Distributed Constraint Reasoning 1

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Where are We?

Agent architectures (inc. BDI architecture)

Logics for MAS

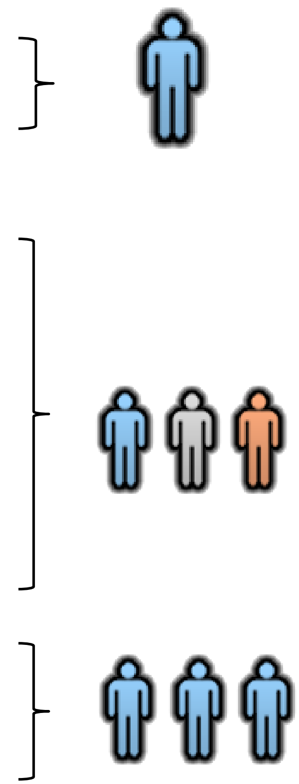
Non-cooperative game theory

Cooperative game theory

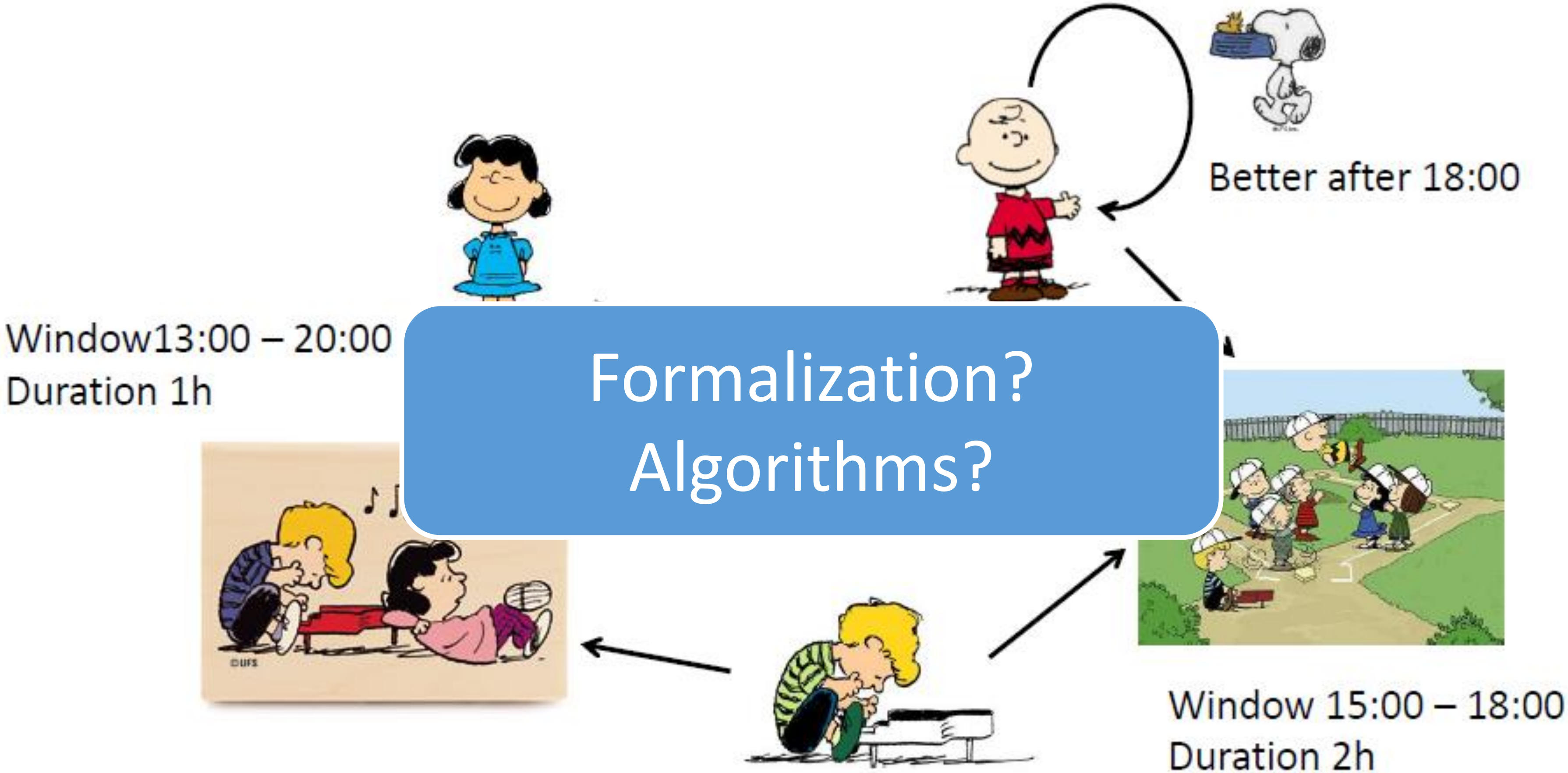
Auctions

Social choice

Distributed constraint reasoning



Motivating Example: Meeting Scheduling



Formalization?
Algorithms?

Lecture Objectives

At the end of 2-lecture series you will

- will be able to express problems as distributed constraint reasoning problems
- will be able to use basic solution algorithms for constraint satisfaction and optimization

Lecture Outline

1. Introduction
2. Definitions and Examples
3. Solution Methods
4. Asynchronous Backtracking Algorithm
5. Bottom-up Algorithms
6. Summary and Outlook

Introduction

Distributed Constraint Reasoning 1

Constraint Reasoning

Constraints pervade our lives (time, money, energy, ...) and usually perceived as elements that **limit solutions** to the problems we face.

From a **computational point of view**, they:

- **reduce the space** of possible solutions
- **encode knowledge** about the problem at hand
- are **key components** for efficiently solving hard problems

Hard computational problems can often be **made tractable** by carefully **considering the constraints** that define the **structure of the problem**.

General framework: applies to planning and scheduling, operational research, automated reasoning and decision theory, computer vision... and multiagent systems

