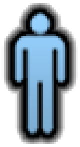
Auctions

Michal Jakob
Agent Technology Center,
Dept. of Computer Science and Engineering,
FEE, Czech Technical University

AE4M36MAS Autumn 2015 - Lecture 12

Where are We?

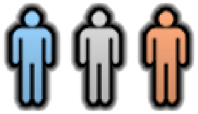
Agent architectures (inc. BDI architecture)



Logics for MAS

Non-cooperative game theory

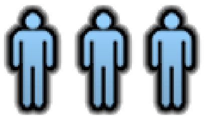
Cooperative game theory



Resource allocation and Auctions

Social choice

Distributed constraint reasoning



What is an Auction?

*An **auction** is a protocol that allows agents (=bidders) to indicate their **interests** in one or more **resources** and that uses these indications of interest to determine both an **allocation** of resources and a set of **payments** by the agents.* [Shoham & Leyton-Brown 2009]

Lecture Online

Properties of Single Item Auctions

Multi-Item Auctions

Exchanges

Basic Single-Item Auction Mechanisms

English

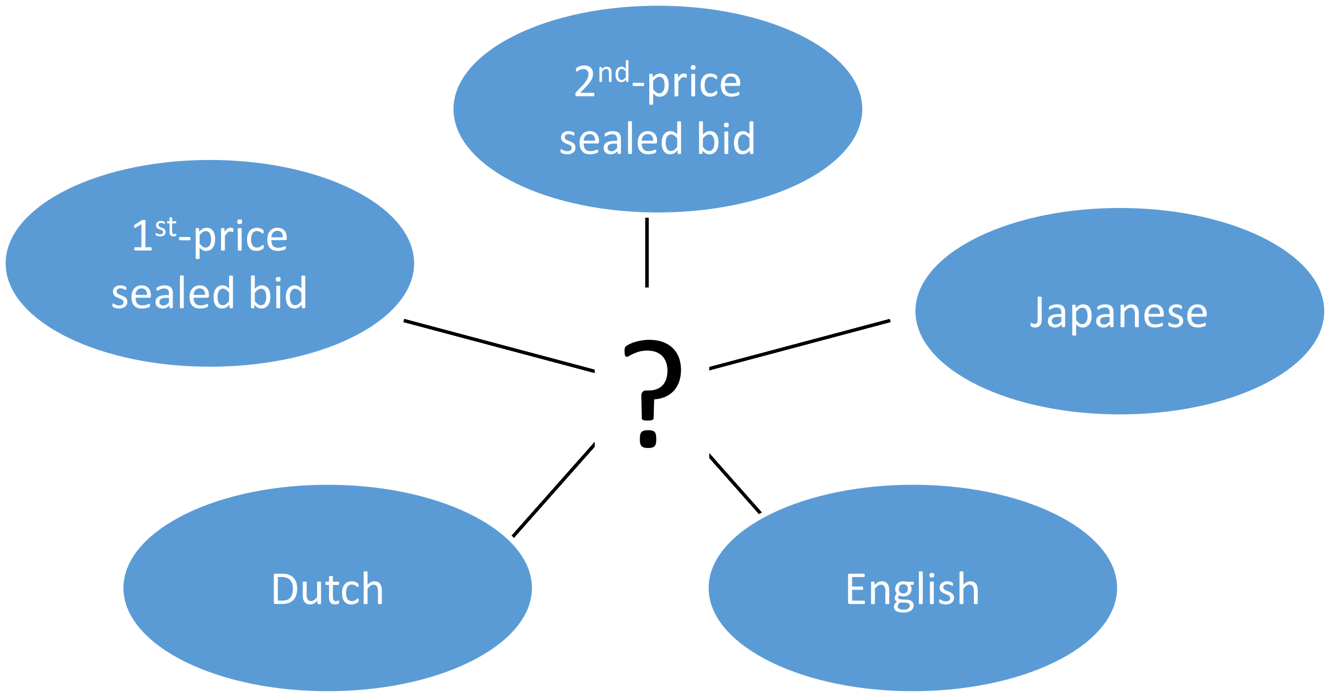
Japanese

Dutch

First-Price sealed bid

Second-Price sealed bid

Analysing Auctions



Are there fundamental similarities / differences between mechanisms described?

Two Problems

Auction **mechanism analysis**

- determine the properties of a given auction mechanism
- methodology: treat auctions as (extended-form) *Bayesian games* and analyse players' (i.e. bidders') strategies

Auction **mechanism design**

- design the auction mechanism (i.e. the game for the bidders) with the desirable properties
- methodology: apply mechanism design techniques

Bayesian Game

Definition (Bayesian Game)

A Bayesian game is a tuple $\langle N, A, \Theta, p, \boldsymbol{u} \rangle$ where

- N is the set of **players**
- $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, Θ_i is the **type space** of player i
- $A = A_1 \times A_2 \times \cdots \times A_n$ where A_i is the **set of actions** for player i
- $p: \Theta \mapsto [0,1]$ is a **common prior over types**
- $\boldsymbol{u} = (u_1, \dots, u_n)$, where $u_i: \Theta \mapsto \mathbb{R}$ is the **utility function** of player i

We assume that all of the above is **common knowledge** among the players, and that each **agent knows** his **own type**.

