A4M33MAS - Multiagent Systems
Distributed Constraint Optimization

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OTEVŘENÁ INFORMATIKA

In parts based on Multi-agent Constraint Programming, Boi Faltings, Laboratoire d'Intelligence Artificielle, EPFL

Multiagent Constraint Optimization (DCOP)

Given $< X, D, C, A >$ where:

- $X = \{x_1, .., x_n\}$ is a set of n variables.
- $D = \{d_1, .., d_n\}$ is a set of n domains.
- $C = \{c_1, .., c_m\}$ is a set of m constraints.
- $A = \{a_1, .., a_n\}$ is a set of n agents, not necessarily all different.

Find solution $= (x_1 = v_1 \in d_1, ..., x_n = v_n \in d_n)$ such that for all the overall cost of the assignment is minimized

$$\text{Cost}\ (\{v_1, .., v_n\}) = \sum_{\forall c_i \in C} c_i(\{v_1, .., v_n\})$$

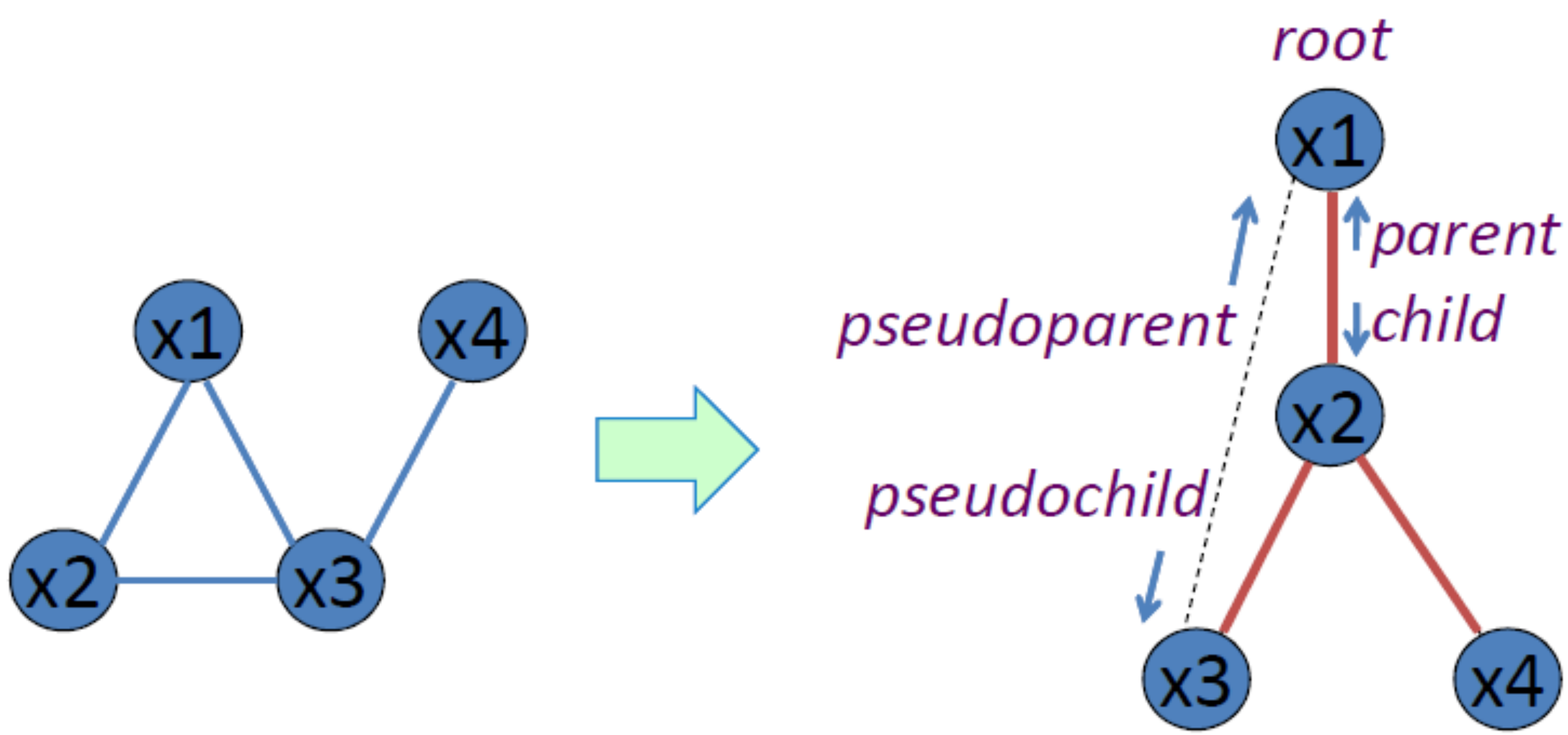
$C =$ represented as a list of cost functions on 1 … n variables in X and their values from D, so that $\mathcal{P}(X,D) \rightarrow \mathbf{R}$

ABT for DCOP

- Nogoods give lower bounds on costs.
- Compute total cost of all lower priority agents by summing nogoods.
- Nogood tags must exactly cover all lower-priority variables, otherwise some variables are not counted or counted multiple times.
- If we can prevent this from happening, then ABT works fine for optimization as well.

ADOPT

- ADOPT assumes that agents are arranged in a DFS tree:
  - constraint graph rooted graph (select a node as root)
  - some links form a tree / others are back edges
  - two constrained nodes must be in the same path to the root by tree links (same branch)
- Every graph admits a DFS tree



ADOPT: Description

- Asynchronous algorithm
- Each time an agent receives a message:
  - Processes it (the agent may take a new value)
  - Sends VALUE messages to its children and pseudochildren
  - Sends a COST message to its parentd
- **View**: set of variable value pairs (as ABT agent view) of ancestor agents, in the same branch. Current context:
  - Updated by each VALUE message

ADOPT: Description

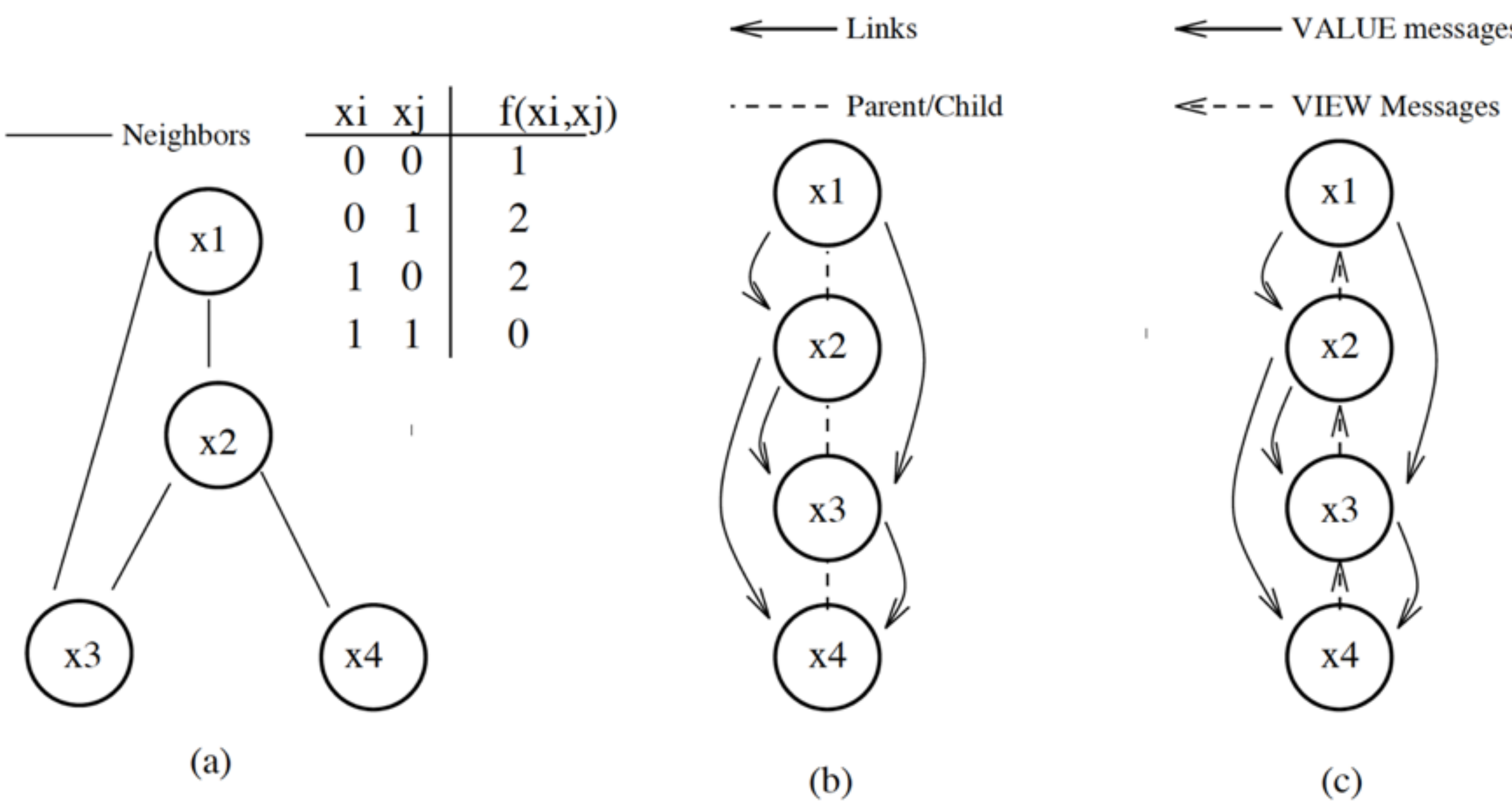
01

Definition: The *local cost* δ incurred at x_i, wrt to a given view vw is defined as

$$\delta(x_i, vw) = \sum_{x_j \in V} f_{ij}(d_i, d_j) \text{ , } where\ x_i \leftarrow d_i, \quad x_j \leftarrow d_j\ in\ vw$$

Definition: A *view* is a set of variable/value pairs of the form $\{(x_i, d_i), (x_j, d_j) ...\}$. A variable can appear in a view no more than once. Two views are *compatible* if they do not disagree on any variable assignment.

