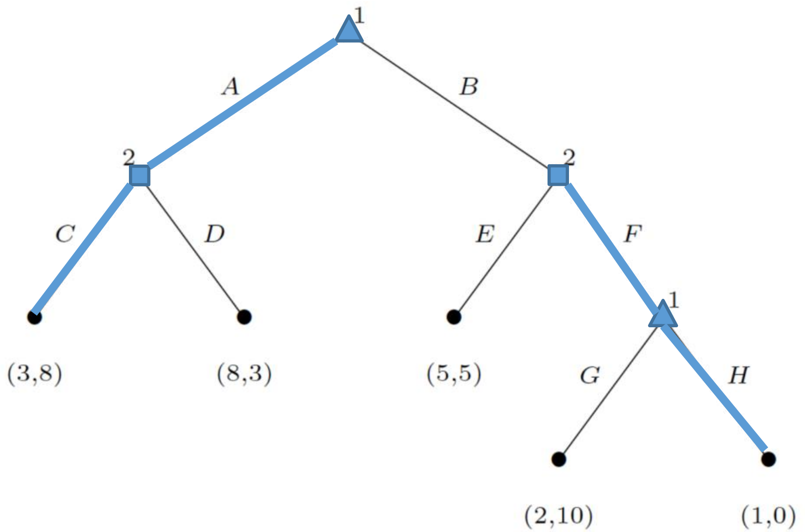


- Refinements of NE
  - Solution concepts that pose further assumptions on the strategy profile
  - Sub-game-perfect equilibria
  - Sequential equilibria
  - Quasi-perfect equilibria
  - ….
- Sub-game perfect
  - sub-game of G rooted at some node h is the restriction of G to the descendants of h
  - Strategy profile is a sub-game perfect NE, if it is a NE for every sub-game of G
  - Every SPE is NE

Nash Equilibrium in EFGs – SPE?