Bayes-Nash equilibrium: rational, risk-neutral players are seeking to maximize their expected payoff, given their beliefs about the other players' types.

Relation to Auctions

Sealed bid auction under IPV is a Bayesian game in which

- player i's actions correspond to his bids $\widehat{v_i}$
- player types Θ_i correspond to player's private valuations v_i over the auctioned item(s)
- the payoff of a player i corresponds to his/her valuation of the item v_i – its bid $\widehat{v_i}$

(Desirable) Properties

Truthfulness: bidders are incentivized to bid their *true* valuations, i.e.

$$v_i = \widehat{v_i} \ \forall i \forall v_i$$

Efficiency: the aggregated value of bidders is maximized, i.e.

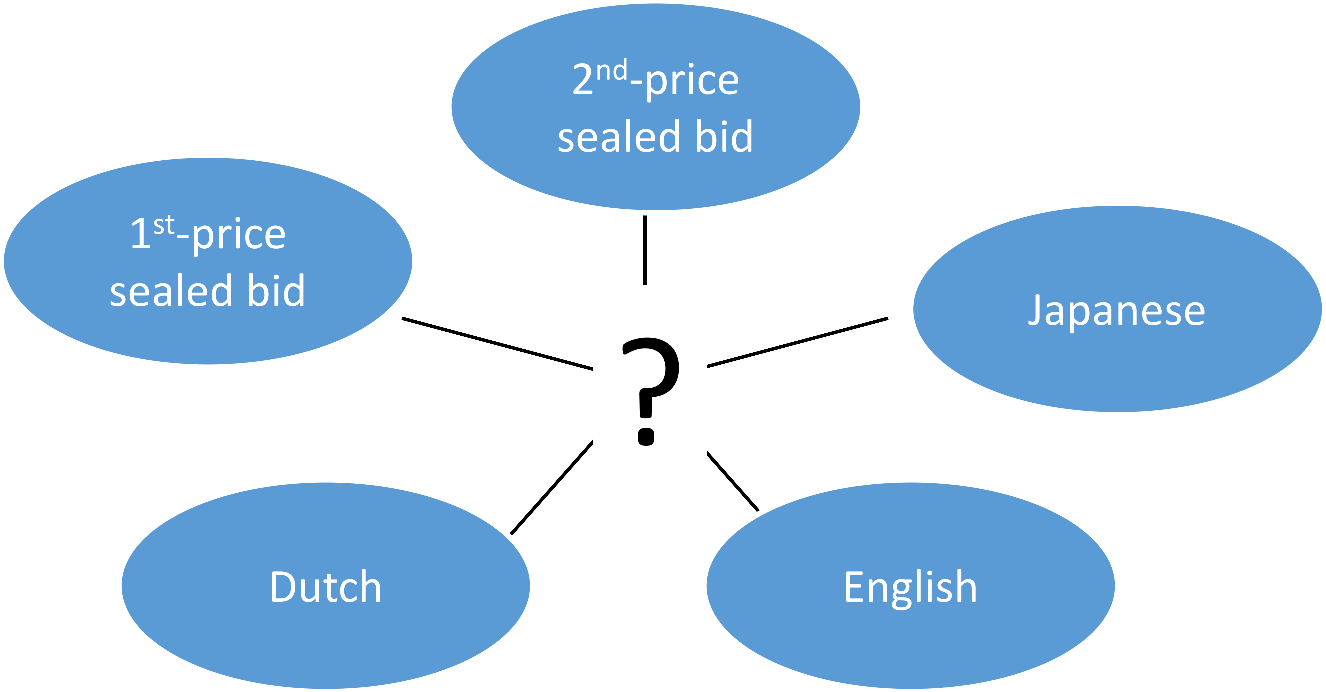
$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x')$$

Optimality: maximization of seller's revenue

Strategy: existence of dominant strategy

Manipulation vulnerability: lying auctioner, shills, bidder collusion

Other consideration: communication complexity, private information revelation, ...



Are there fundamental similarities / differences between mechanisms described?