ADOPT: Example



xi	xj	f(xi,xj)
0	0	1
0	1	2
1	0	2
1	1	0

ADOPT: Messages

- **value**(parent $\rightarrow$ children & pseudochildren, value):
  parent informs descendants that it has taken value a
- **view**(child $\rightarrow$ parent, cost, view):
  child informs parent of the best cost of its assignement; attached context to detect obsolescence;

Simple-ADOPT Algorithm

Initialize: $Currentvw \leftarrow \{\}$;$d_i \leftarrow$ null;
  $\forall d \in D_i$:
    $c(d) \leftarrow 0$
  hill_climb;

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procedure hill_climb
$\forall d \in D_i$:
 # e(d) is x_i's estimate of cost if it chooses d
$e(d) \leftarrow \delta(x_i, Currentvw \cup \{(x_i, d)\}) + c(d)$;
choose d that minimizes $e(d)$
prefer current value d_i for tie;
$d_i \leftarrow d$;
SEND (**VALUE**, (x_i, d_i)) to all linked descendents;
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parent;

Simple-ADOPT Algorithm

when received (**VALUE**, (x_j, d_j))
  add (x_j, d_j) to $Currentvw$;
  # context change
  if $Currentvw$ changed then
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  end if;
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when received (**VIEW**, vw, $cost$)
  $d \leftarrow$ value of x_i in vw
  if vw is compatible with
  $Currentvw \cup \{(x_i, d)\}$ then
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    if $c(d)$ changed then
      hill_climb;
    end if;
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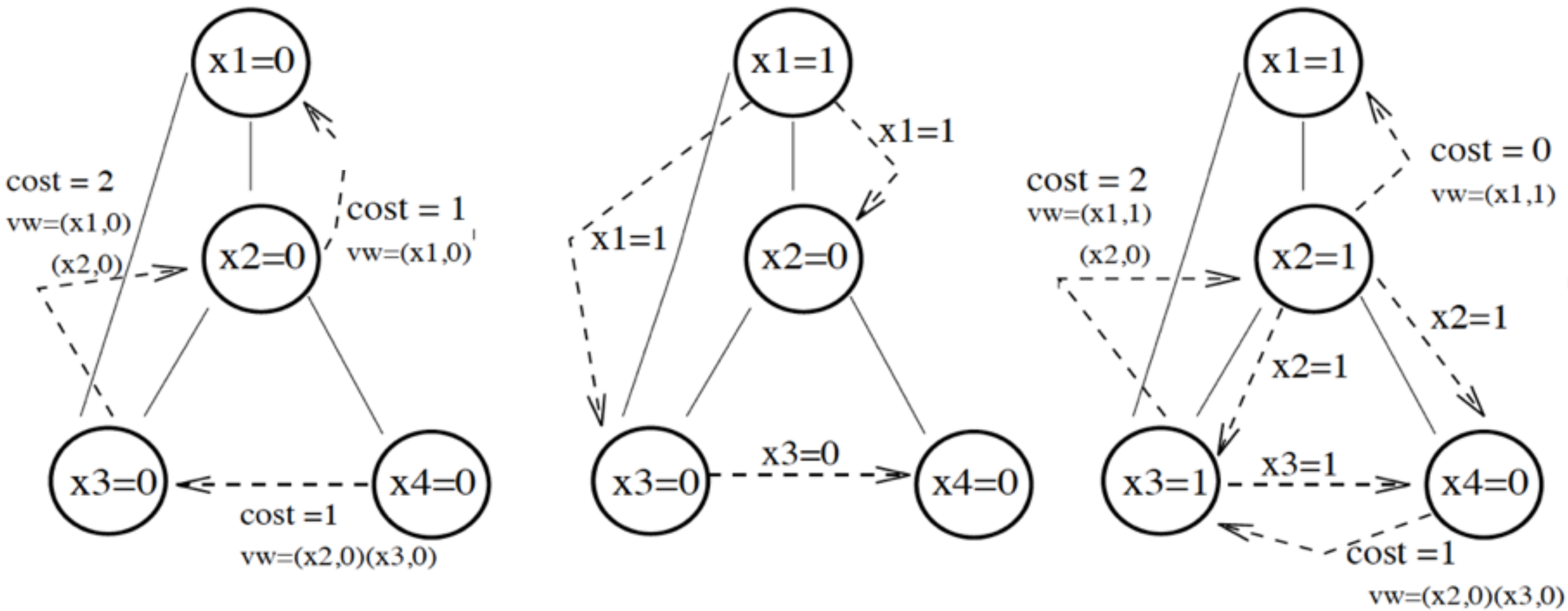

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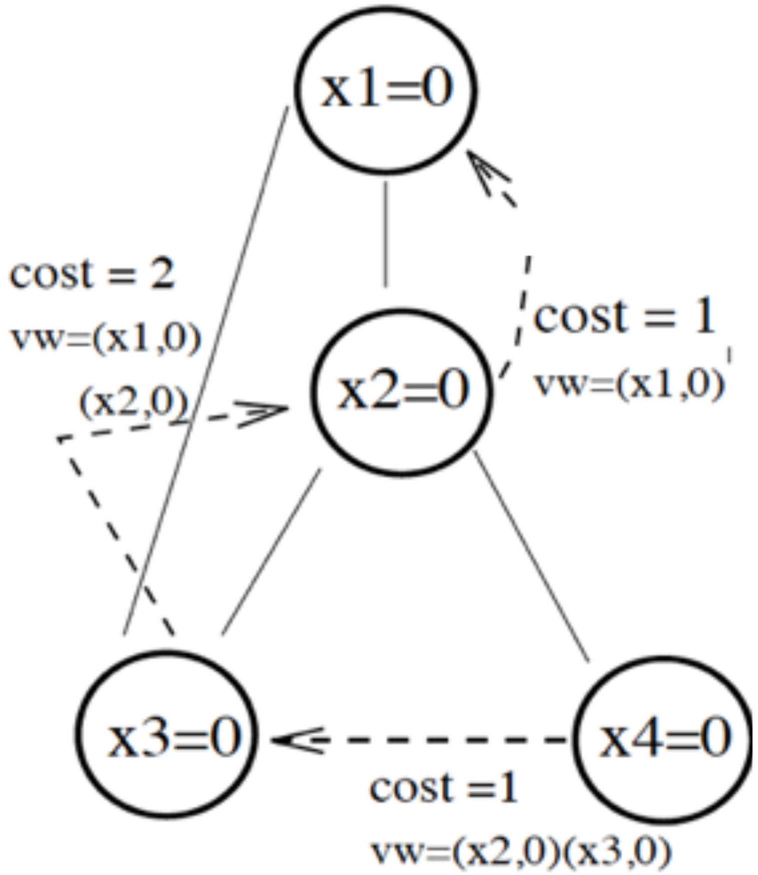

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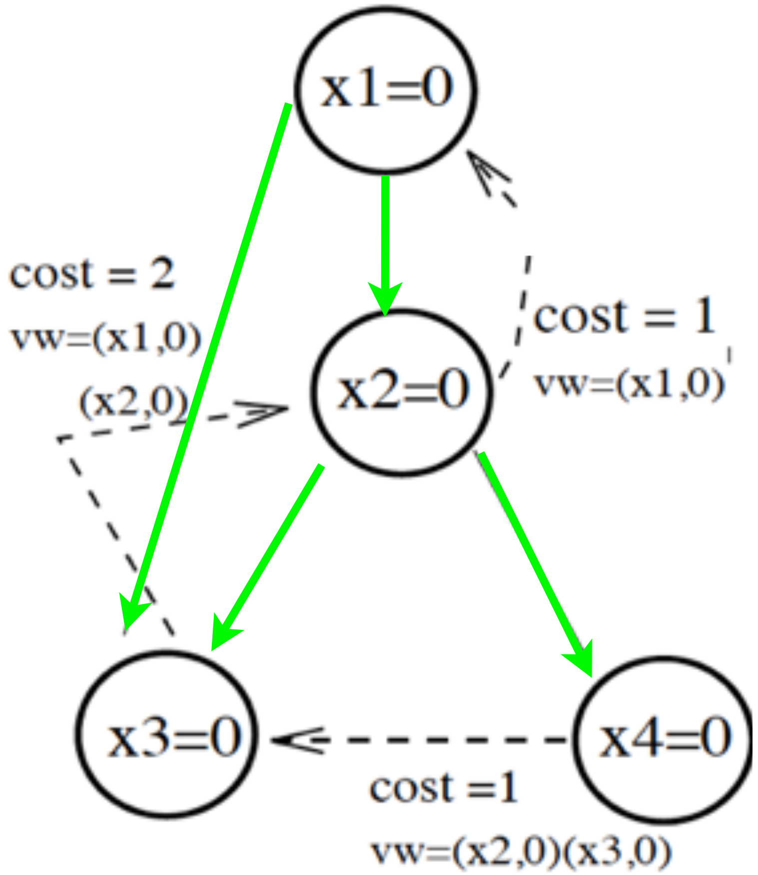

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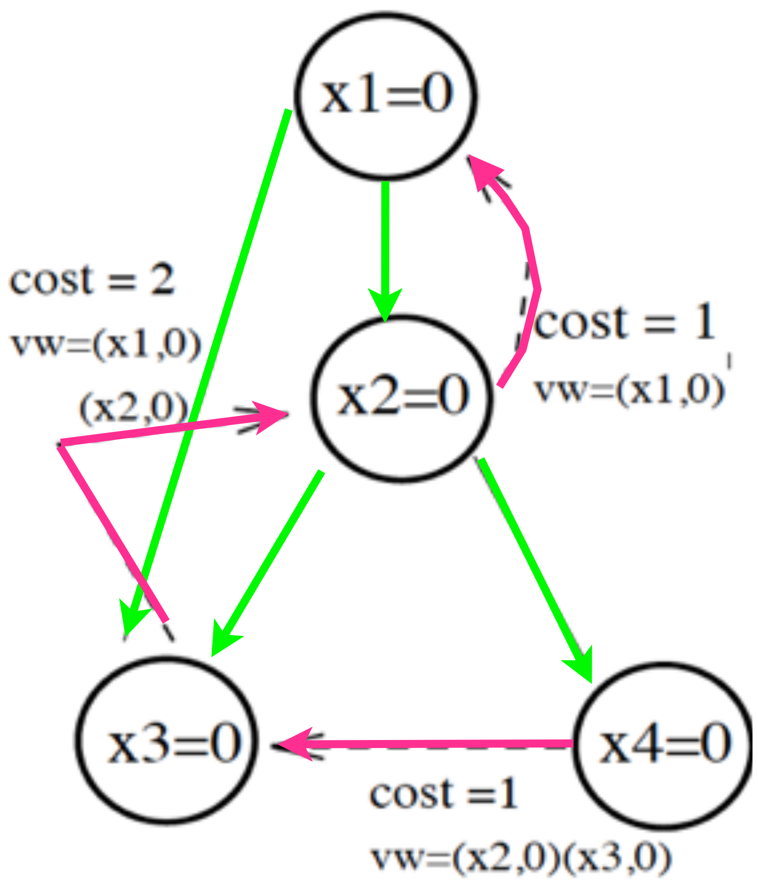