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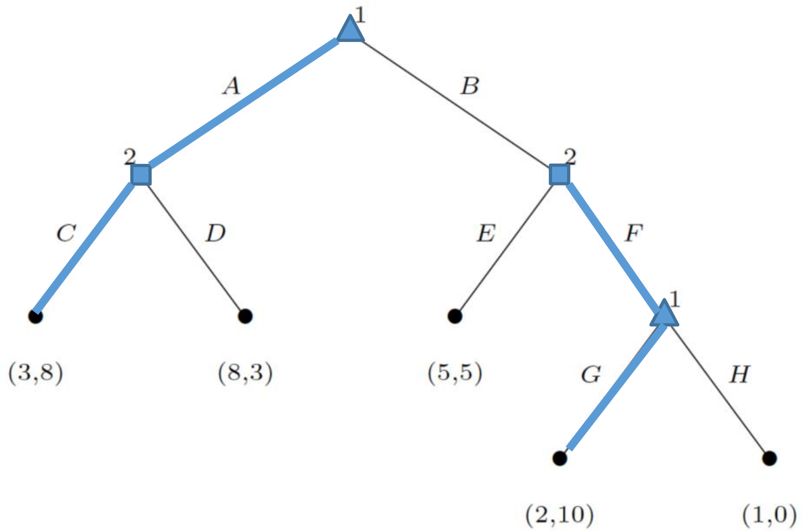
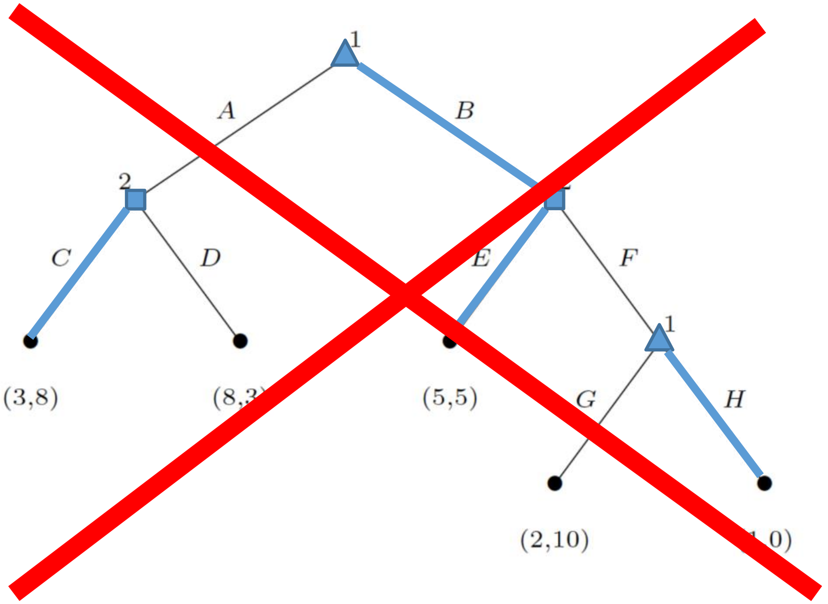
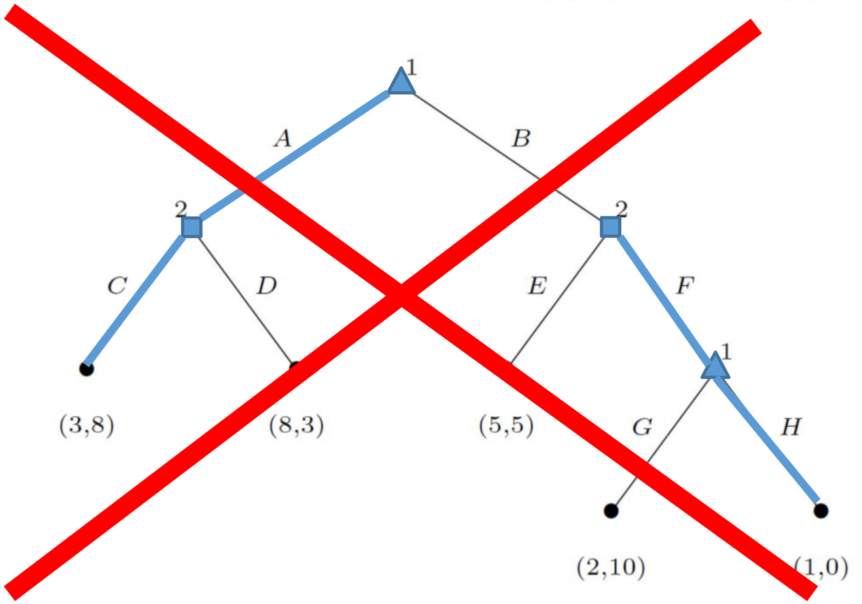
Nash Equilibrium in EFGs – SPE?







