Multiagent Resource Allocation 1

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Where are We?

Agent architectures (inc. BDI architecture)

Logics for MAS

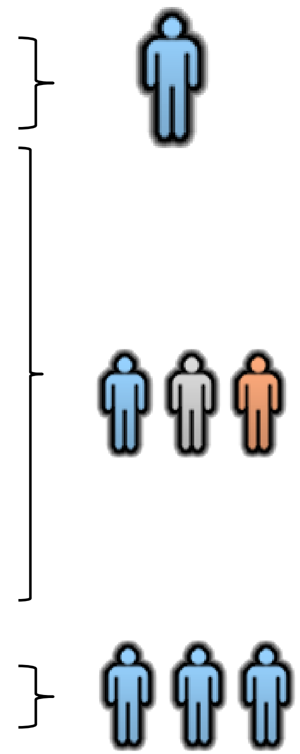
Non-cooperative game theory

Cooperative game theory

Resource allocation and Auctions

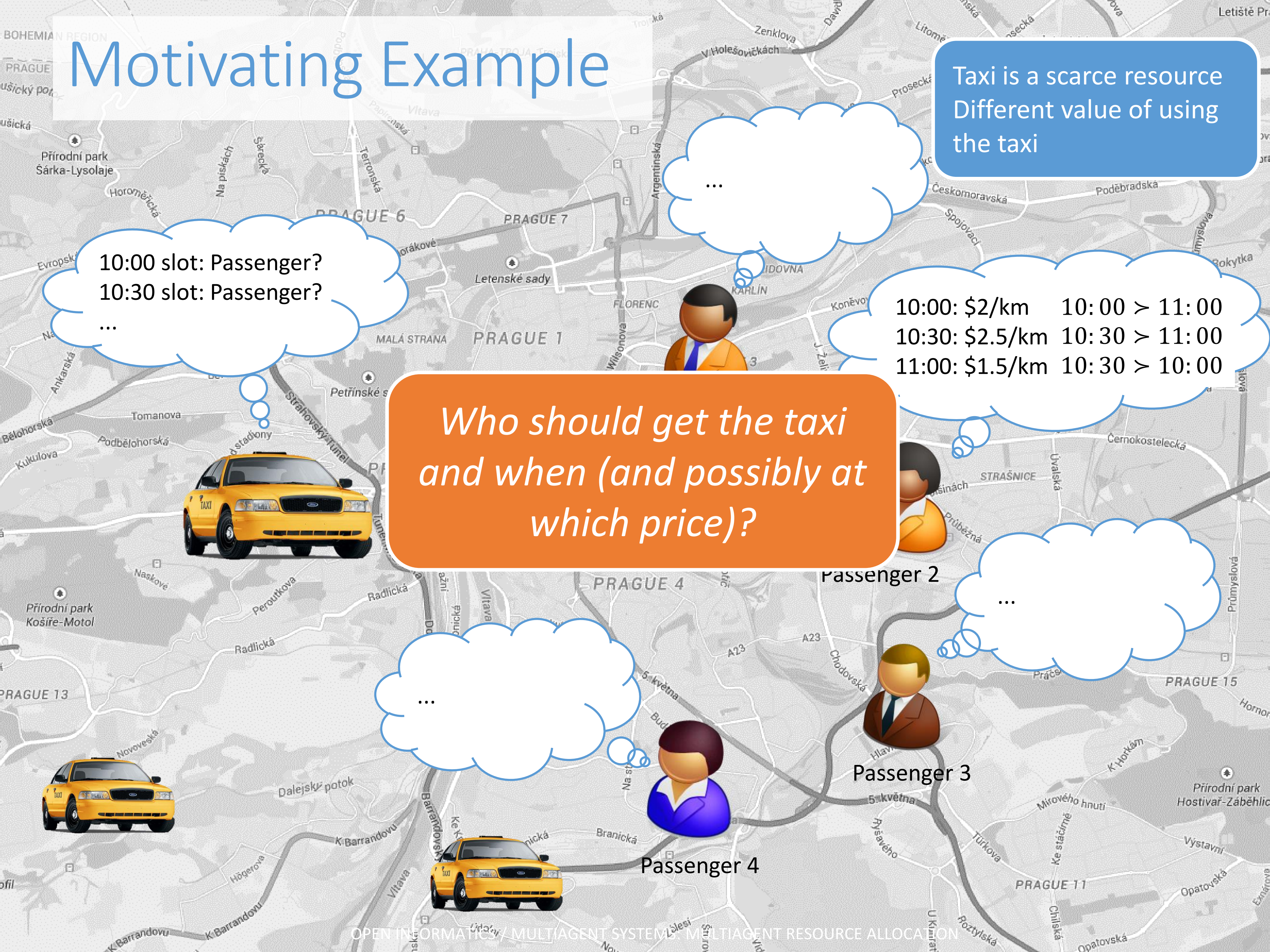
Social choice

Distributed constraint reasoning



Motivating Example

Taxi is a scarce resource
Different value of using the taxi



Who should get the taxi and when (and possibly at which price)?