Second-Price Sealed Bid

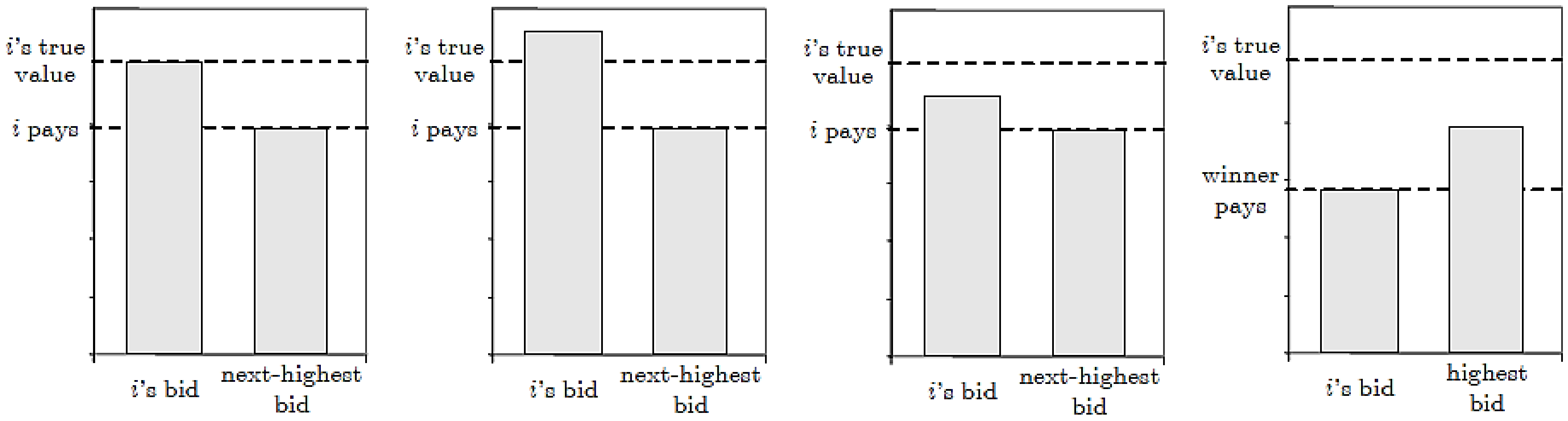
Theorem

Truth-telling is a **dominant strategy** in a second-price sealed bid auction (assuming independent private values (IPV) model and risk neutral bidders).

Proof: Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

- Bidding honestly, i would win the auction
- Bidding honestly, i would lose the auction

Second-Price Sealed Bid Proof



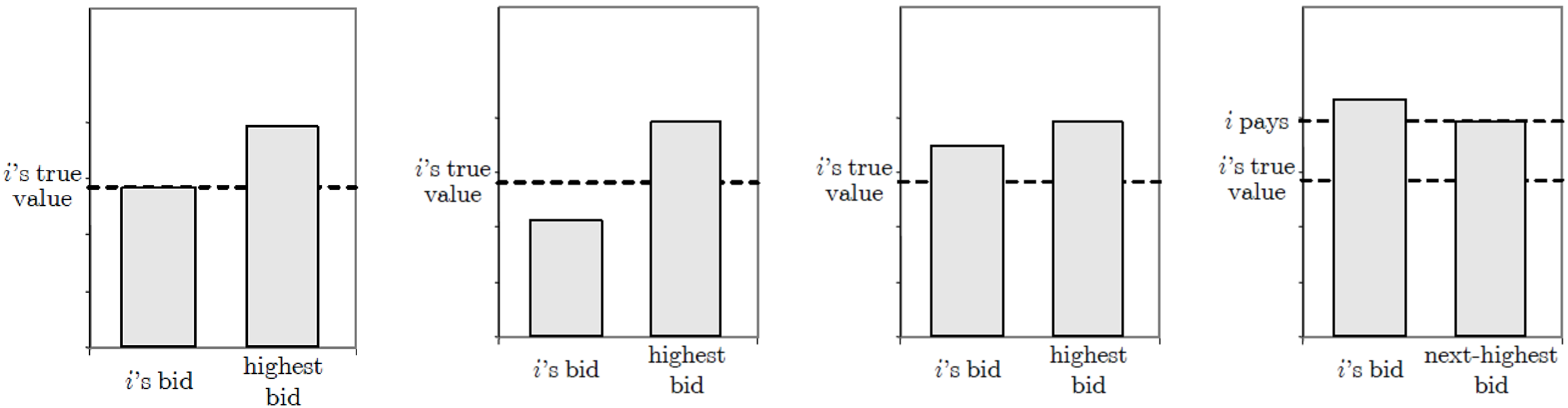
Bidding honestly, i is the winner

If i bids higher, he will still win and still pay the same amount

If i bids lower, he will either still win and still pay the same amount. . .

... or lose and get the payoff of zero.

Second-Price Sealed Bid Proof



Bidding honestly, i is not the winner

If i bids lower, he will still lose and still pay nothing

If i bids higher, he will either still lose and still pay nothing...

... or win and pay more than his valuation (⇒ negative payoff).

Second-Price Sealed Bid

Advantages:

- Truthful bidding is dominant strategy
- No incentive for counter-speculation
- Computational efficiency

Disadvantages:

- Lying auctioneer
- Bidder collusion self-enforcing
- Not revenues maximizing

Unfortunately, the auction is not very popular in real life due to its counter-intuitiveness

- but very successful in computational auction systems (e.g. Adwords)

Dutch and First-price Sealed Bid

Strategically equivalent: an agent bids without knowing about the other agents' bids

- a bidder must decide on the amount he's willing to pay, conditional on having placed the highest bid

Differences

- First-price auctions can be held **asynchronously**
- Dutch auctions are **fast**, and require **minimal communication**

Bidding in Dutch / First Price Sealed Bid?