Constraint Reasoning in/for MAS

Constraint reasoning can be used to address **coordination and optimization** problems in MAS.

- Set of agents must come to some **agreement**, typically via some form of **negotiation**, about **which action** each agent should take in order to jointly obtain the **best solution** for the **whole system**.

We will consider **Distributed Constraint Reasoning Problems** where:

- Each agent **negotiates locally** with just a subset of other agents (usually called **neighbours**) that are those that can **directly influence** his/her behaviour.

Suitable for the **cooperative setting**

- use game theory otherwise

Definitions

Distributed Constraint Reasoning 1

Constraint Network

A **constraint network** $\mathcal{N}$ is formally defined as a triple $\langle X, D, C \rangle$ where:

- $X = \{x_1, \dots, x_n\}$ is a set of **variables**;
- $D = \{D_1, \dots, D_n\}$ is a set of **variable domains**, which enumerate all possible values of the corresponding variables; and
- $C = \{C_1, \dots, C_m\}$ is a set of **constraints**; where a constraint C_i is defined on a subset of variables $S_i \subseteq X$ which comprise the **scope of the constraint**
  - $r_i = |S_i|$ is the **arity** of constraint i

Hard vs. Soft Constraints

Hard constraint C_i^h is a Boolean **predicate** P_i that defines **valid joint assignments** of variables in the scope

$$P_i: D_{i_1} \times \cdots \times D_{i_r} \rightarrow \{\mathrm{F}, \mathrm{T}\}$$