Lecture Online

Introduction

Multiagent Resource Allocation

- Type of resources
- Preference representation
- Social Welfare

Auction Mechanisms

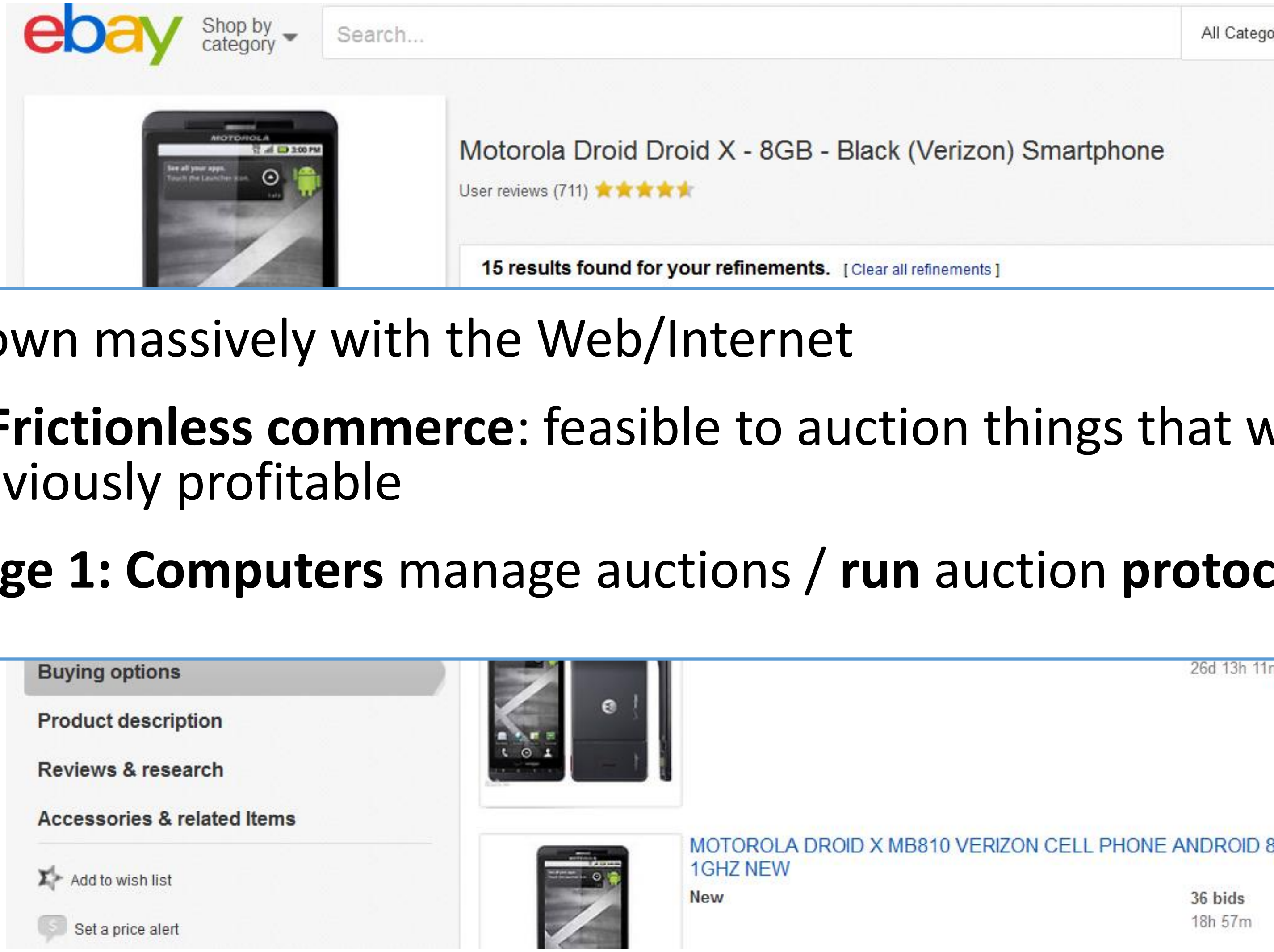
- Basic Definitions
- Single-good auction mechanisms
- Analysis of auction mechanisms

Auctions: Traditional

Auctions used in Babylon as early as 500 B.C. but used to be rare (not so long ago)

Stage 0: No automation

Auctions: Partial Automation

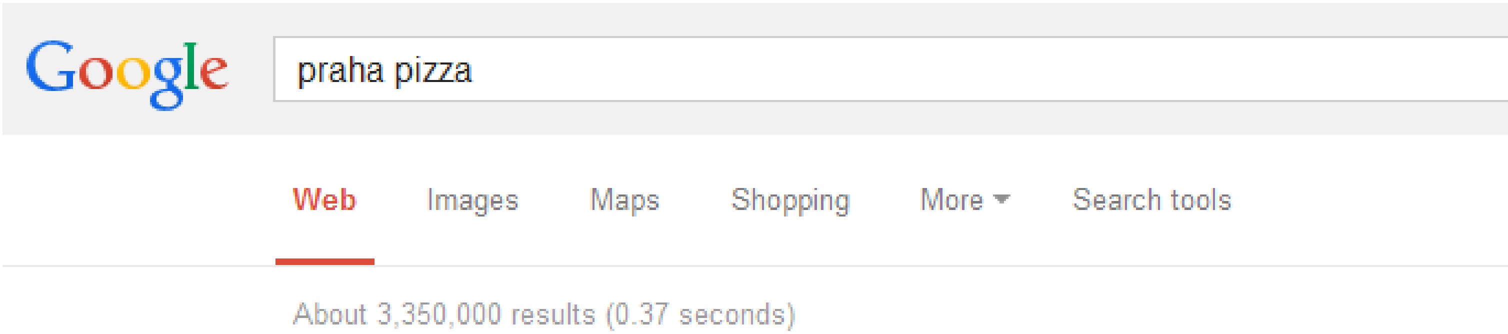


Grown massively with the Web/Internet

→ **Frictionless commerce**: feasible to auction things that weren't previously profitable

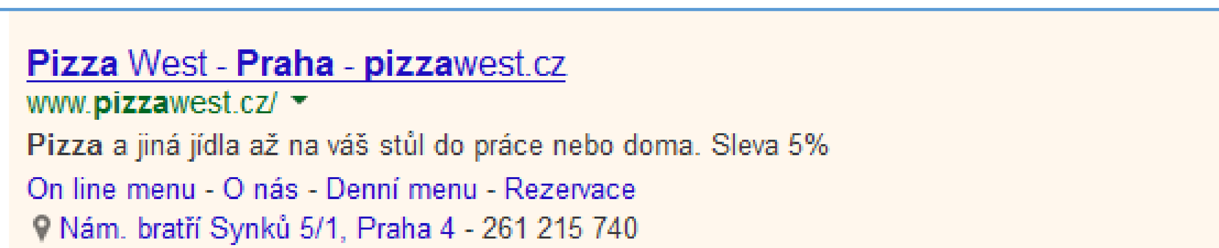
Stage 1: Computers manage auctions / **run** auction **protocols**

Auctions: (Almost) Full automation



Stage 2: Computers also automate the decision making of bidders

Concerns:
(1) the most relevant adds are shown and
(2) auctioner's **profit is maximized**



Lots of Applications

Industrial procurement

Transport and logistics

Energy markets

Cloud and grid computing

Internet auctions

(Electromagnetic spectrum allocation)

... and counting!

Multiagent Resource Allocation (MARA)

What is Multiagent Resource Allocation?

Multiagent Resource Allocation (MARA) is the process of distributing a number of items amongst a number of agents.

- **What** kind of items (resources) are being distributed?
- **How** are they being distributed?
- **Why** are they being distributed?