Bidders don't have a **dominant strategy** any more:

- there's a **trade-off** between **probability of winning** vs. **amount paid** upon winning
- **individually optimal** strategy depends on **assumptions** about **others' valuations**

Assume a **first-price auction** with **two risk-neutral bidders** whose valuations are drawn independently and **uniformly** at random from the interval [0, 1] - what is the equilibrium strategy?

→ $\left(\frac{1}{2}v_1, \frac{1}{2}v_2\right)$ is the Bayes-Nash equilibrium strategy profile

⇒ Dutch / FPSB auctions **not incentive compatible,** i.e., there are incentives to counter-speculate.

Bidding in Dutch / First Price Sealed Bid?

Theorem

In a first-price sealed bid auction with n **risk-neutral** agents whose valuations $v_1, v_2, \dots, v_n$ are **independently** drawn from a **uniform distribution** on the **same** bounded **interval** of the real numbers, the **unique** symmetric **equilibrium** is given by the **strategy profile** $\left(\frac{n-1}{n}v_1, \dots, \frac{n-1}{n}v_n\right)$.

For non-uniform valuation distributions: Each bidder should bids the expectation of the second-highest valuation, conditioned on the assumption that his own valuation is the highest.

English and Japanese Auctions Analysis

A much more complicated **strategy space**

- extensive-form game
- bidders are able to condition their bids on information revealed by others
- in the case of English auctions, the ability to place jump bids

Intuitively, though, the **revealed information** doesn't make any **difference** in the **independent-private value** (IPV) setting.

- proxy bidding

English and Japanese Auctions Analysis

Theorem

Under the IPV model, it is a **dominant strategy** for bidders to bid **up to** (and not beyond) their valuations in both Japanese and English auctions.

In correlated-value auctions, it can be worthwhile to counter-speculate

Revenue Equivalence

Which auction should an auctioneer choose?

To some extent, it doesn't matter...

Theorem (Revenue Equivalence)

Assume that each of n **risk-neutral** agents has an **independent private valuation** for a single good at auction, drawn from a **common cumulative distribution** $F(v)$ that is **strictly increasing** and **atomless** on $[\underline{v}, \overline{v}]$. Then any auction mechanism in which

1. the good will be allocated to the agent with the highest valuation; and
2. any agent with valuation $\underline{v}$ has an expected utility of zero

yields the **same expected revenue**, and hence results in any bidder with valuation v making the same expected payment.

What about Efficiency?

Efficiency in single-item auctions: the item allocated to the agent who values it the most.

With independent private values (IPV):

Auction	Efficient
English (without reserve price)	yes
Japanese	yes
Dutch	no
Sealed bid second price	yes
Sealed bid first price	no

Efficiency (often) lost in the correlated value setting.