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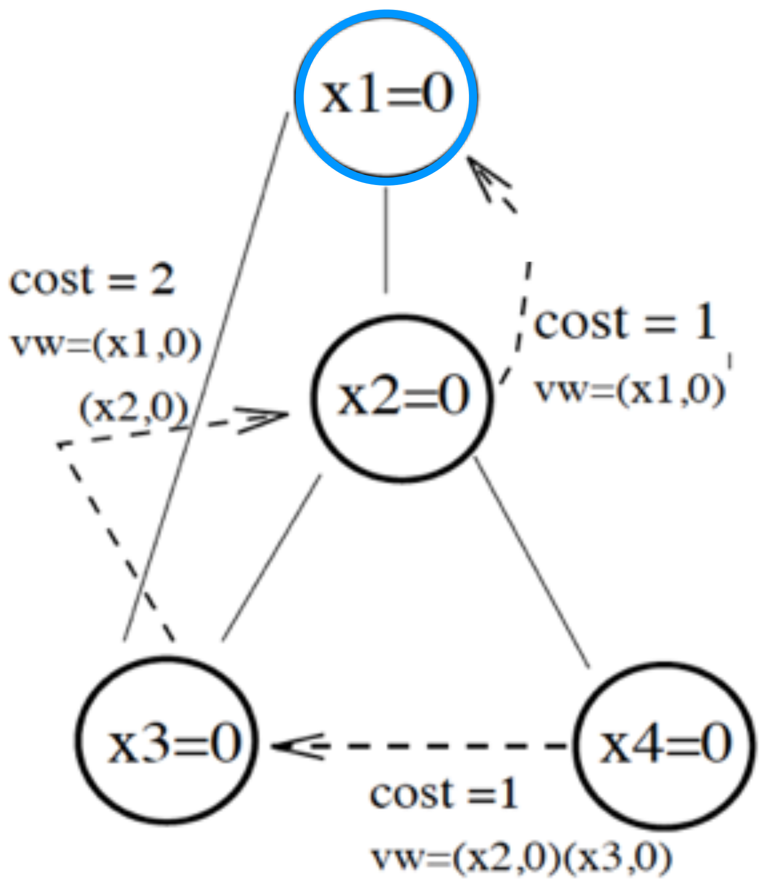

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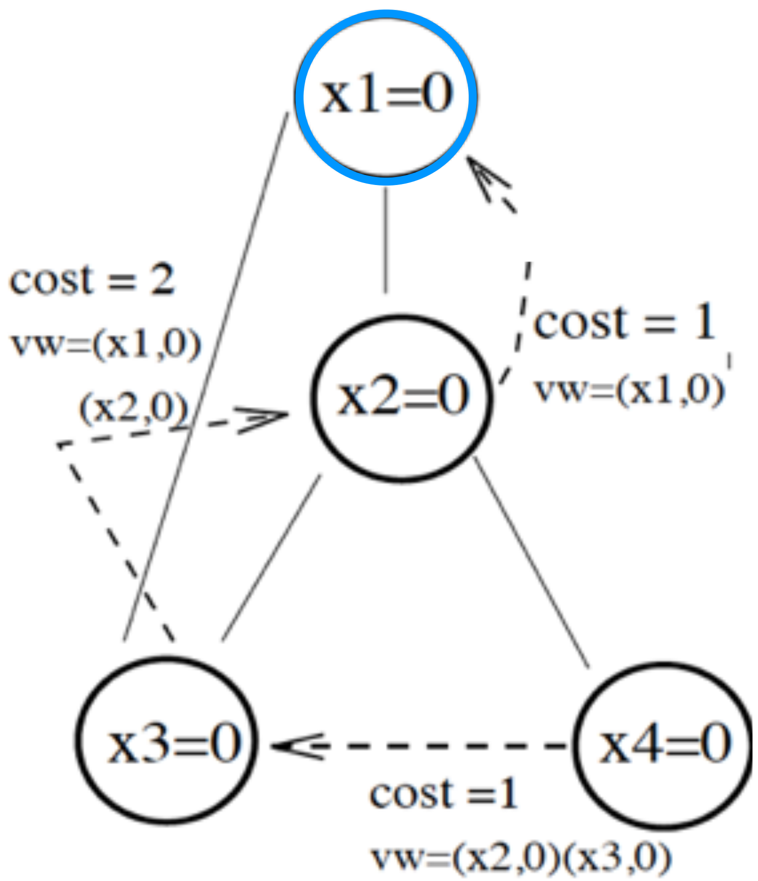

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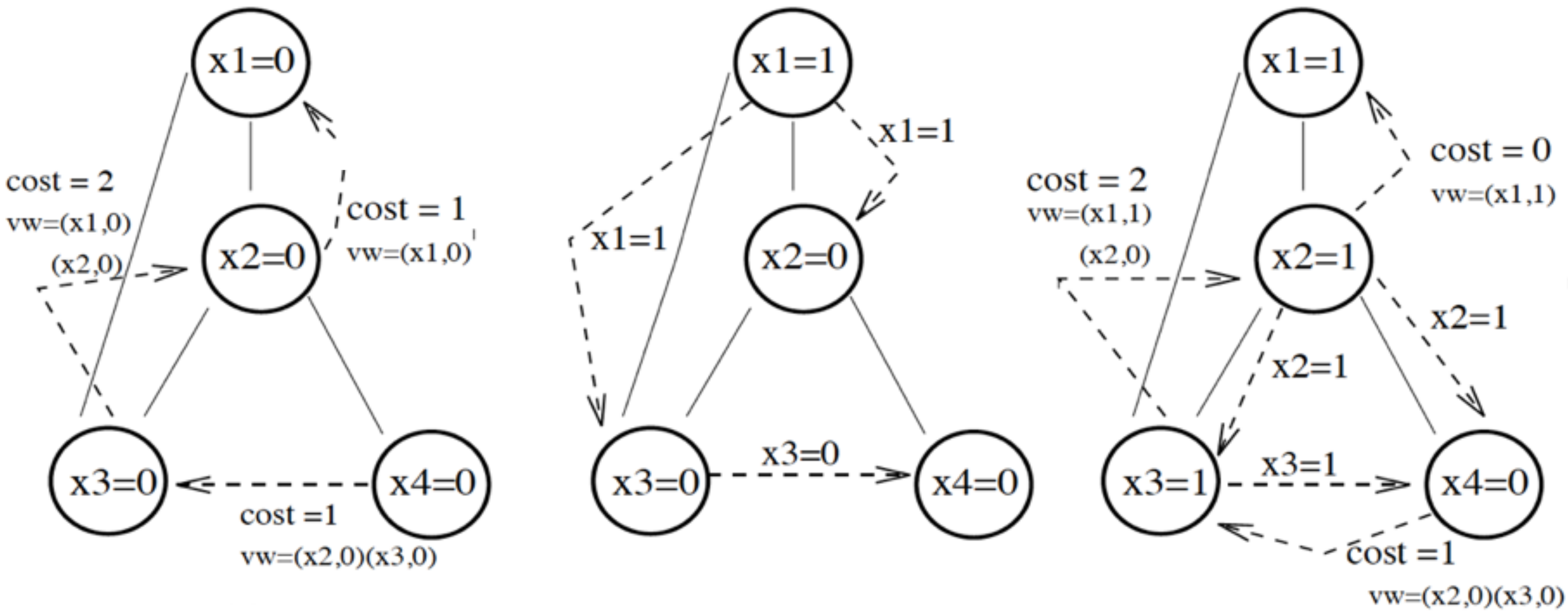

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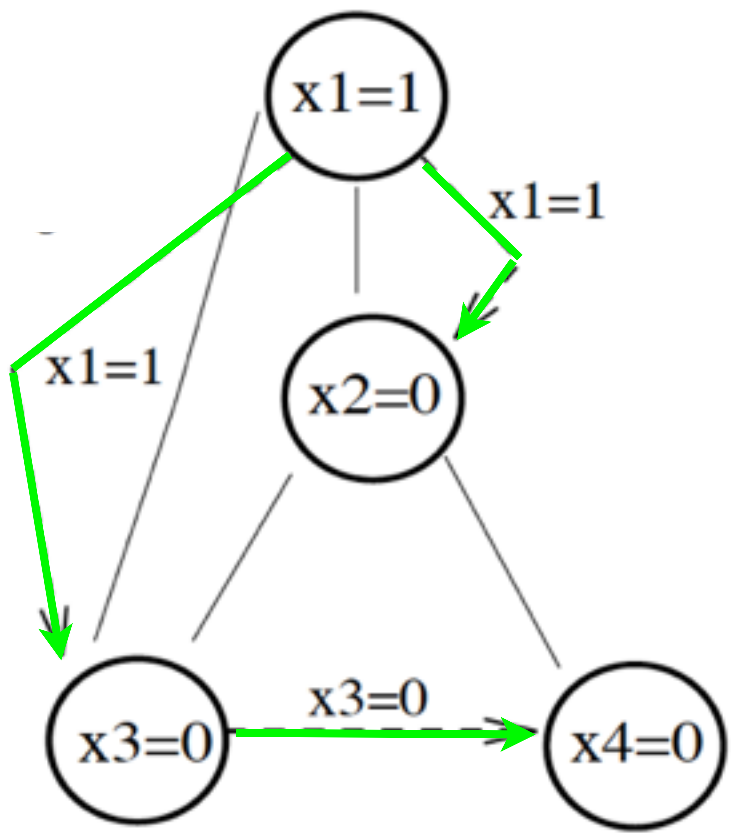