Classification of MARA

1. Resources (What)
2. Agent (i.e. individual) preferences (Why)
3. Social (i.e. collective) welfare (Why)
4. Allocation mechanism (How)

Link to **social choice**: allocations are alternatives agents express preference over.

Link to **game theory**: allocation mechanisms are games.

Type of Resources

Central parameter in any resource allocation problem.

Different **types** of resources may require different resource allocation **techniques**.

Inherent **properties** of the **resource** vs. **characteristics** of the chosen **mechanism**.

Terminology: **resource** ~ **goods**.

Types of Resources

Continuous vs. **Discrete**

Divisible vs. **Indivisible**

- discrete resources indivisible; continuous can be treated either way

Sharable vs. **Non-Sharable**

- sharable: e.g. a path in a network

Static vs. **Non-Static**

- non-static: e.g. perishable goods

Single-Unit vs. **Multi-Unit**

Resources vs. Tasks

Tasks may be considered resources with **negative** utility (cost).

Task allocation may be regarded a multiagent resource allocation problem.

- However, tasks are often coupled with **constraints** regarding their **coherent combination** (timing and ordering).

Preference Representation

Preference Representation

Agents may have **preferences** over

- the bundle of resources they receive
- the bundles of resources received by others (**externalities**)

What are suitable **languages** for representing agent **preferences**?

Preference Representation Languages

Expressive power

Succinctness

Complexity

Cognitive relevance

Elicitation

Cardinal vs. Ordinal Preferences

A **preference structure** represents an agent's preferences over a set of alternatives $\mathcal{X}$ (i.e. allocations in the MARA case).

- **Cardinal preference** structure is a function $u\colon \mathcal{X} \mapsto Val$, where Val is usually a set of numerical values such as $\mathbb{N}$ or $\mathbb{R}$.
- **Ordinal preference** structure is a *binary relation* $\preccurlyeq$ over the set of alternatives, that is *reflexive* and *transitive* (and connected).

If the alternatives over which agents have to express preferences are *bundles of indivisible resources* from the set $\mathcal{R}$, then we have $\mathcal{X} = 2^{\mathcal{R}}$.

Example

Hanging a picture with a frame (f), a hammer (h) and a nail (n)

Cardinal

X	$u(X)$
$\{\}$	0
$\{f\}$	0
$\{h\}$	0
$\{n\}$	10
$\{f,h\}$	0
$\{f,n\}$	20
$\{h,n\}$	15
$\{f,h,n\}$	50

Ordinal

$\succcurlyeq$	$\{\}$	$\{f\}$	$\{h\}$	$\{n\}$	$\{f,h\}$	$\{f,n\}$	$\{h,n\}$	$\{f,h,n\}$
$\{\}$	1	0	0	0	0	0	0	0
$\{f\}$	1	1	1	0	1	0	0	0
$\{h\}$	1	1	1	0	1	0	0	0
$\{n\}$	1	1	1	1	1	0	0	0
$\{f,h\}$	1	1	1	0	1	0	0	0
$\{f,n\}$	1	1	1	1	1	1	1	0
$\{h,n\}$	1	1	1	1	1	0	1	0
$\{f,h,n\}$	1	1	1	1	1	1	1	1

Preferences Properties

	Cardinal	Ordinal
Intrapersonal comparison	yes	Yes
Interpersonal comparison	yes	No
Preference intensity	yes	No
Cognitive relevance	lower	higher
Explicit representation	$\mathcal{O}(\lvert\mathcal{X}\rvert)$	$\mathcal{O}(\lvert\mathcal{X}\rvert^2)$

Social Welfare

A third parameter in the specification of a MARA problem concerns our goals: **what kind of allocation do we want to achieve?**

We use the term **social welfare** in a very broad sense to describe **metrics** for assessing the **quality** of an **allocation** of resources.

Efficiency and Fairness

Two key indicators of social welfare.

Aspects of **efficiency*** include:

- The chosen agreement should be such that there is no alternative agreement that would be better for some and not worse for any of the other agents (**Pareto optimality**).
- If preferences are quantitative, the sum of all payoffs should be as high as possible (**utilitarianism**).

Aspects of **fairness** include:

- No agent should prefer to take the bundle allocated to one of its peers rather than keeping their own (**envy-freeness**).
- The agent that is going to be worst off should be as well off as possible (**egalitarianism**).

*not in the computational sense

Notation

Set of **agents** $\mathcal{A} = \{1, \ldots, n\}$

Agents have **preferences over allocations** $X \in \mathcal{X}$:

- **ordinal**: $X \preccurlyeq_i X'$ means agent i likes the allocation X no less than X'
- **cardinal**: $u_i(A) = u \in \mathbb{R}$ means agent i assigns utility u to allocation X

Utilitarian Social Welfare

Utilitarian Collective Utility

The **utilitarian** collective utility function sw_u is defined as the sum of individual utilities:

$$sw_u(X) = \sum_{i \in \mathcal{A}} u_i(X)$$

Maximizing utilitarian CUF improves **efficiency**.

The utilitarian CUF is **zero-independent**: adding a constant value to your utility function won't a affect social welfare judgements.

Egalitarian Social Welfare

Egalitarian Collective Utility

The **egalitarian** collective utility function sw_e is defined as the sum of individual utilities:

$$sw_e(X) = \min\{u_i(X) | i \in \mathcal{A}\}$$

Maximising this function amounts to improving the situation of the weakest members of society (→ **fairness**).

Allocation X' is strictly preferred over allocation X (by society) iff $sw_e(X) < sw_e(X')$ holds (so-called **maximin**-ordering).

Nash Product Social Welfare

Nash Collective Utility

The **Nash** collective utility function sw_e is defined as the sum of individual utilities:

$$sw_e(A) = \prod_{i \in \mathcal{A}} u_i(A)$$

This is a useful measure of social welfare as long as all utility functions can be assumed to be **positive**.

Nash CUF favours increases in overall utility, but also inequality-reducing redistributions ($2 \cdot 6 < 4 \cdot 4$).

The Nash CUF is **scale independent**: whether a particular agent measures their own utility in euros or dollars does not affect social welfare judgements.

Social Welfare Curve Illustration

Allocation Procedures

Allocation Procedures

Protocols: What messages do agents have to exchange and in which order?

Strategies: What strategies may an agent use for a given protocol? How can we give incentives to agents to behave in a certain way?

Algorithms: How do we solve the computational problems faced by agents when engaged in negotiation?