Optimal Auctions

Optimal Auction Design

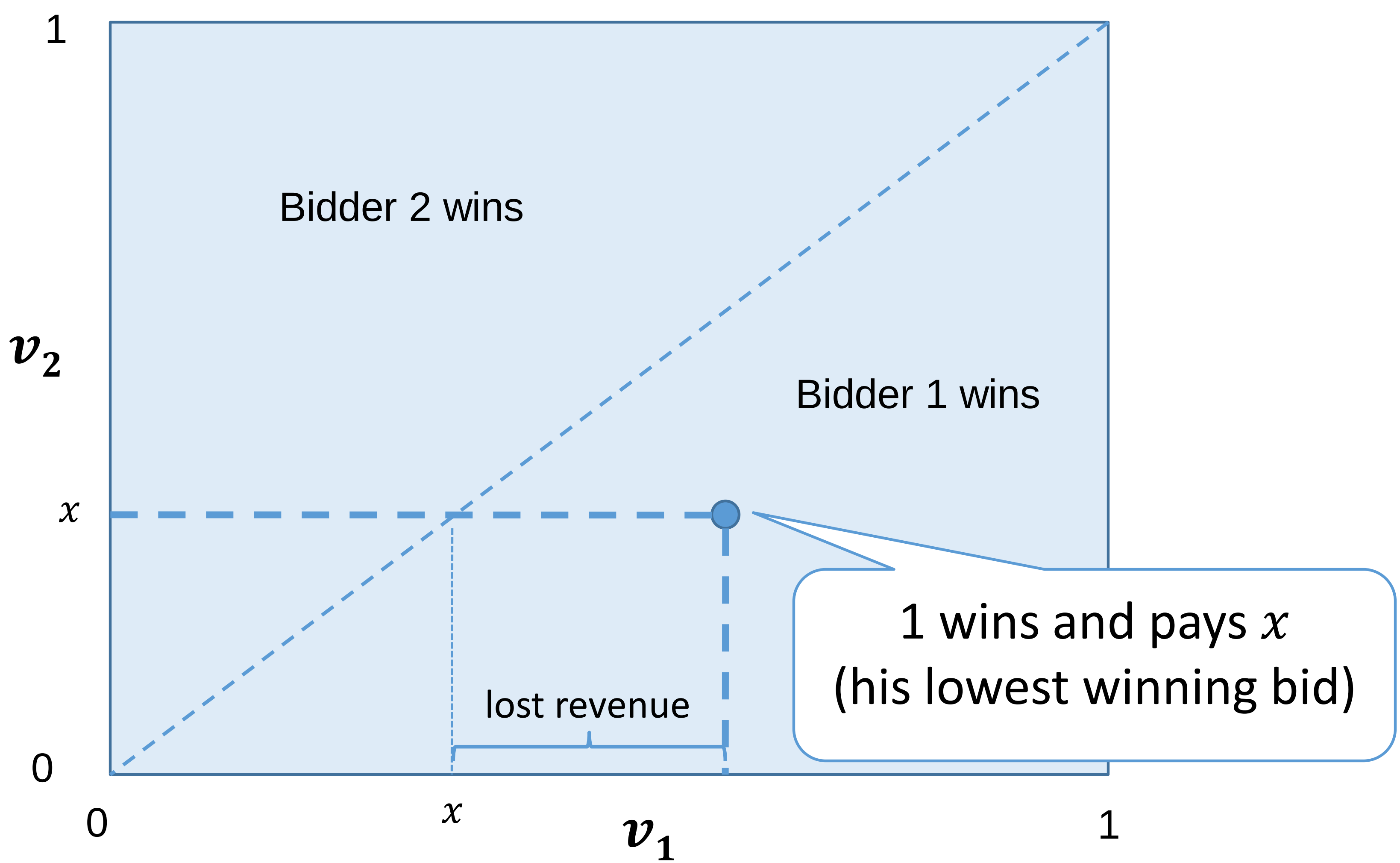
The seller's problem is to **design an auction mechanism** which has a Nash equilibrium giving him the **highest possible expected utility**.

- assuming individual rationality

Second-prize sealed bid auction **does not maximize** expected revenue → not a very good choice if profit maximization is important.

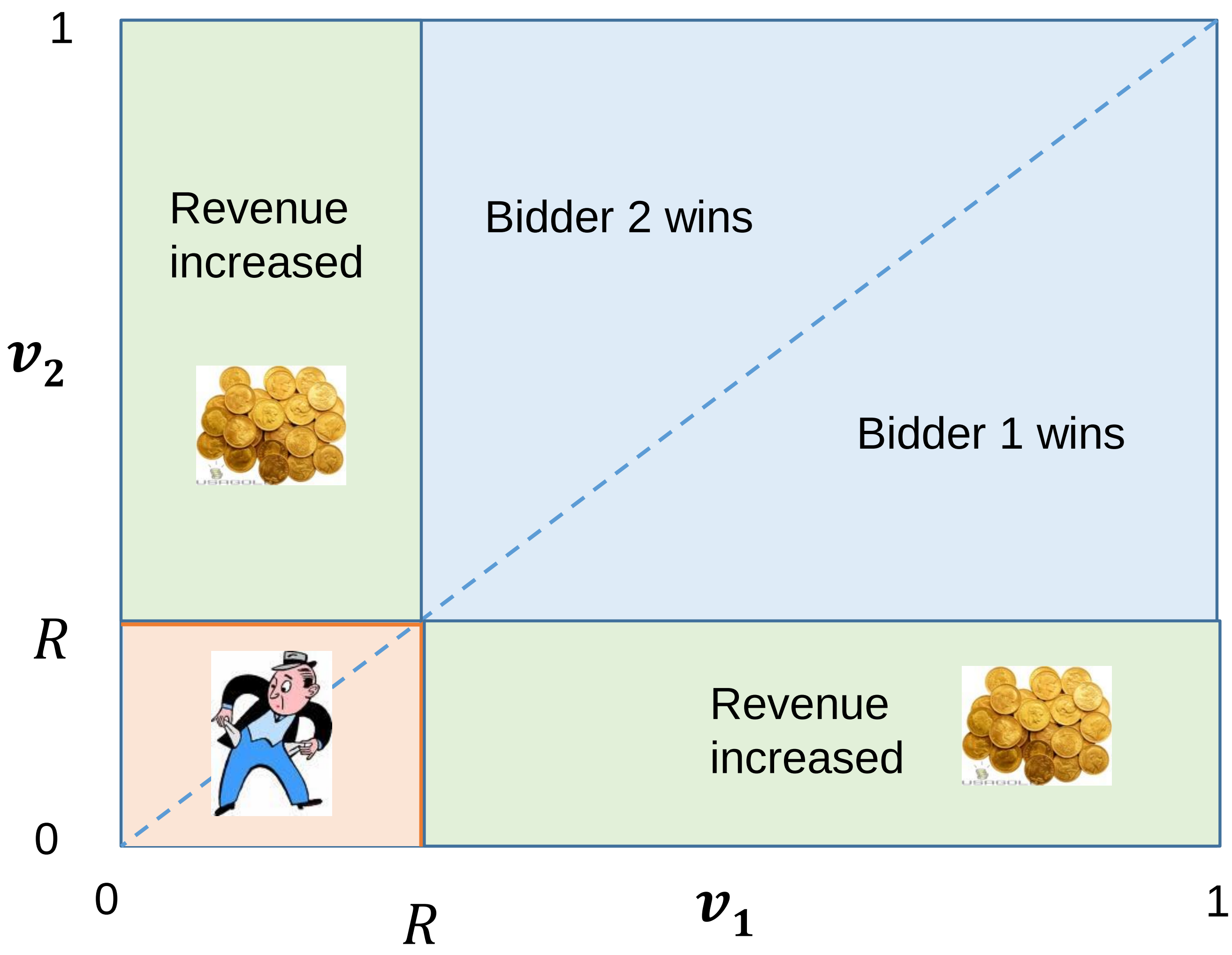
Can we get better revenue?