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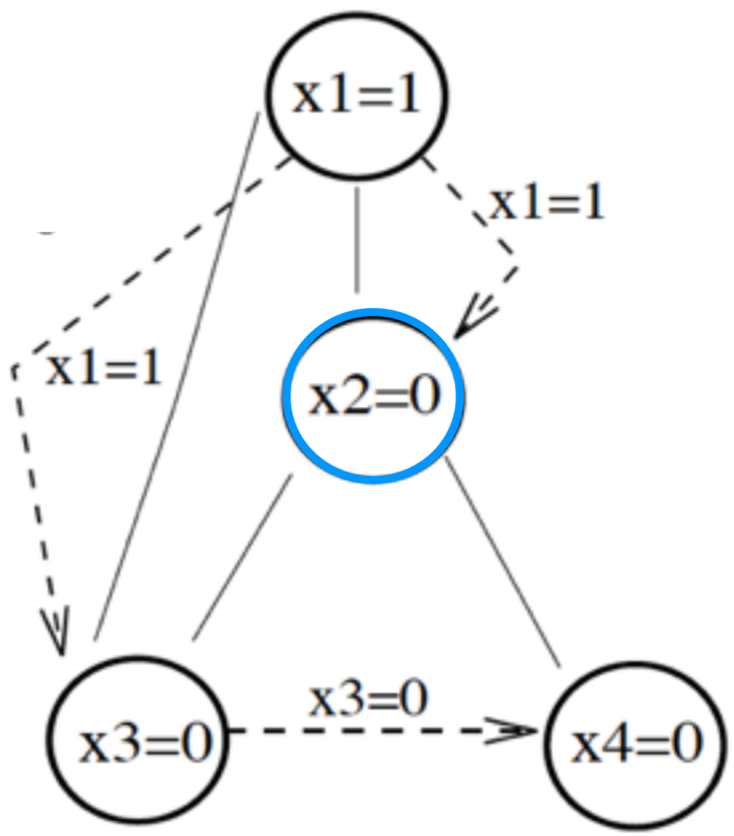

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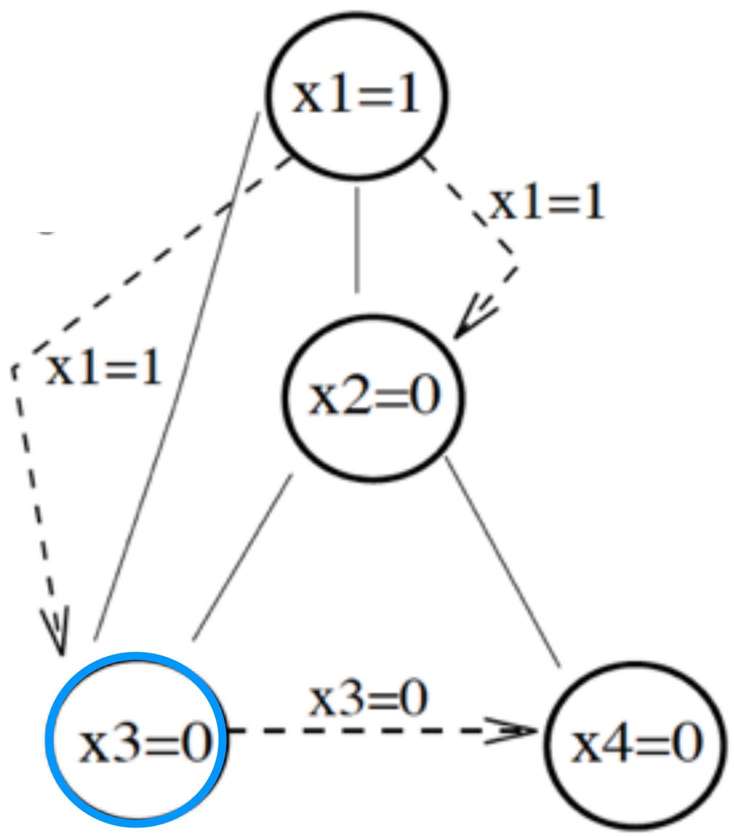

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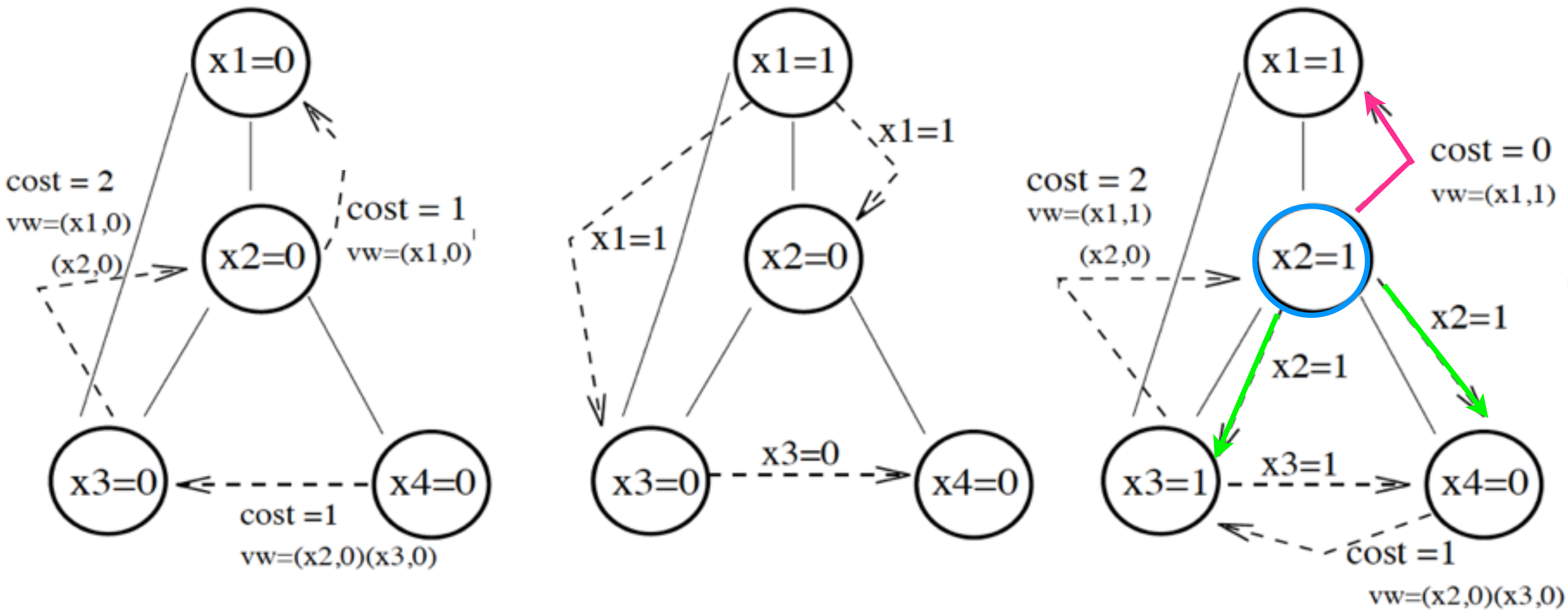

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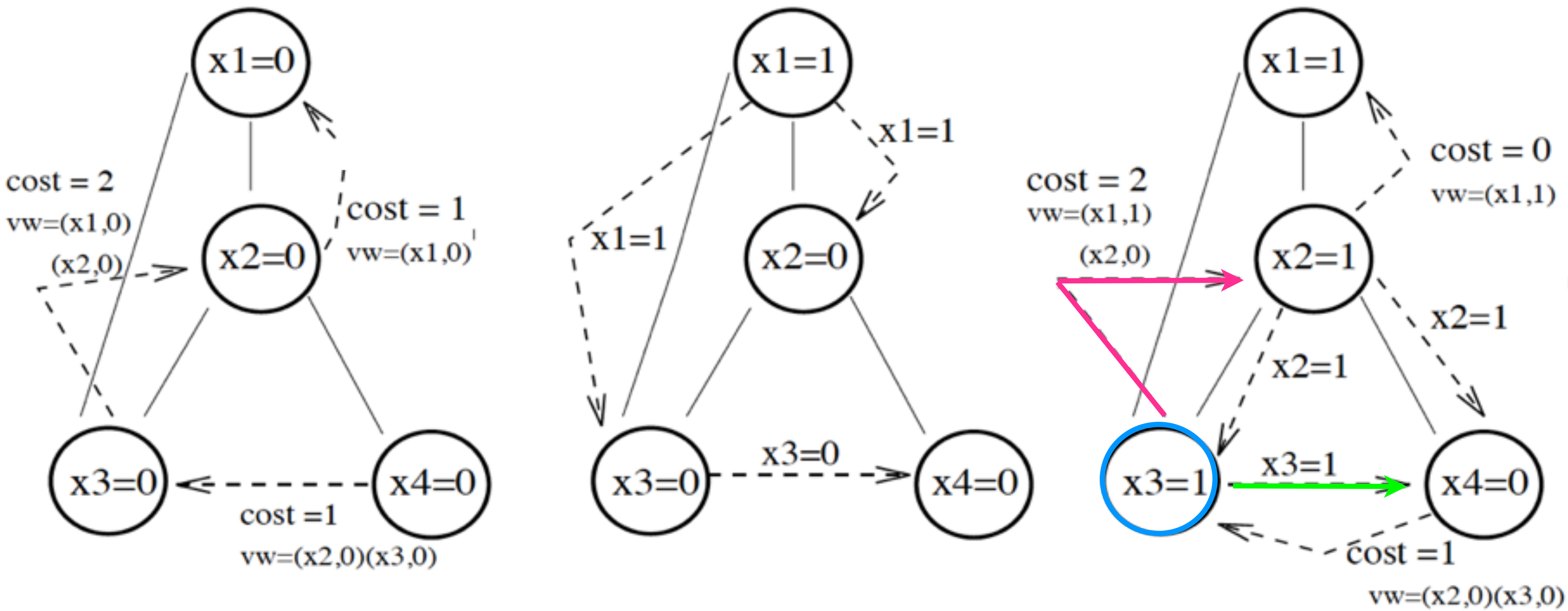
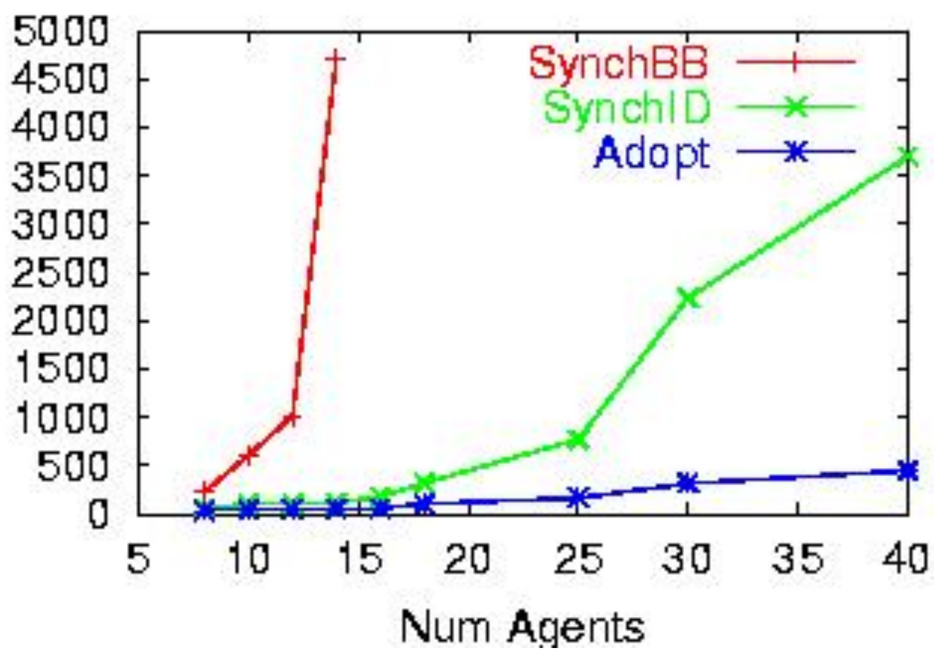