Centralised vs. Distributed Allocation

Centralised case

- A **single entity decides** on the final allocation, possibly after having elicited the preferences of the other agents.
- Example: auctions

Distributed case

- **Allocations emerge** as the result of a sequence of local negotiation steps.
- Such local steps may or may not be subject to structural restrictions (say, bilateral deals).

Which approach is appropriate under what circumstances?

Centralised vs. Distributed Comparison

Centralised	Distributed
• The **communication** protocols required are relatively **simple**. • Many **results** from **economics** and **game theory** , in particular on mechanism design, can be exploited. • **Powerful algorithms** for winner determination in combinatorial auctions. • Possible **trust** issues. • Difficult to deal with **unbounded problems**.	• Avoids **trust** issues. • Inherently **scalable**. • Can take an **initial allocation** into account. • More natural to model **step-wise improvements** over the status quo. • Can deal with **unbounded domains**. • More **complex** protocols significantly more **difficult** to analyse (convergence etc.)

Introduction to Auctions

What is an Auction?

*An **auction** is a protocol that allows agents (=bidders) to indicate their **interests** in one or more **resources** and that uses these indications of interest to determine both an **allocation** of the resources and a set of **payments** by the agents.* [Shoham & Leyton-Brown 2009]

Auctions use employ **cardinal preferences** to express interest.

Auctions are **games** of a specific structure.

English Auction

Auctioneer starts the bidding at some **reservation price**

Bidders then shout out **ascending** prices

- minimum increments

Once bidders stop shouting, the *high bidder* gets the good at that price

Why Auctions?

Market-based price setting: for objects of unknown value, the value is dynamically assessed by the market!

Flexible: any object type can be allocated

Can be **automated**

- use of simple rules reduces complexity of negotiations
- well-suited for computer implementation

Revenue-maximising and **efficient allocations** are achievable

Auctions Rules

Auction mechanism is **specified** by **auction rules**

- *rules of the game*

Bidding rules: How offers are made:

- by whom
- when
- what their content is

Clearing rules: Who gets which goods (**allocation**) and what money changes hands (**payment**).

Information rules: What information about the state of the negotiation is *revealed* to whom and when.

Valuation Models

Agent's payoff from participating in an auction

- if winner: payoff = item's valuation – price paid for the item
- if not winner: payoff = zero

Common value: the good has the *same* value to *all* agents

- a 100 dollar note

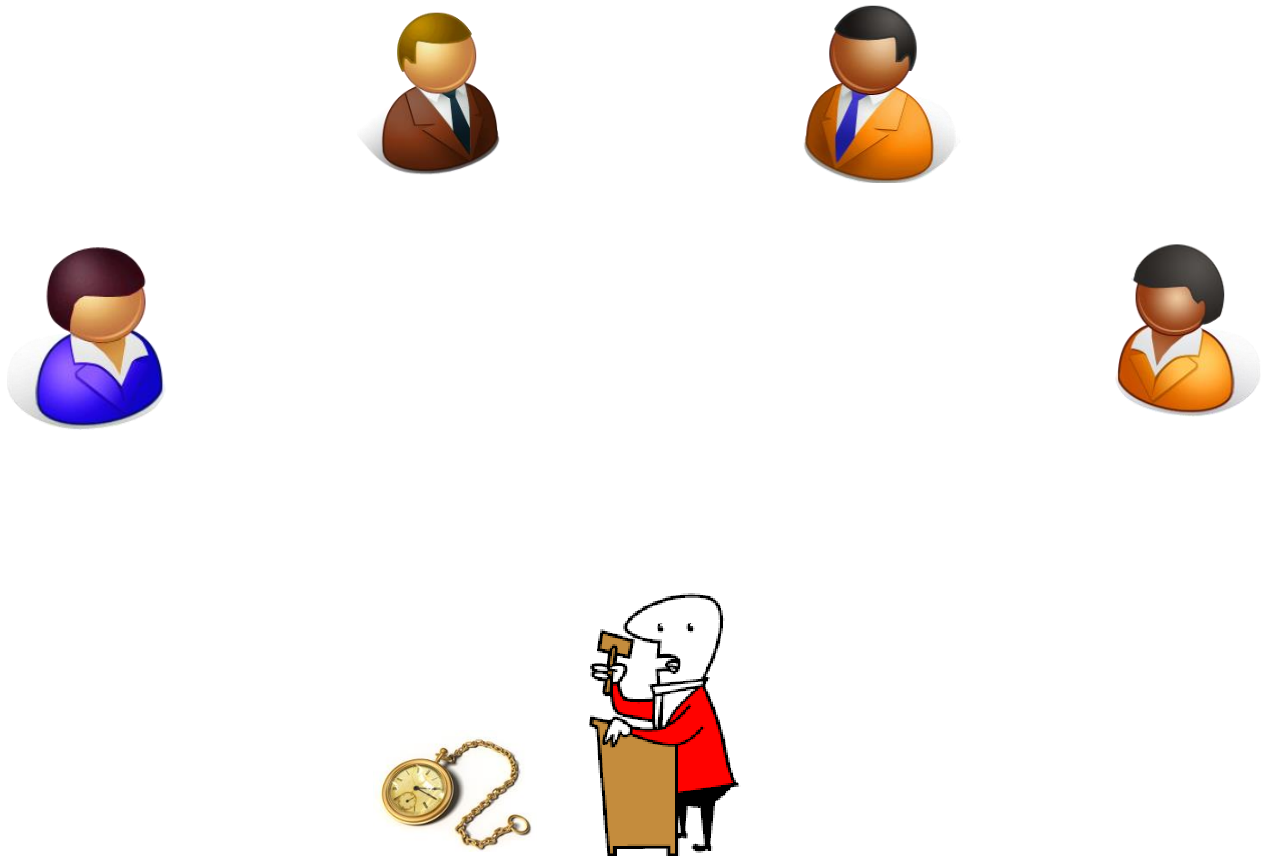
Private value: an agent A's valuation of the good is *independent* from other agent's valuation of the good