Soft constraint C_i^s is a **function** F_i that maps every possible joint assignment of all variables in the scope to a real value

$$F_i: D_{i_1} \times \cdots \times D_{i_r} \rightarrow \mathfrak{R}$$

Binary Constraint Networks

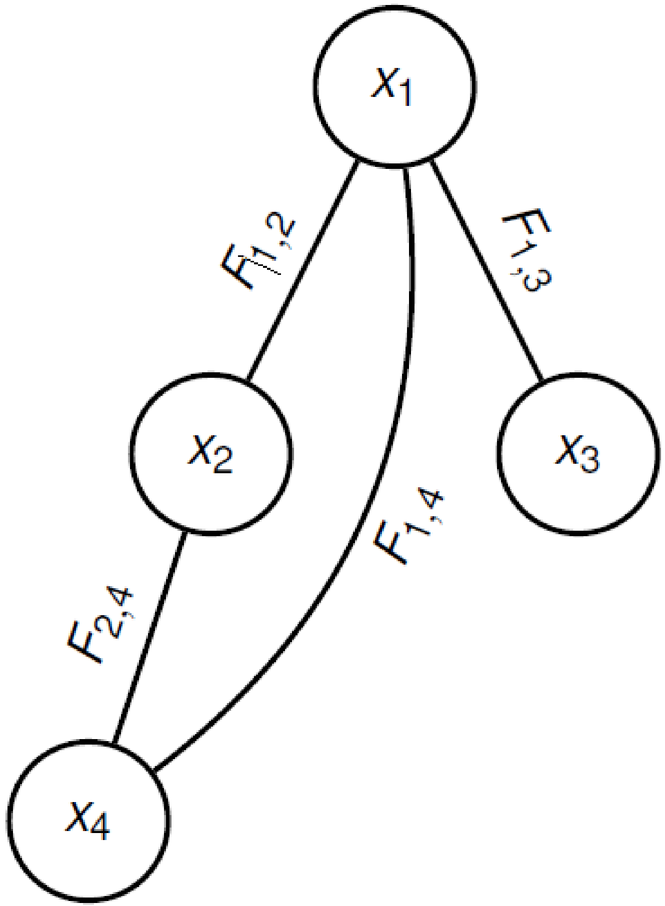
Binary constraint networks are those where each **constraint** (soft or hard) is defined **over two variables**

Binary constraint networks can be represented by a **constraint graph**

Every constraint network can be **mapped to a binary** constraint network

- requires the addition of variables and constraints
- may increase the size of the model

Algorithms explained for binary constraints but can be extended to n-ary.



Types of Constraint Reasoning Problems

Constraint Satisfaction Problem (CSP)

- **Objective**: find an **assignment** for all the variables in the network that **satisfies all constraints**.
- Extension to **MaxCSP/MinCSP**: Maximize the number of satisfied constraints / minimize the number of violated constraints.

Constraint Optimization Problem (COP)

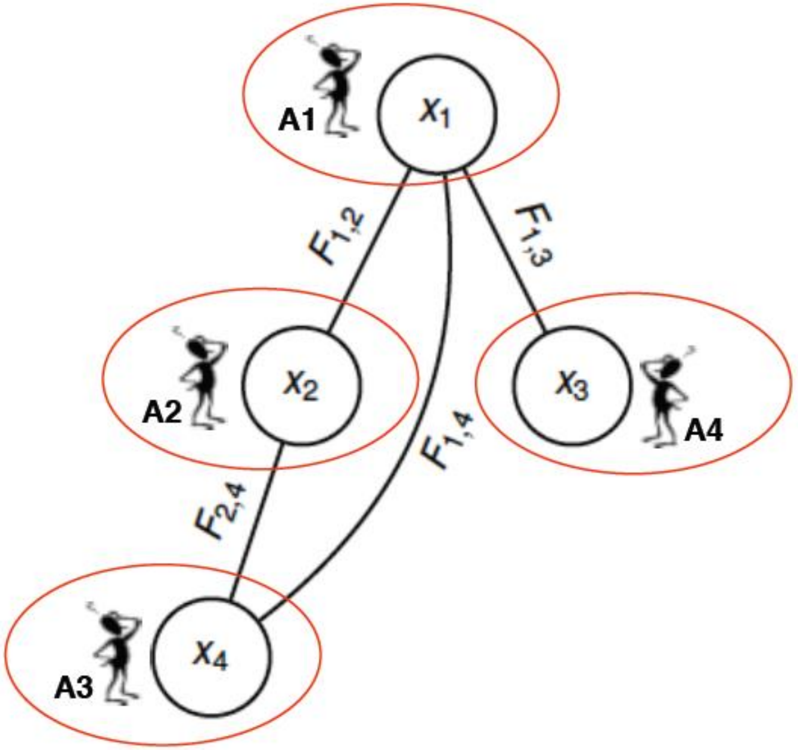
- **Objective**: find an **assignment** for all the variables in the network that **satisfies all constraints** and **optimizes a global function**.
- **Global function** = aggregation (typically sum) of constrain functions, i.e., $F = \sum F_i$