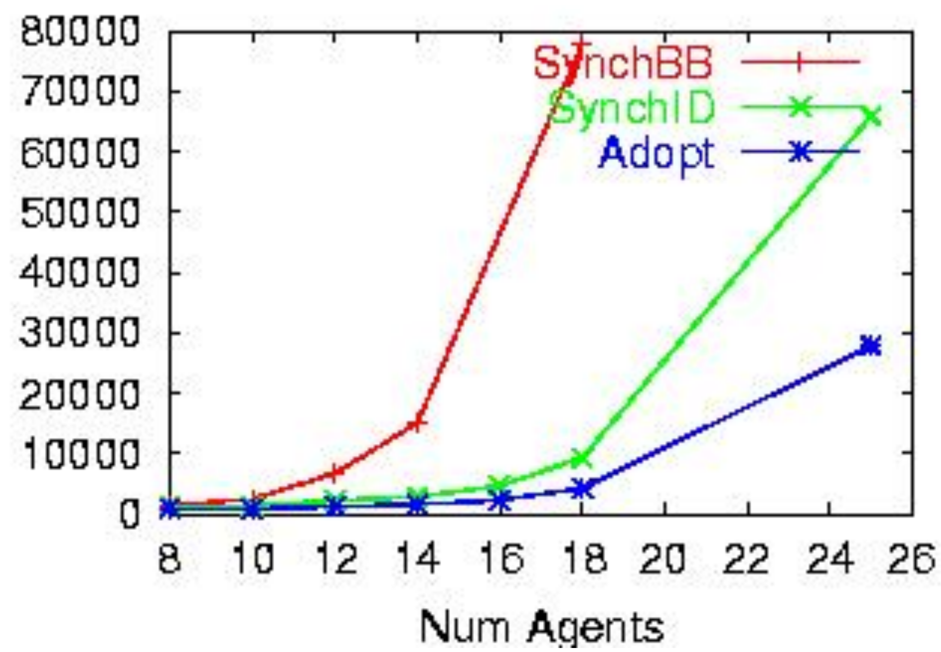
ADOPT: Properties

- For finite DCOPs with binary non-negative constraints, ADOPT is guaranteed to terminate with the globally optimal solution.
- An ADOPT agent takes the value with minimum cost:
  - Best-first search with eager behavior:
  - Agents may constantly change value
- Graph coloring benchmark:

Avg. number of cycles, link density = 2



Avg. number of cycles, link density = 3



ADOPT: Key Ideas

- Optimal, asynchronous algorithm for DCOP
  - polynomial space at each agent
- Weak Backtracking
  - lower bound based search method
  - Parallel search in independent subtrees
- Efficient reconstruction of abandoned solutions
  - backtrack thresholds to control backtracking
- Bounded error approximation
  - sub-optimal solutions faster
  - bound on worst-case performance

Dynamic Programming Optimization Protocol (DPOP)

- Principle: replace variables by constraints.
- Consider variable x having constraint with y.
- For each value of x, there may be a consistent value of y.

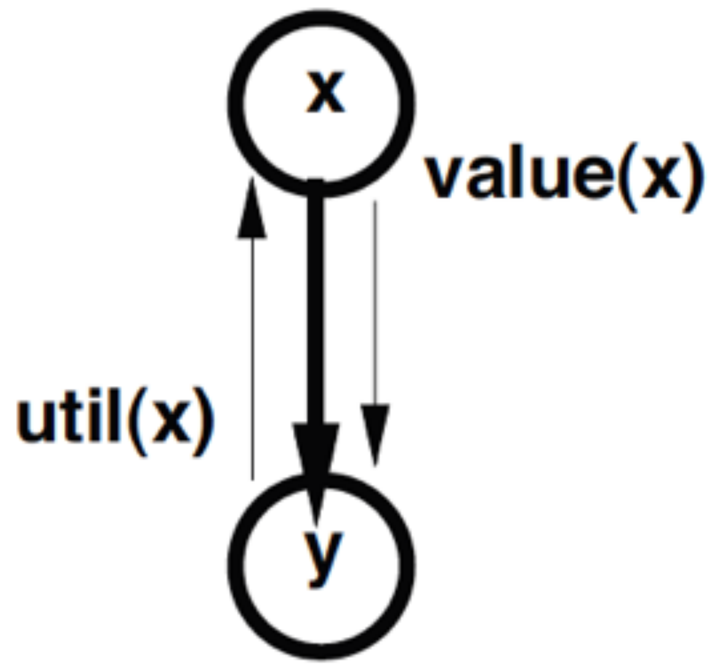
$\Rightarrow$ replace y by a constraint on x:

x=v is allowed if there is a consistent value of y.

- Optimization version:

utility(x=v) = utility(x=v,y=w); w = best possible value of y given x=v.

Dynamic Programming Optimization Protocol (DPOP)



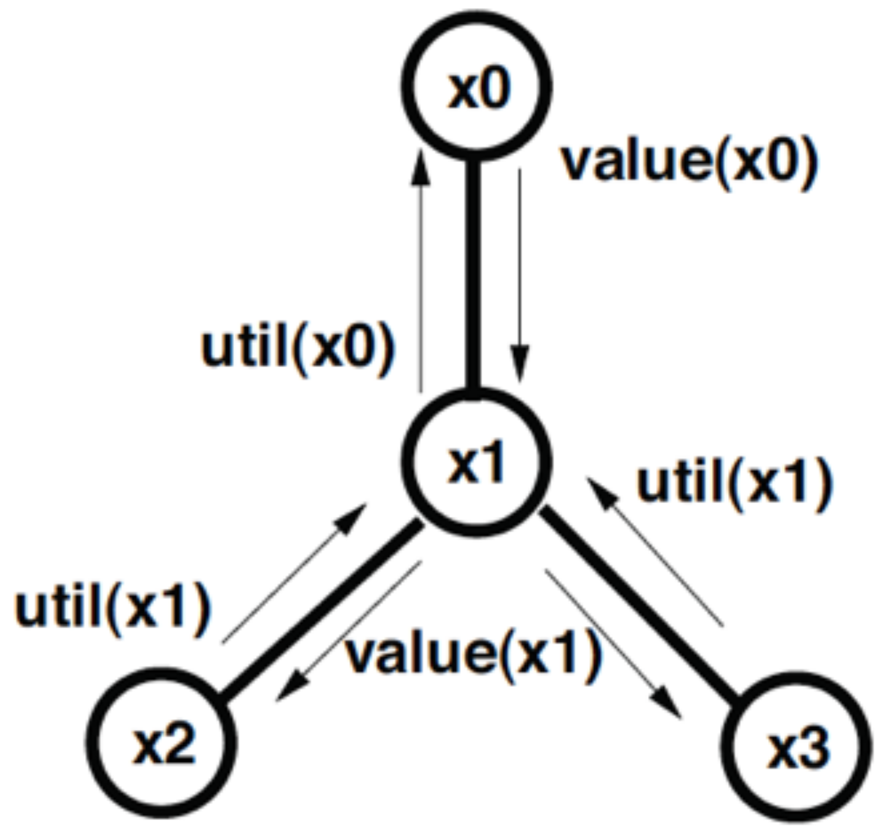
- y sends constraint in util(x) message.

$\Rightarrow$ x can decide (best) value locally.

- x informs y of value using value(x) message.

Dynamic Programming Optimization Protocol (DPOP)

- Rooted tree: every node has at most one parent
- Nodes send UTIL messages to their parents
- Best values of x2, x3 $\Rightarrow$ unary constraint on x1
- x1 sums up UTIL messages + own constraint $\Rightarrow$ unary constraint on x0
- x0 picks best value v(x0); sends value(x0=v(x0)) $\rightarrow$ x1
- x1 picks best value given x0 and informs x2,x3



Dynamic Programming Optimization Protocol (DPOP)

$c(x_0, x_3)$

x_0 \ x_3	w	b
w	3	0
b	3	3

$c(x_0, x_1)$

x_0 \ x_1	w	b
w	1	0
b	2	2

$c(x_1, x_2)$

x_1 \ x_2	w	b
w	1	0
b	0	1

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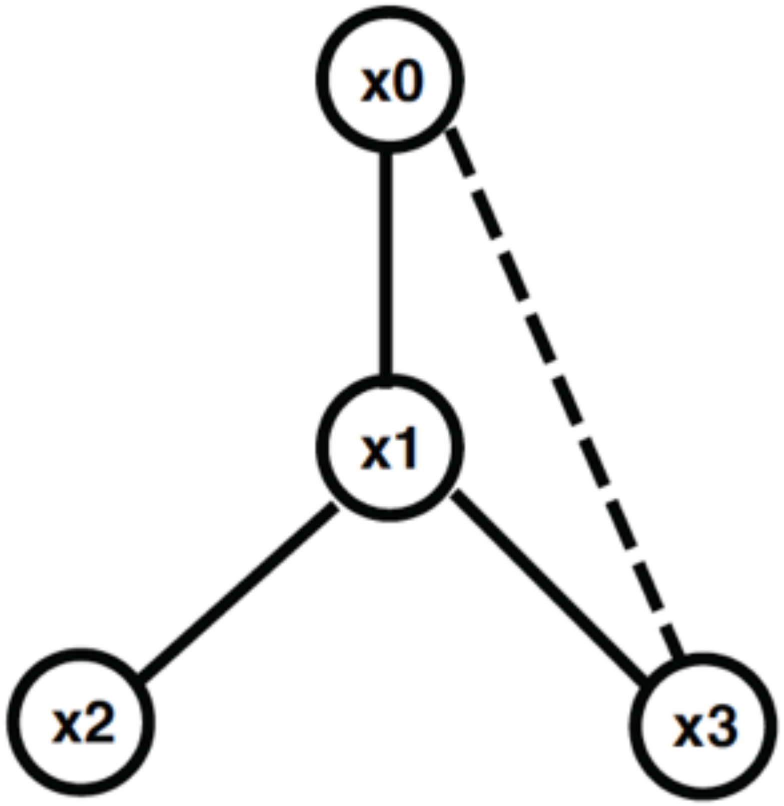
w	2	0
b	0	2