- a taxi ride to the airport

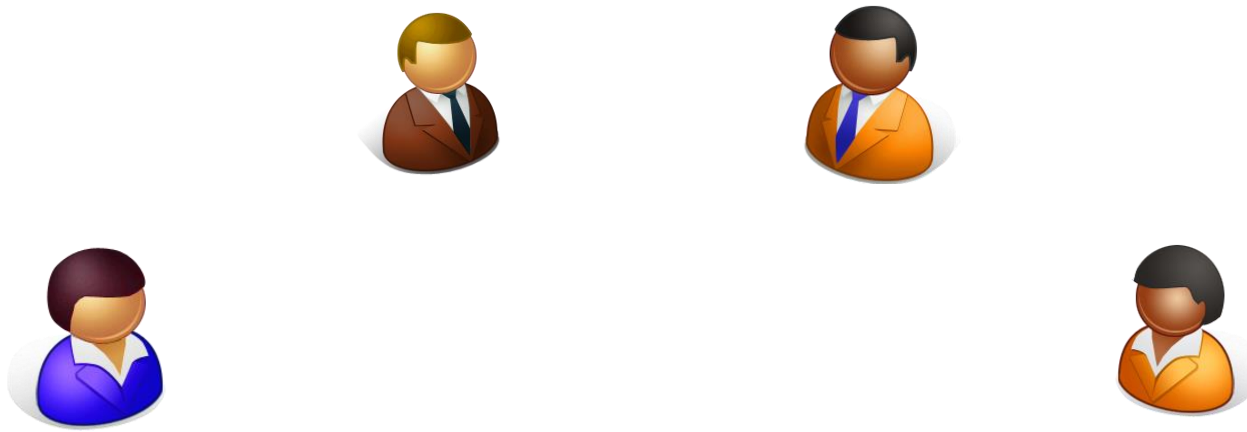
Correlated value: valuations of the good are *related*

- i.e. the more other agents are prepared to pay, the more agent A prepared to pay.
- i.e. purchase of items for later resale

Single Good Auctions



Multi-Unit Auctions



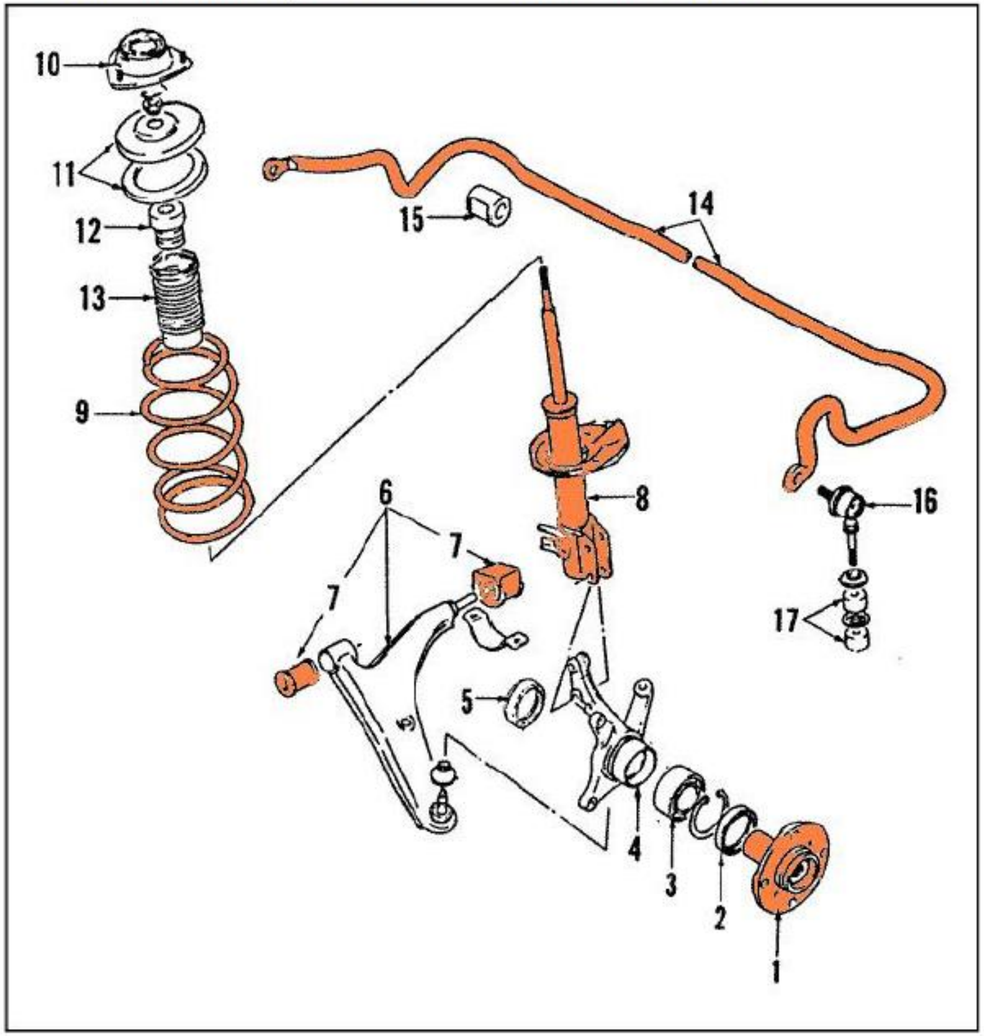
single-unit vs. multiple-unit demand

Indistinguishable items

Multi-Item Auctions

Reverse Auctions

FRONT SUSPENSION, FRONT WHEEL BEARING ACQUISITION



- Goal: Buy parts to produce a front suspension.
- The buyer issues a request for bids to his providers.