Let's have another look at 2^{nd} price auctions:

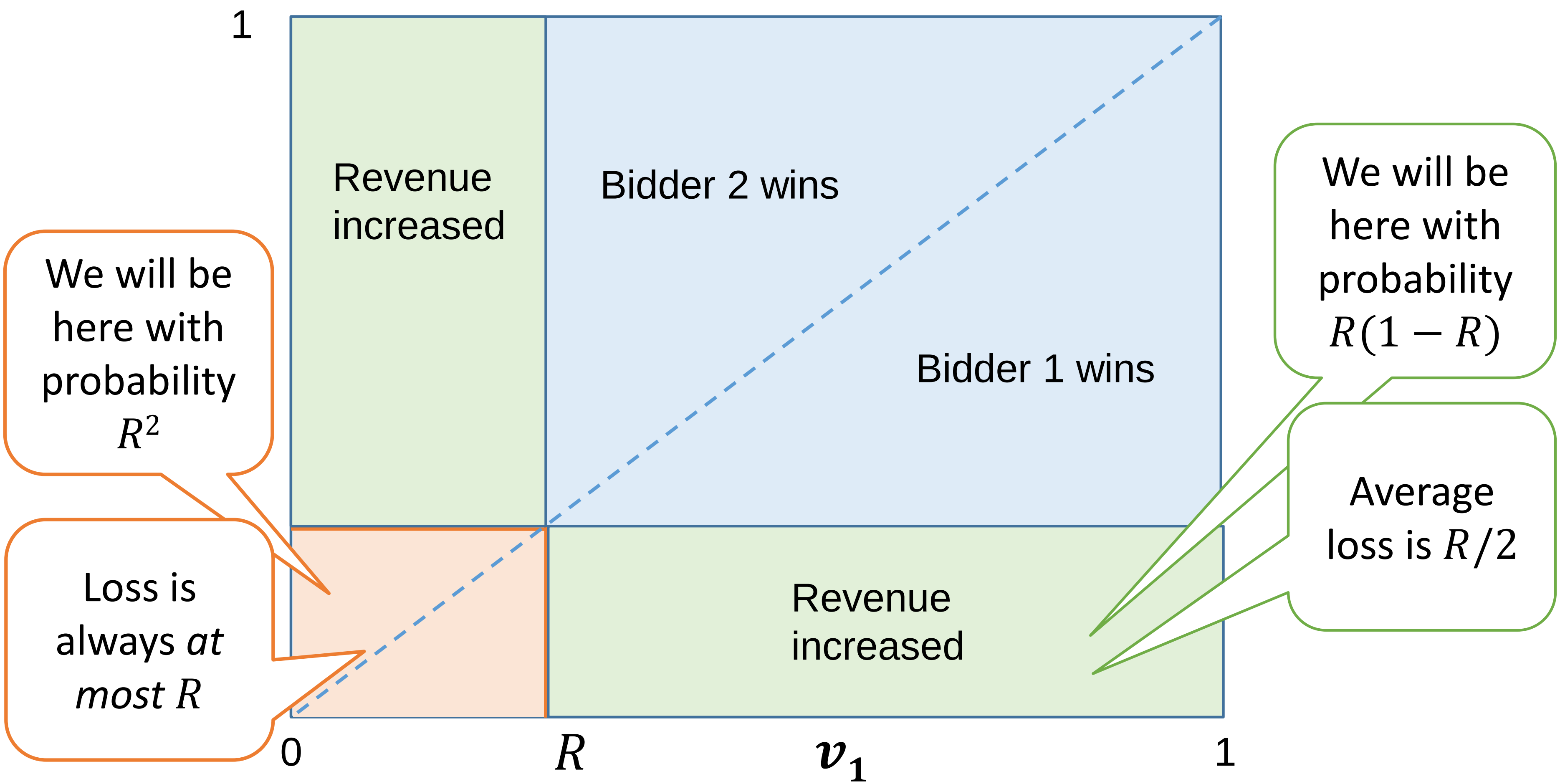


Can we get better revenue?