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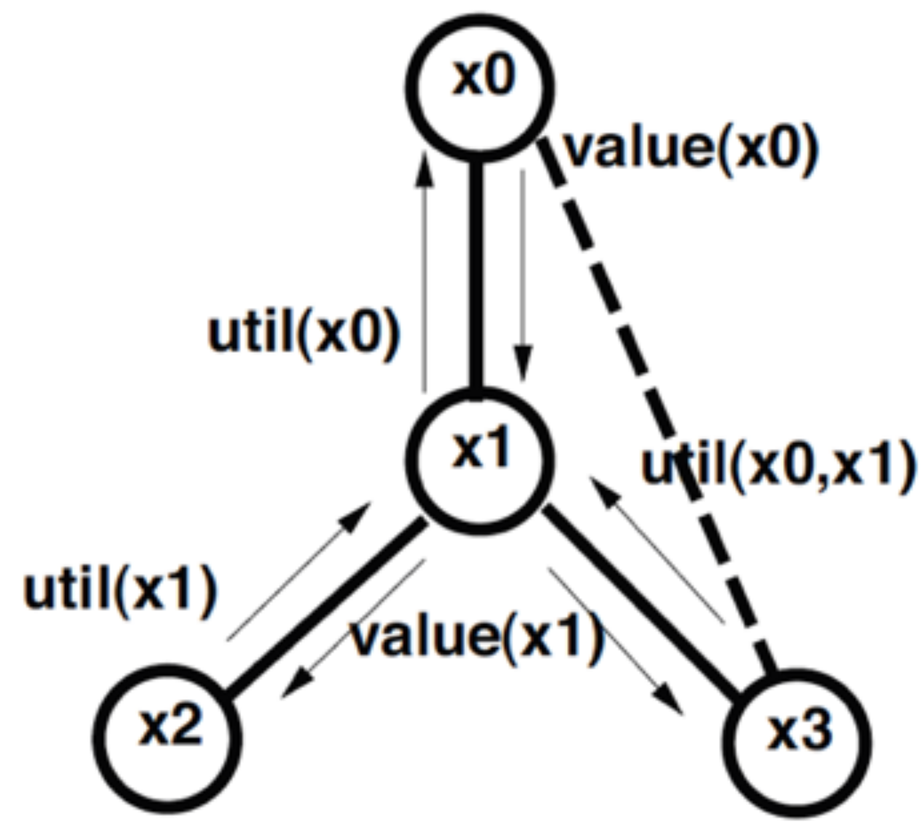

Dynamic Programming Optimization Protocol (DPOP)

$c(x_0, x_3)$

$x_0 \backslash x_3$	w	b
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b	3	3

$c(x_0, x_1)$

$x_0 \backslash x_1$	w	b
w	1	0
b	2	2

$c(x_1, x_2)$

$x_1 \backslash x_2$	w	b
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$x_1 \backslash x_3$	w	b
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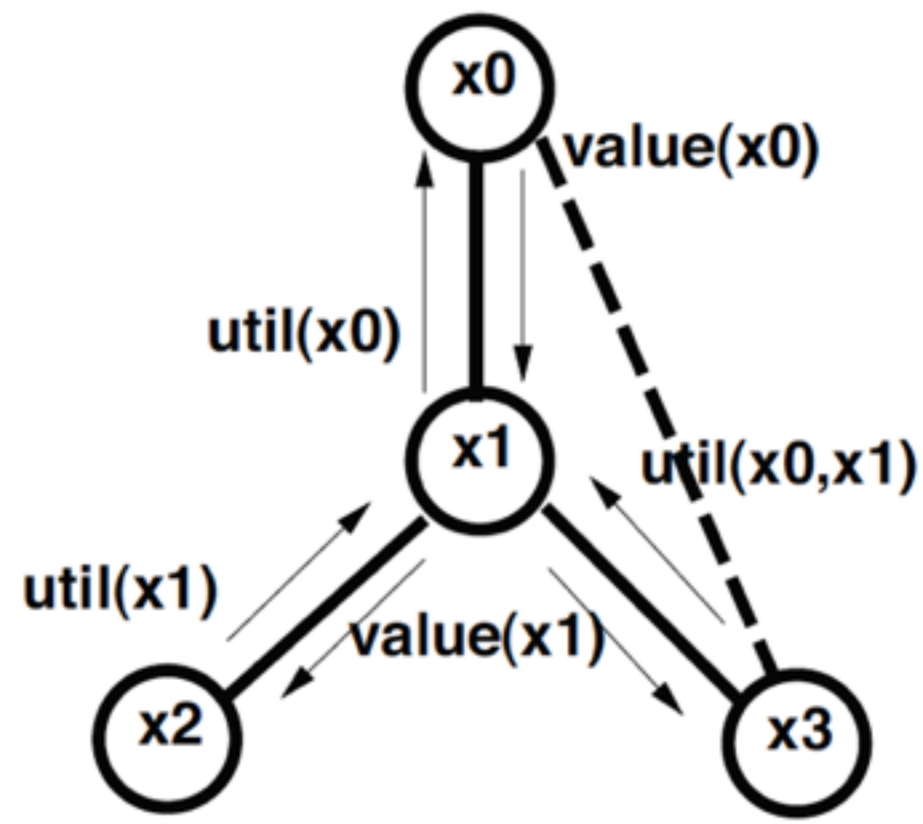

$$UTIL(x_1) = \begin{array}{cc} \multicolumn{2}{c}{x_1} \\ w & b \\ \hline 0 & 0 \end{array}$$

Dynamic Programming Optimization Protocol (DPOP)

$c(x_0, x_3)$

$x_0 \backslash x_3$	w	b
w	3	0
b	3	3

$c(x_0, x_1)$

$x_0 \backslash x_1$	w	b
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$c(x_1, x_2)$

$x_1 \backslash x_2$	w	b
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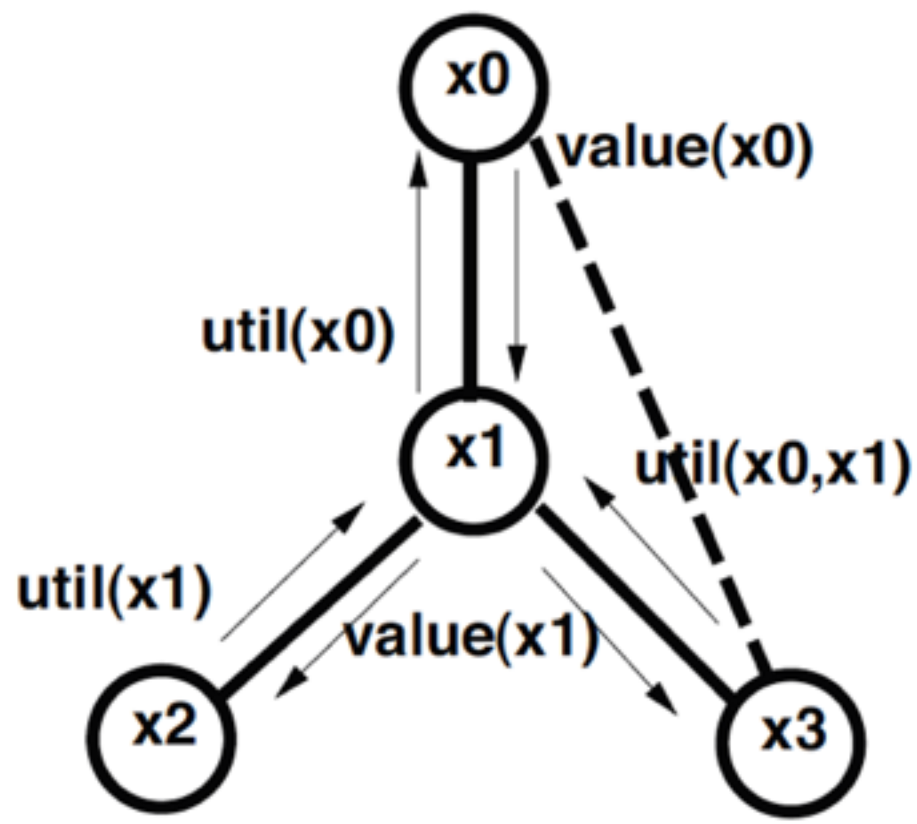

$UTIL(x_1) =$

x_1: w	b
0	0

$UTIL(x_0, x_1) =$

$x_0 \backslash x_1$	w	b
w	0	2
b	3	3

Dynamic Programming Optimization Protocol (DPOP)

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x_0 \ x_3	w	b
w	3	0
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$c(x_0, x_1)$

x_0 \ x_1	w	b
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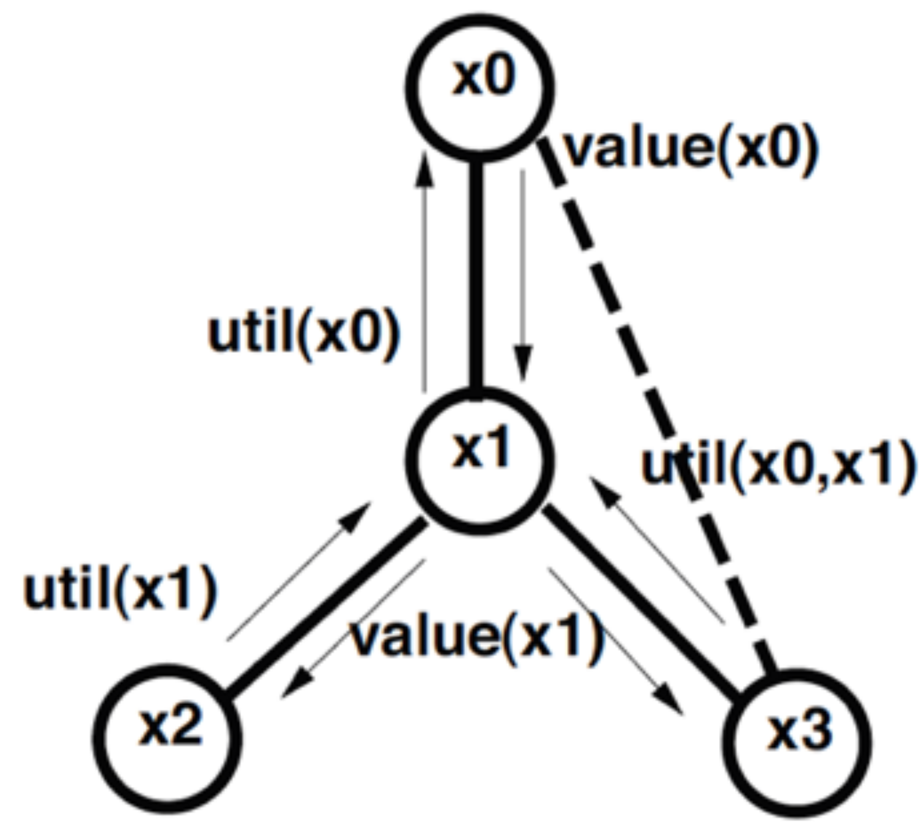