PART #	DESCRIPTION
1	FRONT HUB
7	LOWER CONTROL ARM BUSHINGS
8	STRUT
9	COIL SPRING
14	STABILIZER BAR

Multi-Attribute Auctions

Negotiation over other **attributes in addition to price**

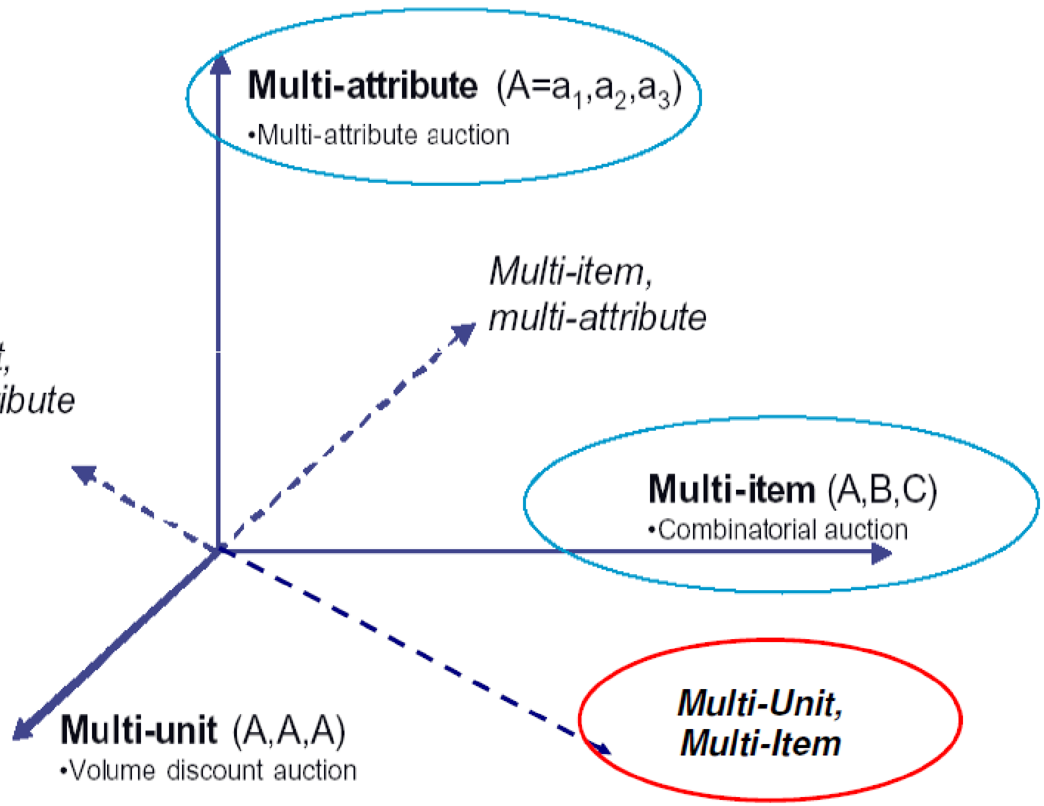
- e.g. color, weight, or delivery time

For instance: Provider John Doe offers to deliver a stainless-steel stabilizer bar that weighs 500 g at the cost of 200 EUR by July 18th 2011.

Promise **higher market efficiency** through a more **effective information exchange** of buyer’s preferences and supplier’s offerings.

Least understood type of auctions.

Auction Mechanism Taxonomy



Other: First-price vs. *k*-th price, open cry vs. sealed bid, single. vs. double-sided, sell-side vs. buy-side

Single-Item Auctions

Basic Auction Mechanisms

English

Japanese

Dutch

First-Price

Second-Price

English Auction

Auctioneer starts the bidding at some **reservation price**

Bidders then shout out **ascending** prices

- minimum increments

Once bidders stop shouting, the *high bidder* gets the good at that price

Japanese Auctions

Same as an English auction except that the auctioneer calls out the prices

All bidders start out **standing**

When the price reaches a level that a bidder is not willing to pay, that bidder **sits down**

Once a bidder sits down, they **can't get back up** the **last** person **standing** gets the good

Dutch Auction

The auctioneer starts a clock at some high value; it descends

At some point, a bidder shouts “mine!" and gets the good at the price shown on the clock

Good when items need to be sold **quickly** (similar to Japanese)

No information is revealed during auction

First-, Second-Price Sealed Bid Auctions

First-price sealed bid auction

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount of **his bid**

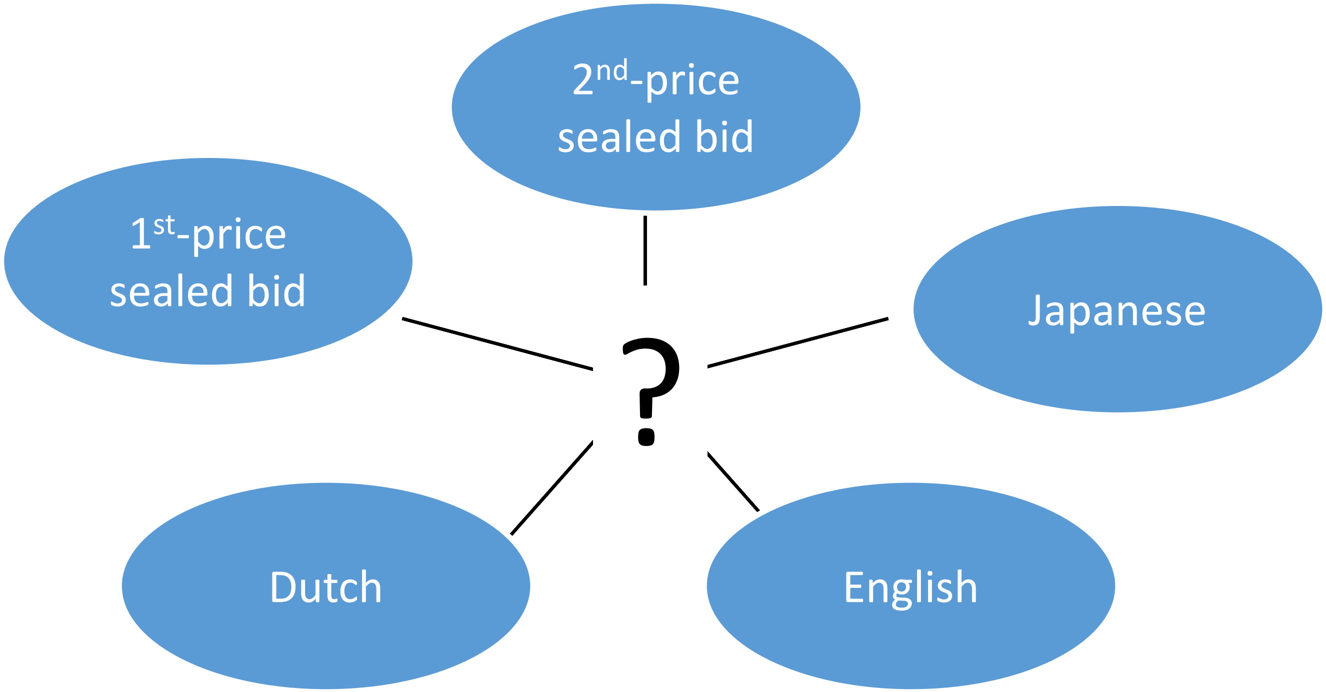
Second-price sealed bid auction (**Vickerey** auction)

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount bid by **the second-highest** bidder

Intuitive Comparison

	English	Dutch	Japanese	1^{st}-Price	2^{nd}-Price
Duration	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
Info Revealed	2^{nd}-highest val; bounds on others	winner's bid	all val's but winner's	none	none
Jump bids	yes	n/a	no	n/a	n/a
Price Discovery	yes	no	yes	no	no

Analysing Auctions



Are there fundamental similarities / differences between mechanisms described?