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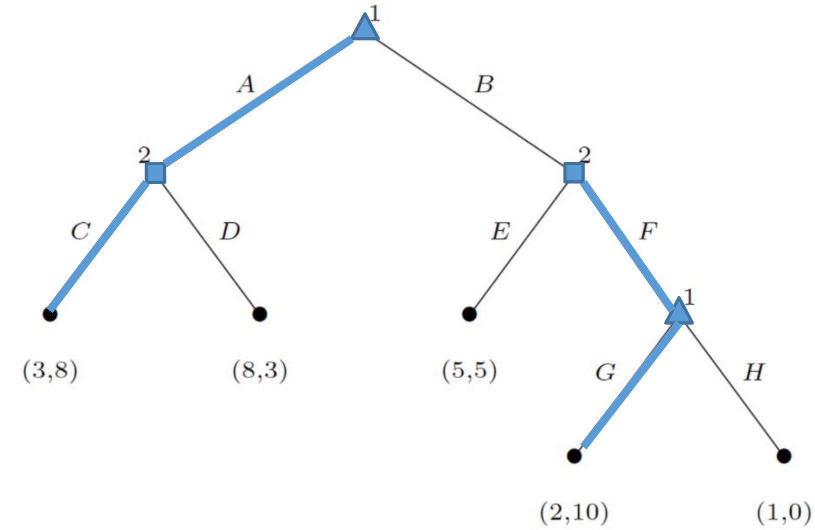
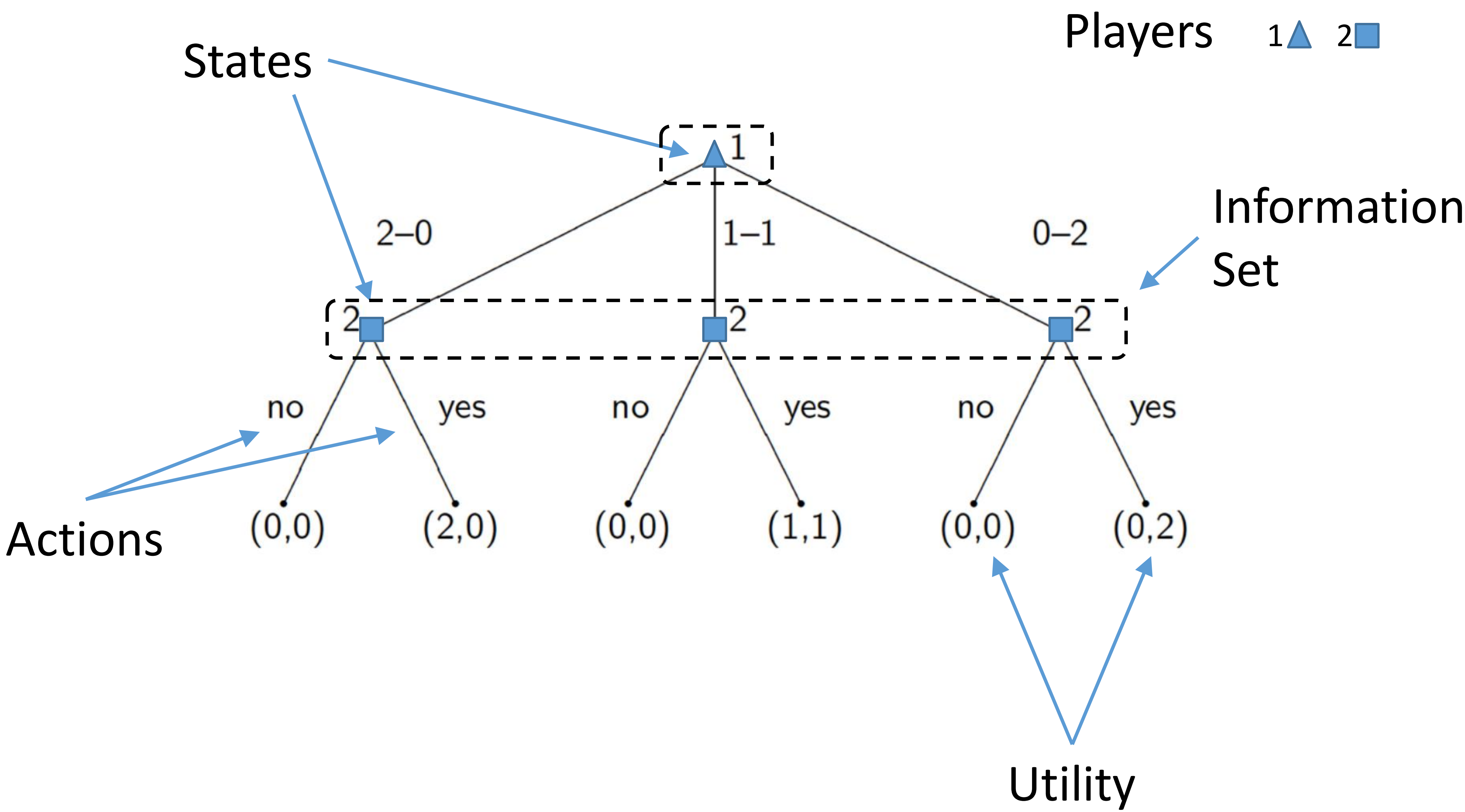
Computing SPE: Backward Induction