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$$UTIL(x_0, x_1) = \begin{array}{c|cc} & \multicolumn{2}{c}{x_1} \\ x_0 & w & b \\ \hline w & 0 & 2 \\ b & 3 & 3 \end{array}$$

$$\boxed{UTIL(x_0) = \begin{array}{cc} \multicolumn{2}{c}{x_0} \\ w & b \\ \hline 1 & 3 \end{array}}$$

Dynamic Programming Optimization Protocol (DPOP)

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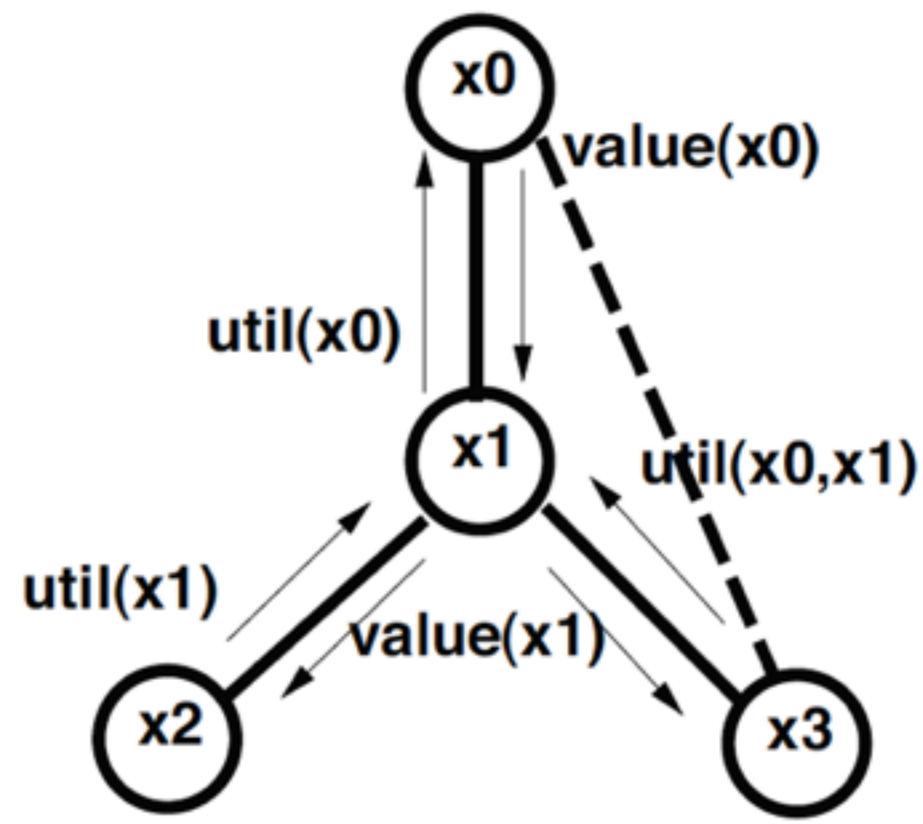

$UTIL(x_1) =$

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0	0

$UTIL(x_0, x_1) =$

x_0 \ x_1	w	b
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$UTIL(x_0) =$

x_0: w	b
1	3

x_0: $\mathtt{w}$; send value$(x_0 = \mathtt{w}) \rightarrow x_1$ x_1: $\mathtt{w}$; send value$(x_0 = \mathtt{w}, x_1 = \mathtt{w}) \rightarrow x_2, x_3$

Dynamic Programming Optimization Protocol (DPOP)

- Two messages per variable (UTIL and VALUE).

$\Rightarrow$ *number* of messages grows linearly with the size of the problem.

- However, the maximum message *size* grows exponentially with the tree-width of the induced graph.
- In many distributed problems, the tree-width is relatively small.

Distributed local search

- Drawbacks of systematic search:
  - need variable ordering
  - no anytime behavior: have to wait for termination.
  - often (too) costly.
- Sacrifice completeness $\Rightarrow$ local search
  - min-conflicts
  - distibuted min-conflicts
  - breakout algorithm
  - random sampling

Min-conflicts

- Assign random value to each variable in parallel (this will conflict with some constraints).
- At each step, find the change in variable assignment which most reduces the number of conflicts.
- Corresponds to search by "hill-climbing".

Distributed min-conflicts

- Neighbourhood of $N(x_i)$ = variables connected to x_i through constraints.
- Change to x_i can happen asynchronously with others as long as there is no other change in the neighbourhood.

  ⇒ two neighbouring agents are not allowed to change simultaneously:

  – highest improvement wins

  – ties broken by fixed ordering

  ⇒ parallel, distributed execution.

- also called MGM

Distributed min-conflicts

- Neighbourhood of $N(x_i)$ = variables connected to x_i through constraints.
- Change to x_i can happen asynchronously with others as long as there is no other change in the neighbourhood.

  $\Rightarrow$ two neighbouring agents are not allowed to change simultaneously:

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  $\Rightarrow$ parallel, distributed execution.

- CAN GET STUCK IN LOCAL MINIMUM

Distributed min-conflicts

- Neighbourhood of $N(x_i)$ = variables connected to x_i through constraints.
- Change to x_i can happen asynchronously with others as long as there is no other change in the neighbourhood.

  ⇒ two neighbouring agents are not allowed to change simultaneously:

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  ⇒ parallel, distributed execution.

Definition 2.6 (Quasi-local minimum). *An agent x_i is in a* quasi-local minimum *if it is violating some constraint and neither it nor any of its neighbors can make a change that results in lower total cost for the system.*

Breakout Algorithm

- To escape *local minima* the algorithm identifies *quasi-local minima* and increases the cost of the constraint violations
- Similar to min-conflict, but assign dynamic priority to every conflict (constraint), initially $=1$
- Modify variable which reduces the most the sum of the priority values of all conflicts.
- When local minimum:
  - increase weight of every existing conflict
- Eventually, new conflicts will have lower weight than existing ones $\Rightarrow$ breakout

Breakout Algorithm: Code

HANDLE-OK?(j, x_j)

$\textit{received-ok}[j] \leftarrow$ TRUE
$\textit{agent-view} \leftarrow \textit{agent-view} + (j, x_j)$
if $\forall_{k \in neighbors}$ $\textit{received-ok}[k] =$ TRUE
    then SEND-IMPROVE()
        $\forall_{k \in neighbors}$ $\textit{received-ok}[k] \leftarrow$ FALSE

SEND-IMPROVE()

$\textit{new-value} \leftarrow$ value that gives maximal improvement
$\textit{my-improve} \leftarrow$ possible maximal improvement
$\forall_{k \in neighbors}$ k.HANDLE-IMPROVE$(i, \textit{my-improve}, \textit{cost})$

HANDLE-IMPROVE$(j, \textit{improve})$

$\textit{received-improve}[j] \leftarrow \textit{improve}$
if $\forall_{k \in neighbors}$ $\textit{received-improve}[k] \neq$ NONE
    then SEND-OK
        $\textit{agent-view} \leftarrow \emptyset$

Breakout Algorithm: Code

$\textit{my-improve} \leftarrow$ possible maximal improvement
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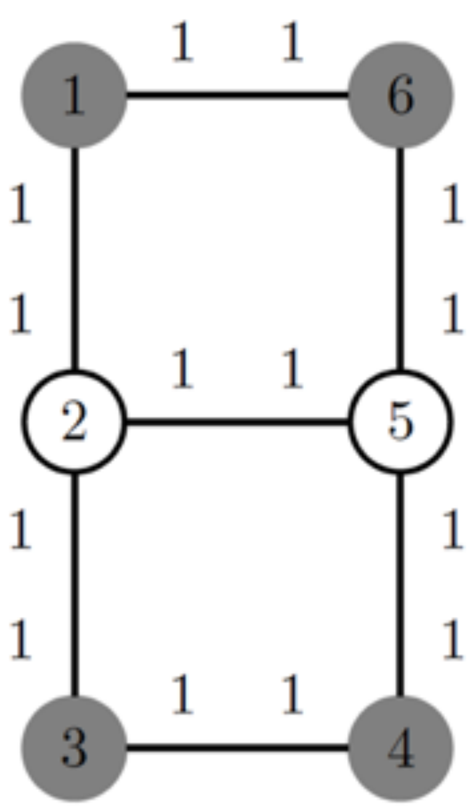
    $\forall_{k \in neighbors}\ \textit{received-improve}[k] \leftarrow$ NONE

SEND-OK$()$

if $\forall_{k \in neighbors}\ \textit{my-improve} \geq \textit{received-improve}[k]$
  then $x_i \leftarrow \textit{new-value}$
if $\textit{cost} > 0 \wedge \forall_{k \in neighbors}\ \textit{received-improve}[k] \leq 0 \triangleright$ quasi-local optimum
  then increase weight of constraint violations
$\forall_{k \in neighbors}\ k.\text{HANDLE-OK?}(i, x_i)$

Breakout Algorithm: Example



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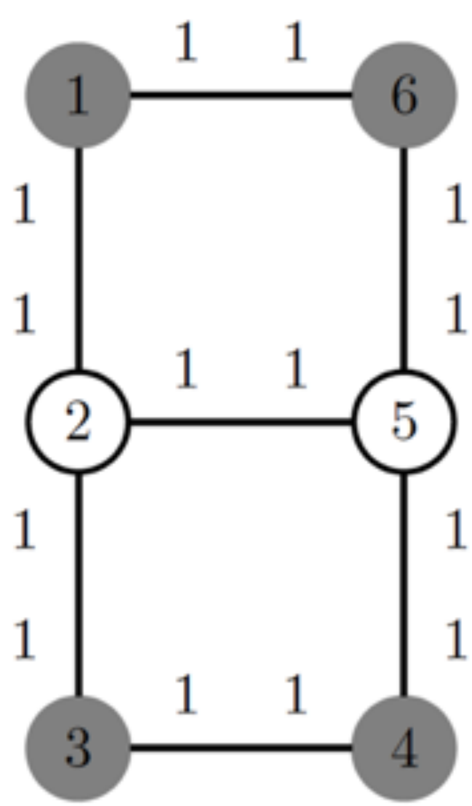
Breakout Algorithm: Example