Two Problems

Auction **mechanism analysis**

- analyse the properties of a given auction mechanism
- methodology: treat auctions as (extended-form) *Bayesian games* and analyse players' (i.e. bidders') strategies

Auction **mechanism design**

- design an auction mechanism (i.e. the game for the bidders) with the desirable properties
- methodology: apply mechanism design techniques

Bayesian Game

Definition (Bayesian Game)

A Bayesian game is a tuple $\langle N, A, \Theta, p, \boldsymbol{u} \rangle$ where

- N is the set of **players**
- $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, Θ_i is the **type space** of player i
- $A = A_1 \times A_2 \times \cdots \times A_n$ where A_i is the **set of actions** for player i
- $p: \Theta \mapsto [0,1]$ is a **common prior over types**
- $\boldsymbol{u} = (u_1, \dots, u_n)$, where $u_i: \Theta \mapsto \mathbb{R}$ is the utility function fo player i

We assume that all of the above is **common knowledge** among the players, and that each **agent knows** his **own type**.

Relation to Auctions

Sealed bid auction under IPV is a Bayesian game in which

- player i's actions correspond to his bids $\widehat{v_i}$
- player types Θ_i correspond to player's private valuations v_i over the auctioned item
- the payoff of a player i corresponds to his valuation of the item v_i – its bid $\widehat{v_i}$

(Desirable) Properties

Truthfulness: bidders are incentivized to bid their true valuations, i.e.

$$v_i = \hat{v}_i \ \forall i \forall v_i$$

Efficiency: the aggregated utility of bidders is maximized, i.e.

$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x')$$

Optimality: maximization of seller's revenue

Strategy: existence of dominant strategy

Manipulation vulnerability: lying auctioner, shills, bidder collusion

Other consideration: communication complexity, private information revelation, ...

Second-Price Sealed Bid

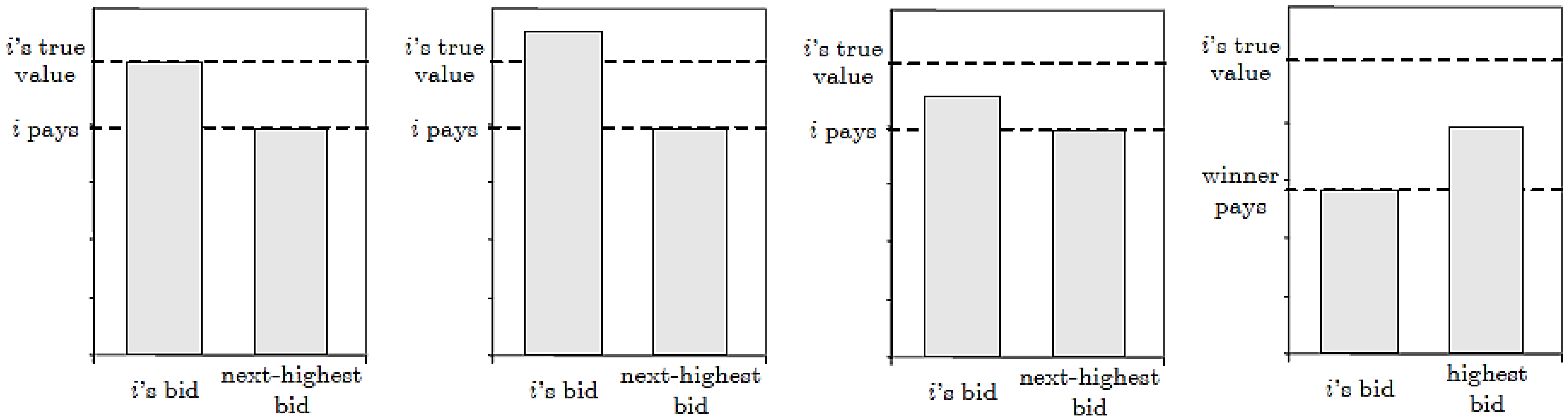
Theorem

Truth-telling is a **dominant strategy** in a second-price sealed bid auction (assuming independent private values (IPV) model and risk neutral bidders).

Proof: Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

- Bidding honestly, i would win the auction
- Bidding honestly, i would lose the auction

Second-Price Sealed Bid Proof



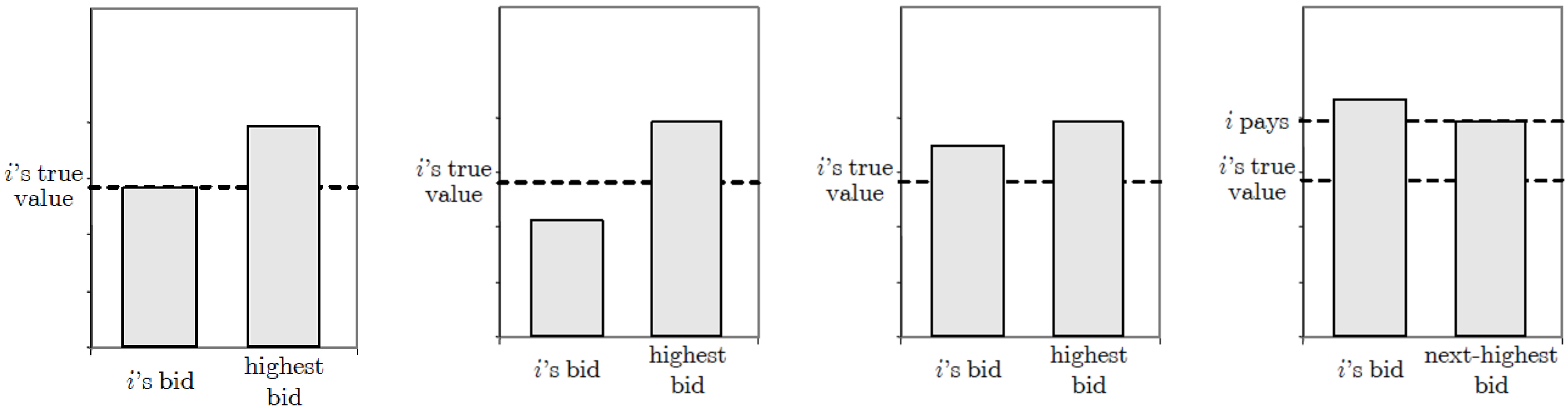
Bidding honestly, i is the winner

If i bids higher, he will still win and still pay the same amount

If i bids lower, he will either still win and still pay the same amount. . .