COP provides more **modelling power** on the expense of more **complex solution** algorithms.

Distributed Constraint Reasoning

When operating in a **decentralized context**:

- a set of **agents control variables**
- agents **interact to find a solution** to the constraint network



Distributed Constraint Reasoning Problem

A distributed constraint reasoning problem consists of a **constraint network** $\langle X, D, C \rangle$ and a **set of agents** $A = \{A_1, \ldots, A_k\}$ where each agent:

- **controls a subset** of the variables $X_i \subseteq \mathrm{X}$
- is only **aware of constraints** that involve variable it controls
- communicates only with its **neighbours**

1:1 agent-to-variable mapping assumed for algorithm explanation (can be generalized)/

Types of DCR Problems

1. Distributed CSP (DCSP)
2. Distributed COP (DCOP)

Examples / Applications

Distributed Constraint Reasoning

Examples

Many standard benchmark problems in computer science can be viewed as DCOPs

- e.g. **graph colouring**

As can many real-world applications

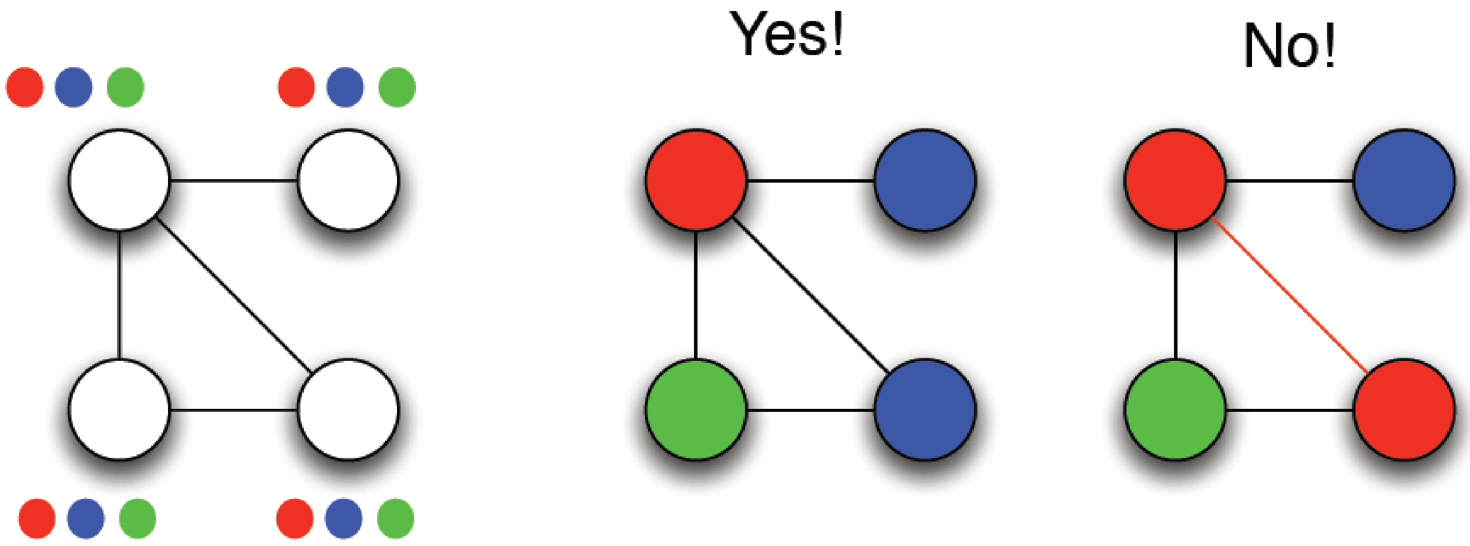
- human-agent organization (e.g. **meeting scheduling**)
- sensor networks and robotics (e.g. **channel allocation**)