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$my\text{-}improve \leftarrow$ possible maximal improvement

$\forall_{k \in neighbors}\ k.$HANDLE-IMPROVE$(i, my\text{-}improve, cost$

HANDLE-IMPROVE$(j, improve)$

$received\text{-}improve[j] \leftarrow improve$

if $\forall_{k \in neighbors}\ received\text{-}improve[k] \neq$ NONE

  then SEND-OK

    $agent\text{-}view \leftarrow \emptyset$

    $\forall_{k \in neighbors}\ received\text{-}improve[k] \leftarrow$ NONE

SEND-OK()

if $\forall_{k \in neighbors}\ my\text{-}improve \geq received\text{-}improve[k]$

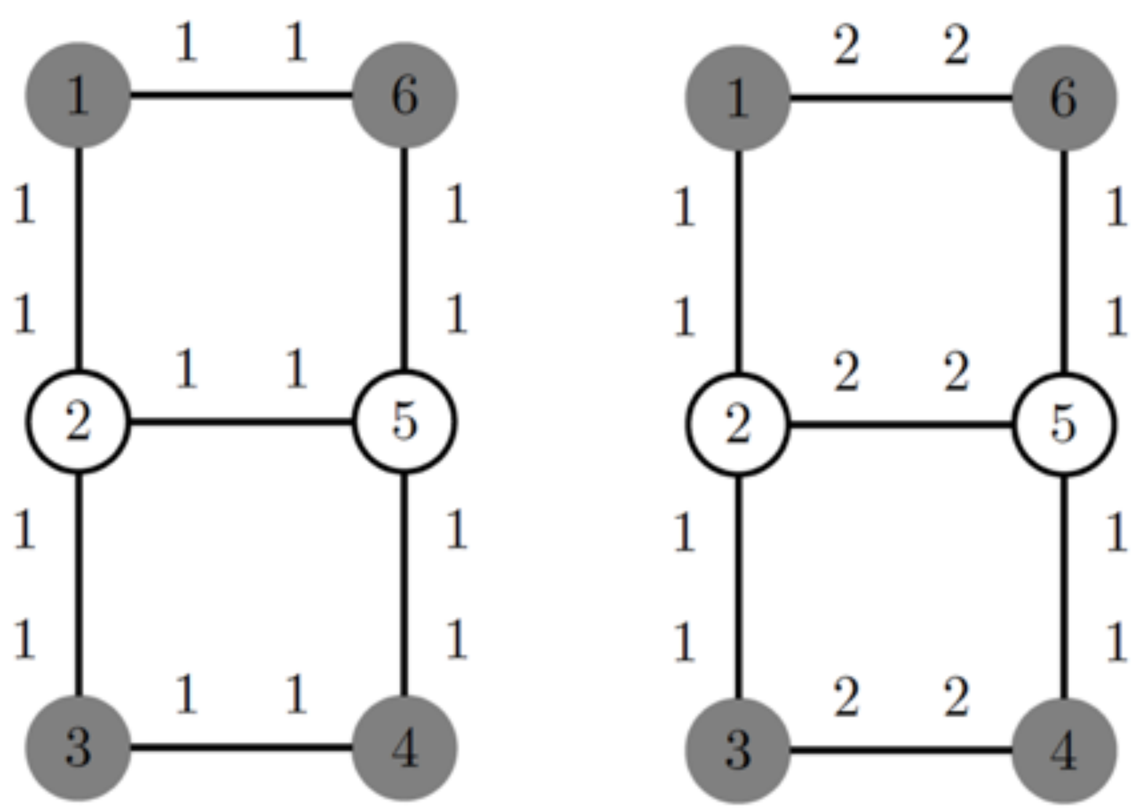
  then $x_i \leftarrow new\text{-}value$

if $cost > 0 \wedge \forall_{k \in neighbors}\ received\text{-}improve[k] \leq 0 \triangleright$

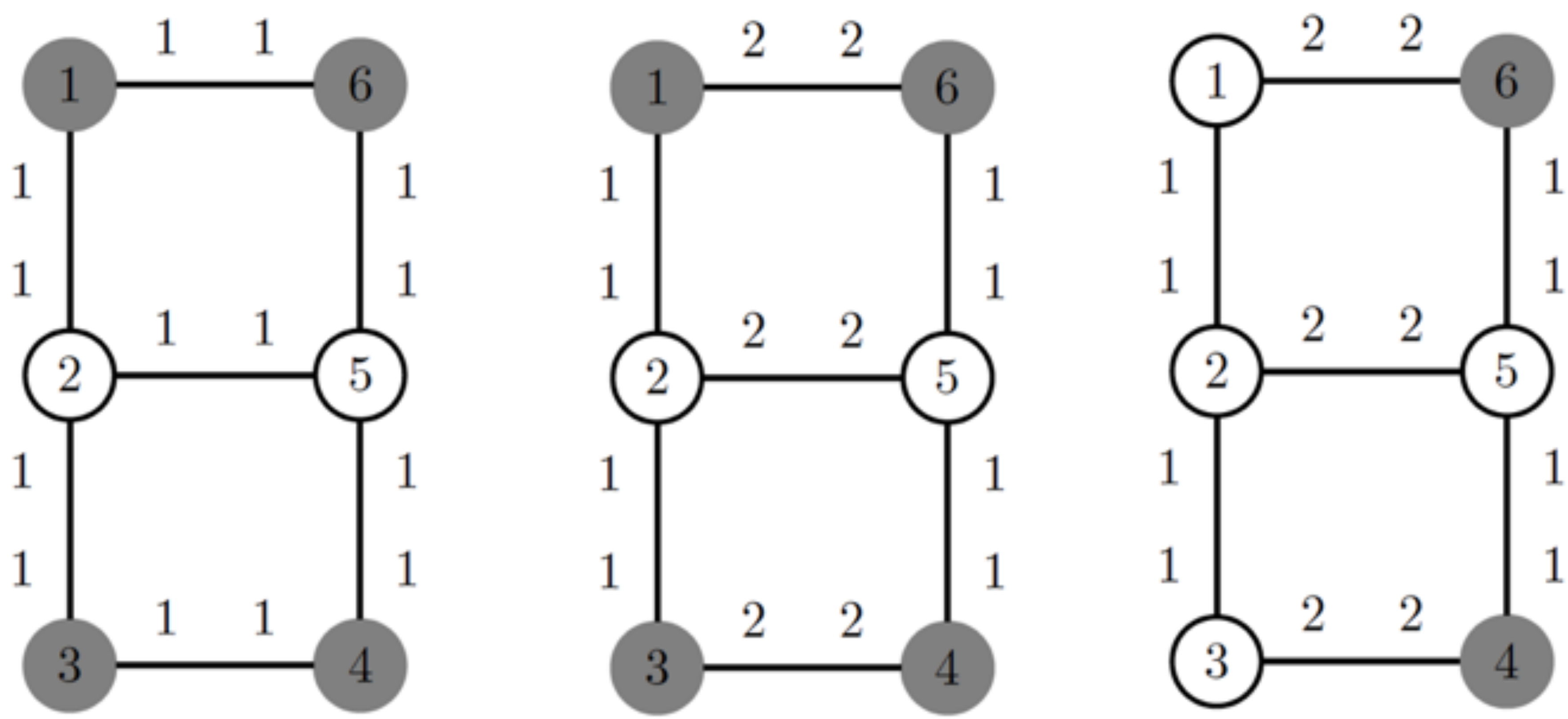
  then increase weight of constraint violations

$\forall_{k \in neighbors}\ k.$HANDLE-OK?$(i, x_i)$

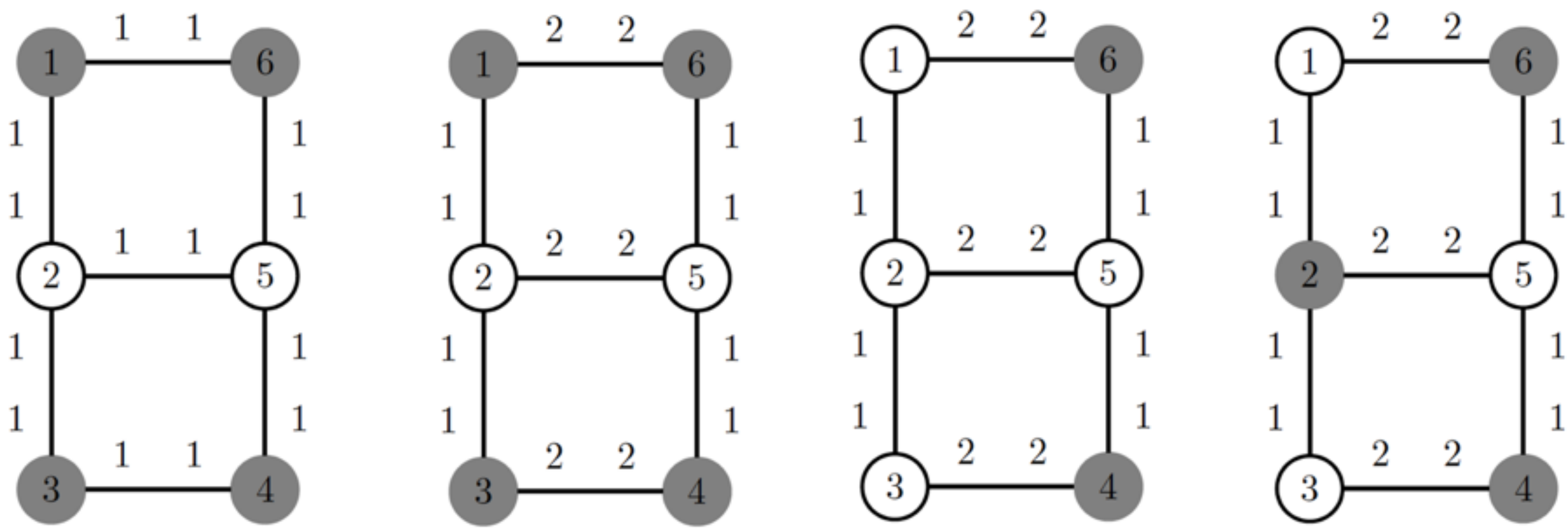
Breakout Algorithm: Example



Breakout Algorithm: Example



Breakout Algorithm: Example



Breakout Algorithm: Properties

- **Theorem (Distributed Breakout is not Complete)**
  Distributed breakout can get stuck in local minimum. Therefore, there are cases where a solution exists and it cannot find it.