function BACKWARDINDUCTION (node h) **returns** $u(h)$
if $h \in Z$ **then**
  return $u(h)$
$best\_util \leftarrow -\infty$
forall $a \in \chi(h)$ **do**
  $util\_at\_child \leftarrow$ BACKWARDINDUCTION$(\sigma(h, a))$
  if $util\_at\_child_{\rho(h)} > best\_util_{\rho(h)}$ **then**
    $best\_util \leftarrow util\_at\_child$
return $best\_util$



- works for general-sum n-players games
- for 2 players zero-sum games:
  - Minimax algorithm
  - Alpha-Beta, NegaScout, …
- Computes pure strategy subgame-perfect equilibrium

Imperfect Information EFGs



Imperfect Information EFGs

Definition:

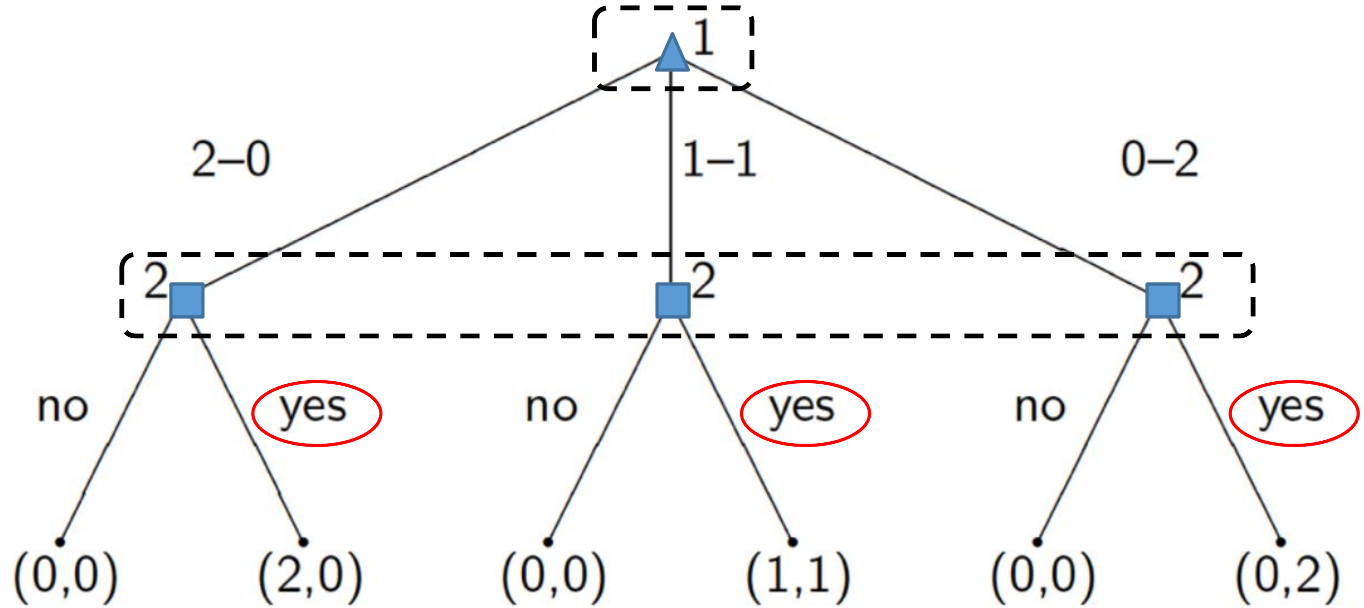
A (finite) imperfect-information game in the extensive form is defined as a tuple $(N, A, H, Z, \chi, \rho, \sigma, u, I)$, where:

- tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$ is a perfect-information extensive-form game
- $I = (I_1, \dots, I_n)$, where $I_i = (I_{i,1}, \dots, I_{i,k})$ is a set of equivalence classes on (i.e., a partition of) choice nodes of a player i with the property that $\chi(h) = \chi(h')$ and $\rho(h) = \rho(h')$, whenever $h \in I_{i,j}$ and $h' \in I_{i,j}$ for some j

II EFGs: Actions and Strategies

Pure strategy: an assignment of an action for each **information set**

Action is uniquely identified by the **information set**, in which it is taken.