Graph Colouring

- Popular benchmark
- Simple formulation
- Complexity controlled with few parameters:
  - Number of available colors
  - Number of nodes
  - Density ($\#nodes/\#constraints$)
- Many versions of the problem:
  - CSP, MaxCSP, COP

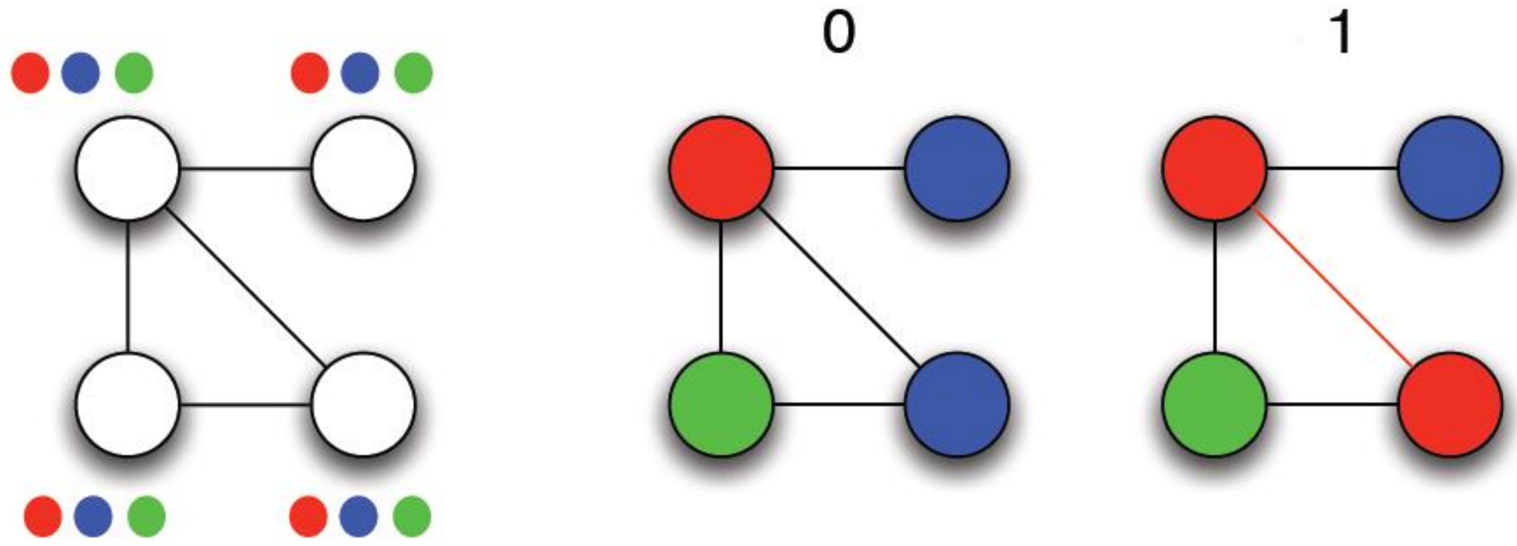
Graph Colouring: CSP

- Nodes can take k colors
- Any two adjacent nodes should have different colors
    - If it happens this is a conflict