... or lose and get utility of zero.

Second-Price Sealed Bid Proof



Bidding honestly, i is not the winner

If i bids lower, he will still lose and still pay nothing

If i bids higher, he will either still lose and still pay nothing...

... or win and pay more than his valuation.

Second-Price Sealed Bid

Advantages:

- Truthful bidding is dominant strategy
- No incentive for counter-speculation
- Computational efficiency

Disadvantages:

- Lying auctioneer
- Bidder collusion self-enforcing

Unfortunately, the auction is not very popular in real life due to its counter-intuitiveness

- but very successful in computational auction systems (e.g. Adwords)

Dutch and First-price Sealed Bid

Strategically equivalent: an agent bids without knowing about the other agents' bids

- a bidder must decide on the amount he's willing to pay, conditional on having placed the highest bid

Differences

- First-price auctions can be held asynchronously
- Dutch auctions are fast, and require minimal communication

Bidding in Dutch / First Price Sealed Bid?

Bidders strategy?

- Bidders would normally bid less than own valuation but just enough to win $\Rightarrow$ **not incentive compatible** and incentive to counter-speculate

Bidders don't have a **dominant strategy** any more:

- there's a **trade-off** between **probability of winning** vs. **amount paid** upon winning
- **individually optimal** strategy depends on **assumptions** about **others' valuations**

Theorem

In a first-price sealed bid auction with n **risk-neutral** agents whose valuations $v_1, v_2, \ldots, v_n$ are **independently** drawn from a **uniform distribution** on the **same** bounded **interval** of the real numbers, the **unique** symmetric **equilibrium** is given by the **strategy profile** $(\frac{n-1}{n}v_1, \ldots, \frac{n-1}{n}v_n)$.

English and Japanese Auctions Analysis

A much more complicated **strategy space**

- extensive form game
- bidders are able to condition their bids on information revealed by others
- in the case of English auctions, the ability to place jump bids

Intuitively, though, the **revealed information** doesn't make any **difference** in the **independent-private value** (IPV) setting.

- proxy bidding

English and Japanese Auctions Analysis

Theorem

Under the IPV model, it is a **dominant strategy** for bidders to bid **up to** (and not beyond) their valuations in both Japanese and English auctions.

In correlated-value auctions, it can be worthwhile to counter-speculate

Revenue Equivalence

Which auction should an auctioneer choose?

To some extent, it doesn't matter...

Theorem (Revenue Equivalence)

Assume that each of n **risk-neutral** agents has an **independent private valuation** for a single good at auction, drawn from a **common cumulative distribution** $F(v)$ that is **strictly increasing** and **atomless** on $[\underline{v}, \overline{v}]$. Then any auction mechanism in which

1. the good will be allocated to the agent with the highest valuation; and
2. any agent with valuation $\underline{v}$ has an expected utility of zero

yields the **same expected revenue**, and hence results in any bidder with valuation v making the same expected payment.

Optimal Auction Design

The seller's problem is to **design an auction game** which has a Nash equilibrium giving him the **highest possible expected utility**.

- Mechanism design problem.

Second-prize sealed bid auction **does not maximize** expected revenue.

Optimal Single Item Auction

Assumptions

- Independent private valuations
- Risk-neutral bidders

Suppose agent i draws valuation from strictly increasing cumulative density function F_i (pdf f_i)

Definition (virtual valuation)

Bidder i's virtual valuation is $\psi_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$.

Definition (bidder-specific reserve price)

Bidder i's bidder-specific reserve price r_i^* is the value for which $\psi_i(r_i^*) = 0$.

Optimal Single Item Auction

Theorem

The optimal (single-good) auction is a sealed-bid auction in which every agent is asked to declare his valuation. The good is sold to the agent $i = \arg\max_i \psi_i(\hat{v}_i)$*, as long as* $v_i > r_i^*$*. If the good is sold, the winning agent* i *is charged the smallest valuation that he could have declared while still remaining the winner:* $\inf\{v_i^* : \psi_i(v_i^*) \geq 0 \text{ and } \forall j \neq i, \psi_i(v_i^*) \geq \psi_j(\hat{v}_j)\}$.

The virtual valuations also increase weak bidders' bids, making them more competitive.

- Low bidders can win, paying less.
- However, bidders with higher expected valuations must bidmore aggressively

Symmetric case: second-price auction with reservce price r^* satisfying: $r^* - \frac{1-F_i(r^*)}{f_i(r^*)} = 0$

Auctions Summary

Auctions are mechanisms for **allocating scarce resource** among **self-interested agent**

Mechanism-design and game-theoretic perspective

Vast range of auctions mechanisms: English, Dutch, Japanese, First-price sealed bid, Second-price sealed bid

Desirable properties: truthfulness, efficiency, optimality, ...

Rapidly expanding list of **applications** worth billions of dollars

How to get around impossibility results

Mechanisms with money

Measure not just that a preferred to b, but also "by how much"...

Each individual j (or player j) has a "valuation" for each alternative a in A. Denoted as $v_j(a)$

Also, each player values money the same.

So, if we choose alternative a, and give $m to j, then j's "utility" is $v_j(a) + m$

Auction Protocols

Auctions are centralised mechanisms for the allocation of goods amongst several agents. Agents report their preferences (bidding) and the auctioneer decides on the final allocation (and on prices).

- Distinguish *direct* and *reverse* auctions (auctioneer buying).
- Bidding may be *open-cry* (English) or by *sealed bids*.
- Open-cry: *ascending* (English) or *descending* bids (Dutch).
- Pricing rule: *first-price* or *second-price* (Vickrey).
- *Combinatorial auctions:* several goods, sold/bought in bundles.

R.P. McAfee and J. McMillan. Auctions and Bidding. *Journal of Economic Literature*, 25:699–738, 1987.

P. Cramton, Y. Shoham, and R. Steinberg (eds.). *Combinatorial Auctions*. MIT Press, 2006.

Where are auctions used nowadays?

Resource allocation

- Treasury auctions
- Right to drill oil, off-shore oil lease
- Use the electromagnetic spectrum
- Private and public goods and services acquisition
- Internet auctions

Market-based computing: The use of a market-based method, such as an auction to compute the outcome of a distributed problem.

- Air-conditioning control
- Production control
- Robot navigation
- Sensor networks