$A_1 = \{2-0, 1-1, 0-2\}$ $A_2 = \{no, yes\}$

$S_1 = \{2-0, 1-1, 0-2\}$ $S_2 = \{no, yes\}$

Strategies in EFGs

Existence of a pure NE is no longer guaranteed for imperfect-information EFGs

Mixed strategies

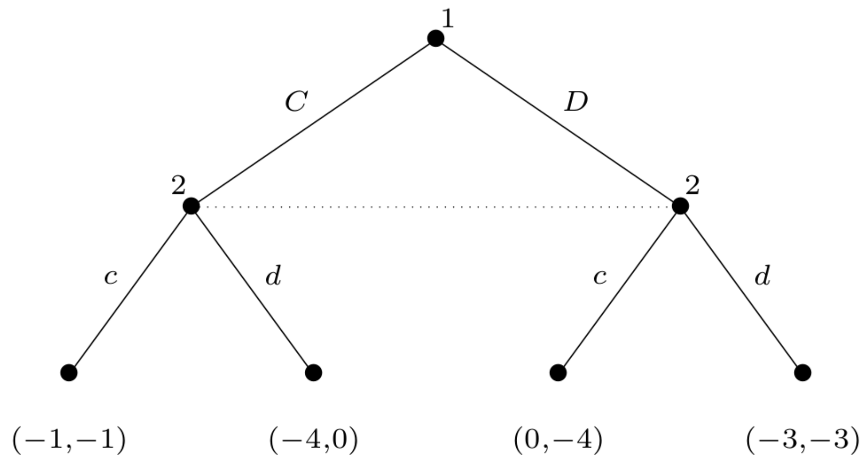
- Probabilistic distribution over pure strategies

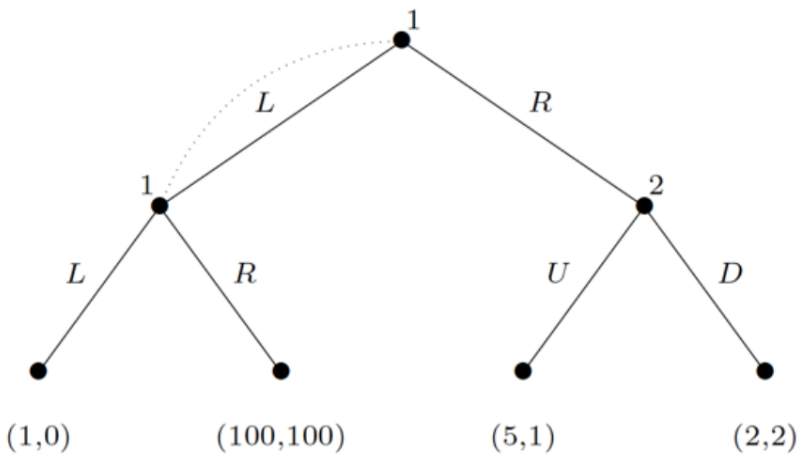
Behavioral Strategies

- Probabilistic distribution over actions to play for each information set

There is a broad class of imperfect-information games in which the expressive power of mixed and behavioral strategies coincides. This is the class of games of **perfect recall**. Intuitively speaking, in these games no player forgets any information he previously knew.

Perfect vs. Imperfect Recall





Remembering all information induces very large game trees

Easier to solve

Strategies can be compactly represented

Smaller trees, unnecessary information can be forgotten

Much harder to solve

Equilibrium in behavior strategies might not exist