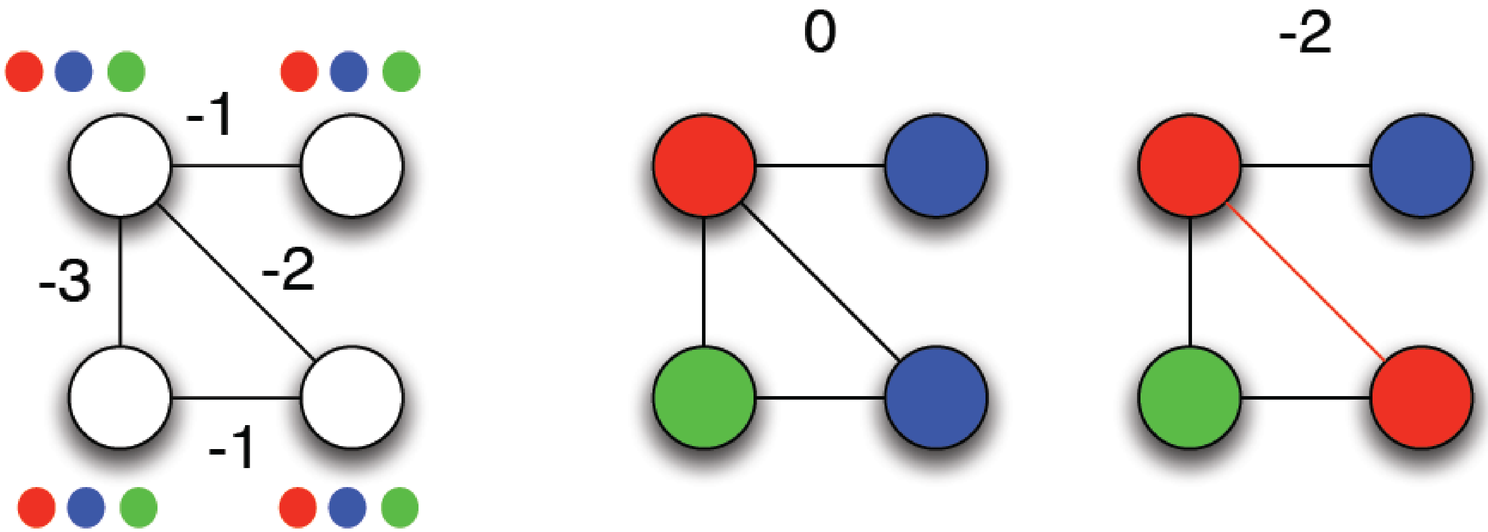
Graph Colouring: Min CSP

- Minimize the number of conflicts



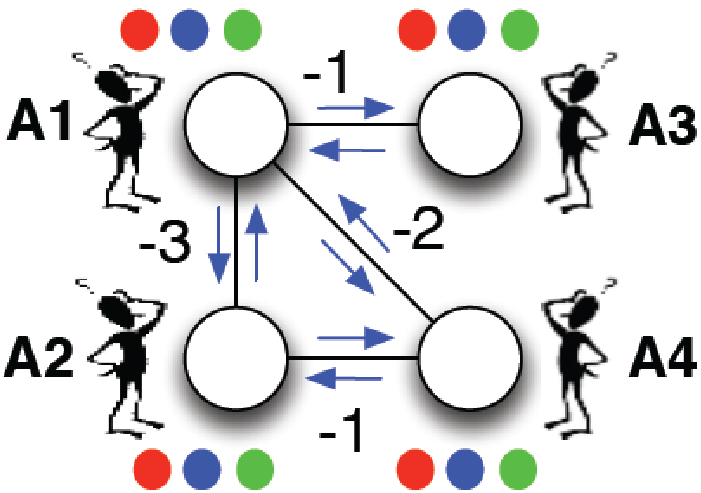
Graph Colouring: COP

- Different weights to violated constraints
- Preferences for different colors



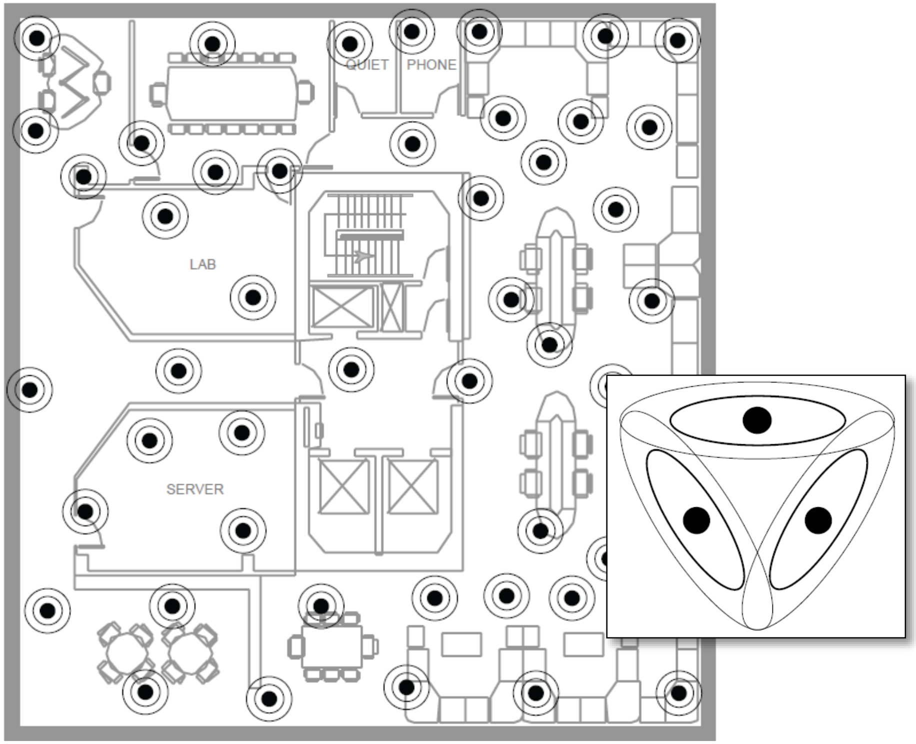
Graph Colouring: DCOP

- Each node:
  - controlled by one agent
- Each agent:
  - Preferences for different colors
  - Communicates with its direct neighbours in the graph



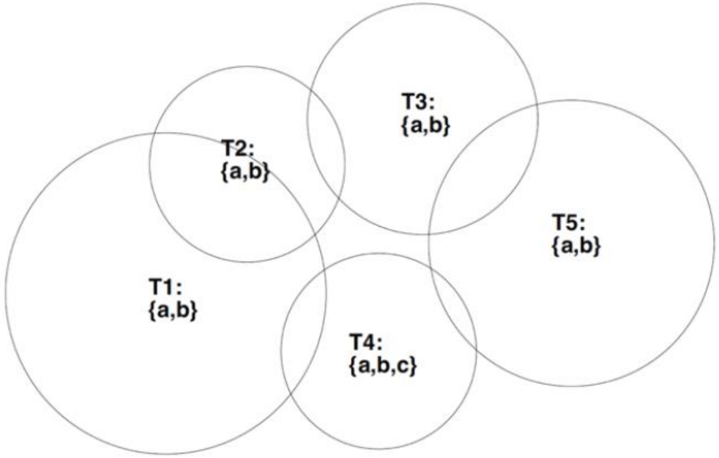
- A1 and A2 exchange preferences and conflicts
- A3 and A4 do not communicate

Channel Allocation in Sensor Networks



Find a **non-conflicting assignment** of communication channels assuming **local communication only**

DCSP Formalization of Channel Allocation



Agents $A = \{A_1, \ldots, A_n\}$ correspond to **sensors**.

Variables $X = \{X_1, \ldots X_n\}$ correspond to **selected** broadcast **channels**:
X_i is the channel on which the sensor A_i broadcasts.

Domains $D = \{D_1, \ldots, D_n\}$ correspond to **available channels**.