Some reserve price improve revenue.



Can we get better revenue?



Gain is at least: $\frac{2R(1-R)R}{2} = R^2 - R^3$

Loss is at most: $R^2\,R = R^3$

When $R^2 - 2R^3 > 0$,

reserve price of R is beneficial.

(for example, $R = 1/4$)

Optimal Single Item Auction

Definition (Virtual valution)

Consider an **IPV setting** where bidders are **risk neutral** and each bidder i's valuation is drawn from some **strictly increasing** cumulative density function $F_i(v)$, having probability density function $f_i(v)$. We then define:
where

- Bidder i's **virtual valuation** is $\psi_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$
- Bidder i's **bidder-specific reserve price** r_i^* is the value for which $\psi_i(r_i^*) = 0$

Example: uniform distribution over $[0,1]$: $\psi(v) = 2v - 1$

Optimal Single Item Auction

Theorem (Optimal Single-item Auction)

The optimal (single-good) auction is a **sealed-bid auction** in which every agent is asked to declare his valuation. The good is sold to the agent $i = \mathrm{argmax}_{\mathrm{i}} \psi_i(\hat{v}_i)$, as long as $\hat{v}_i > r_i^*$.
If the good is sold, the winning agent i is charged the smallest valuation that he could have declared while still remaining the winner:

$$\inf\{v_i^* : \psi_i(v_i^*) \geq 0 \land \forall j \neq i, \psi_i(v_i^*) \geq \psi_j(\hat{v}_j)\}$$

The virtual valuations also increase weak bidders' bids, making them more competitive.

- Low bidders can win, paying less.
- However, bidders with higher expected valuations must bid more aggressively

Second-Price Auction with Reservation Price

Symmetric case: second-price auction with reserve price r^* satisfying: $\psi(r^*) = r^* - \frac{1-F(r^*)}{f(r^*)} = 0$

- Truthful mechanism when $\psi(v)$ is non-decreasing.
- Uniform distribution over $[0, p]$: optimum reserve price $p/2$.

Second-price sealed bid auction with Reserve Price is **not efficient**!

Optimal Auctions: Remarks

Always: **revenue ≤ efficiency**

- Due to **individual rationality**
- More efficiency makes the pie larger!

However, for **optimal revenue** one needs to **sacrifice** some **efficiency**.

Optimal auctions are not **detail-free:**

- they require the seller to incorporate information about the bidders' valuation distributions into the mechanism.

Theorem (Bulow and Klemperer): *revenue* of an efficiency-maximizing auction with *k+1* bidder is at least as high as that of the revenue-maximizing one with *k* bidders.

➔ better to spend energy on attracting more bidders

Multi-Item Auctions

Multi-Item Auctions

Combinatorial Auctions

Auctions for **bundles of goods**

Let $\mathcal{Z} = \{z_1, \ldots, z_n\}$ be a set of items to be auctioned

A **valuation function** $v_i : 2^{\mathcal{Z}} \mapsto \Re$ indicates how much a bundle $Z \subseteq \mathcal{Z}$ is worth to agent i

Properties

- **normalization**: $v(\emptyset) = 0$
- **free disposal**: $Z_1 \subseteq Z_2$ implies $v(Z_1) \leq v(Z_2)$

Combinatorial auctions are interesting when the valuation function is **not additive**

- **complementarity**: $v(Z_1 \cup Z_2) > v(Z_1) + v(Z_2)$ (e.g. left and right shoe)
- **substitutability**: $v(Z_1 \cup Z_2) < v(Z_1) + v(Z_2)$ (e.g. cinema tickets for the same time)

Allocation

Allocation is a list of sets $Z_1, \ldots, Z_n \subseteq \mathcal{Z}$, one for each agent i such that $Z_i \cap Z_j = \emptyset$ for all $i \neq j$ (i.e. not good allocated to more than one agent)

How to define allocation for combinatorial auction?

→ Maximize **social welfare**:

$$U(Z_1, \ldots, Z_n, v_1, \ldots, v_n) = \sum_{i=1}^{n} v_i(Z_i)$$

Winner Determination Problem

Definition

The **winner determination problem** for a combinatorial auctions, given the agents' declared valuations $\hat{v}_i$ is to find the social-**welfare-maximizing allocation** of goods to agents. This problem can be expressed as the following integer program

$$\text{maximize} \sum_{i \in N} \sum_{Z \subseteq \mathcal{Z}} \hat{v}_i(Z) x_{Z,i}$$

$$\text{subject to} \sum_{Z, j \in Z} \sum_{i \in N} x_{Z,i} \leq 1 \quad \forall j \in \mathcal{Z}$$

$$\sum_{Z \subseteq \mathcal{Z}} x_{Z,i} \leq 1 \qquad \forall i \in N$$

$$x_{Z,i} = \{0,1\} \qquad \forall Z \subseteq \mathcal{Z}, i \in N$$

Issues with Winner Determination

Communication complexity

Computation complexity

- Solution 1: Require bids to come from a restricted set, guaranteeing that the WDP can be solved in polynomial time
  - problem: these restricted sets are very restricted...
- Solution 2: Use heuristic methods to solve the problem
  - this works pretty well in practice, making it possible to solve WDPs with many hundreds of goods and thousands of bids.

Exchanges

Exchanges / Two-sided Auctions

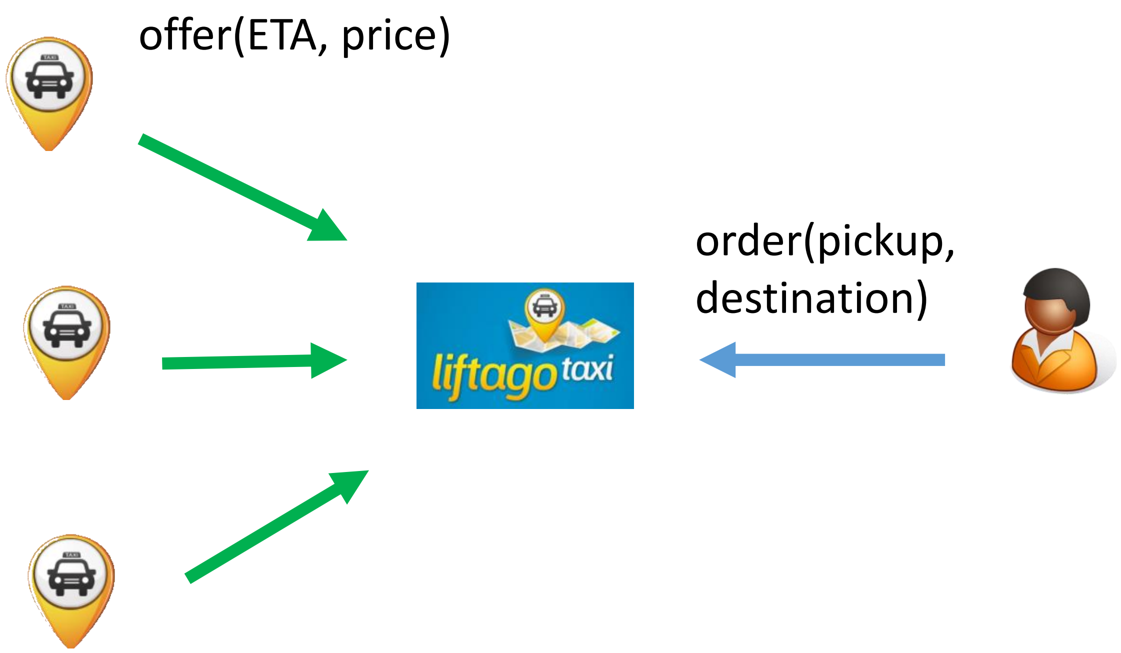
Bidding:

- Each bid consists of a price and quantity (<0: buy; >0: sell)
- Bids put into a central repository: the **order book**

Clearing:

- **Continuous** double auctions trades attempted each time a bid is received
- **Periodic** double auction: clearing at predetermined intervals

Example Application



Essentially a multi-attribute reverse single-good auction

- Which mechanism to use?
- How to select the taxi drivers to address?

Auctions Summary

Auctions are mechanisms for **allocating scarce resource** among **self-interested agent**

Mechanism-design and game-theoretic perspective

Many auction mechanisms: English, Dutch, Japanese, First-price sealed bid, Second-price sealed bid

Desirable properties: truthfulness, efficiency, optimality, ...

Rapidly expanding list of **applications** worth billions of dollars

Reading:

- [Shoham] – Chapter 11

MAS Course Summary

Logics for MAS: Formally describe and analyze (multiple) agents

Agent architectures: acting rationally in an environment

Non-cooperative game theory: acting rationally in strategic interactions

Coalitional game theory: making rational decisions about collaboration

Distributed constraint reasoning: coordinating cooperative action

Social choice: aggregating individual preferences into a collective choice

Multiagent Resource Allocation and Auctions: distributing scarce resources

Many topics not covered: bargaining / negotiation, multiagent learning, multiagent planning, mechanism design, agent-oriented software engineering

Many interconnections

Final Notes

Rapidly evolving field with the exploding number of applications
→ http://agents.cz for (Ph.D.) opportunities

Exam

- 21th Jan (+ 4th Feb?)
- mostly written

Survey/Anketa: be as specific possible: we *do* care