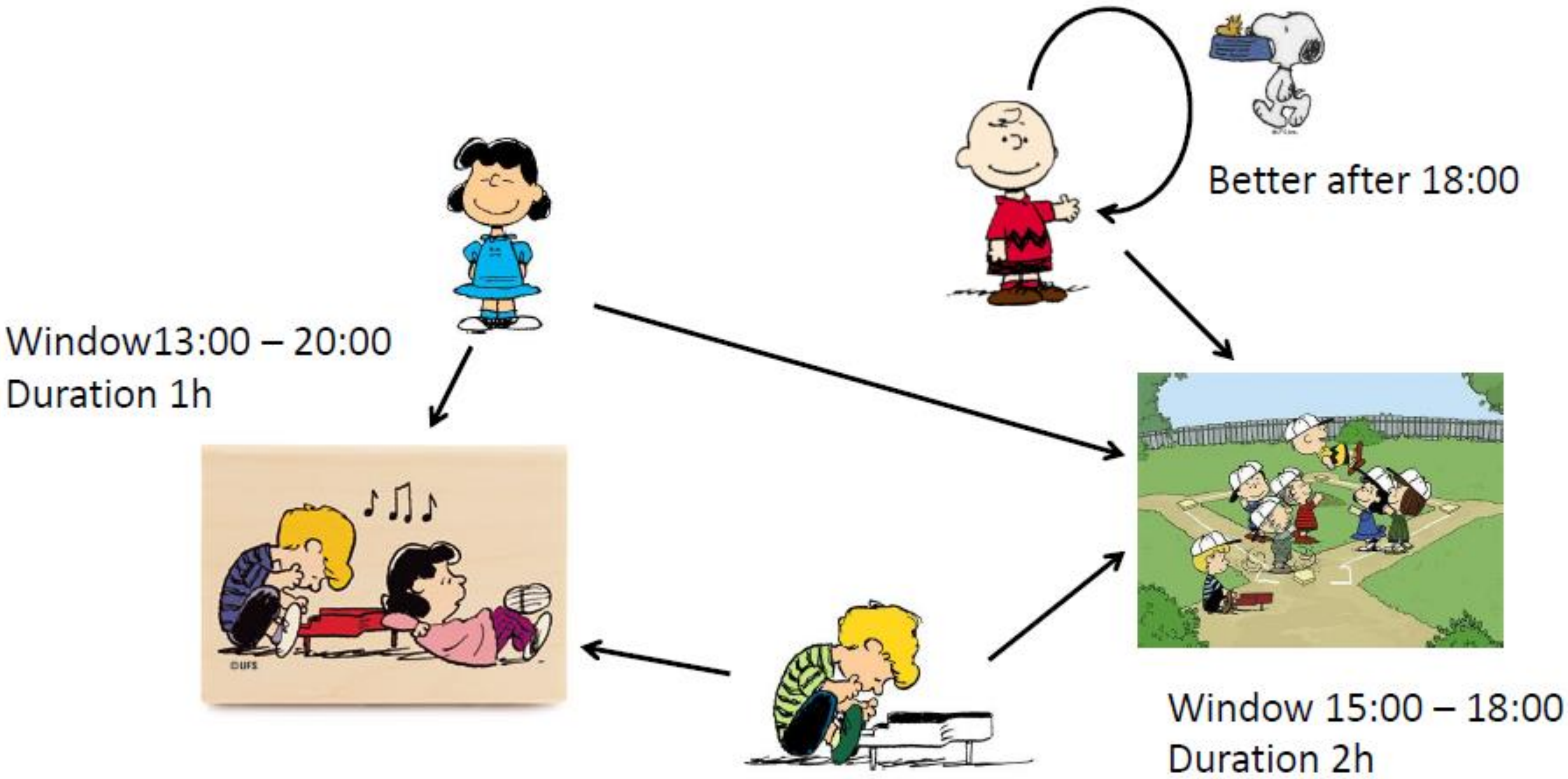
For each pair of sensors i, j that have **overlapping** broadcast **ranges**, there is a corresponding Boolean constraint $P_{i,j}$ so that

$$P_{i,j}(X_i, X_j) = T \text{ iff } X_i \neq X_j$$

i.e. sensors will **overlapping** ranges must use **different** channels.

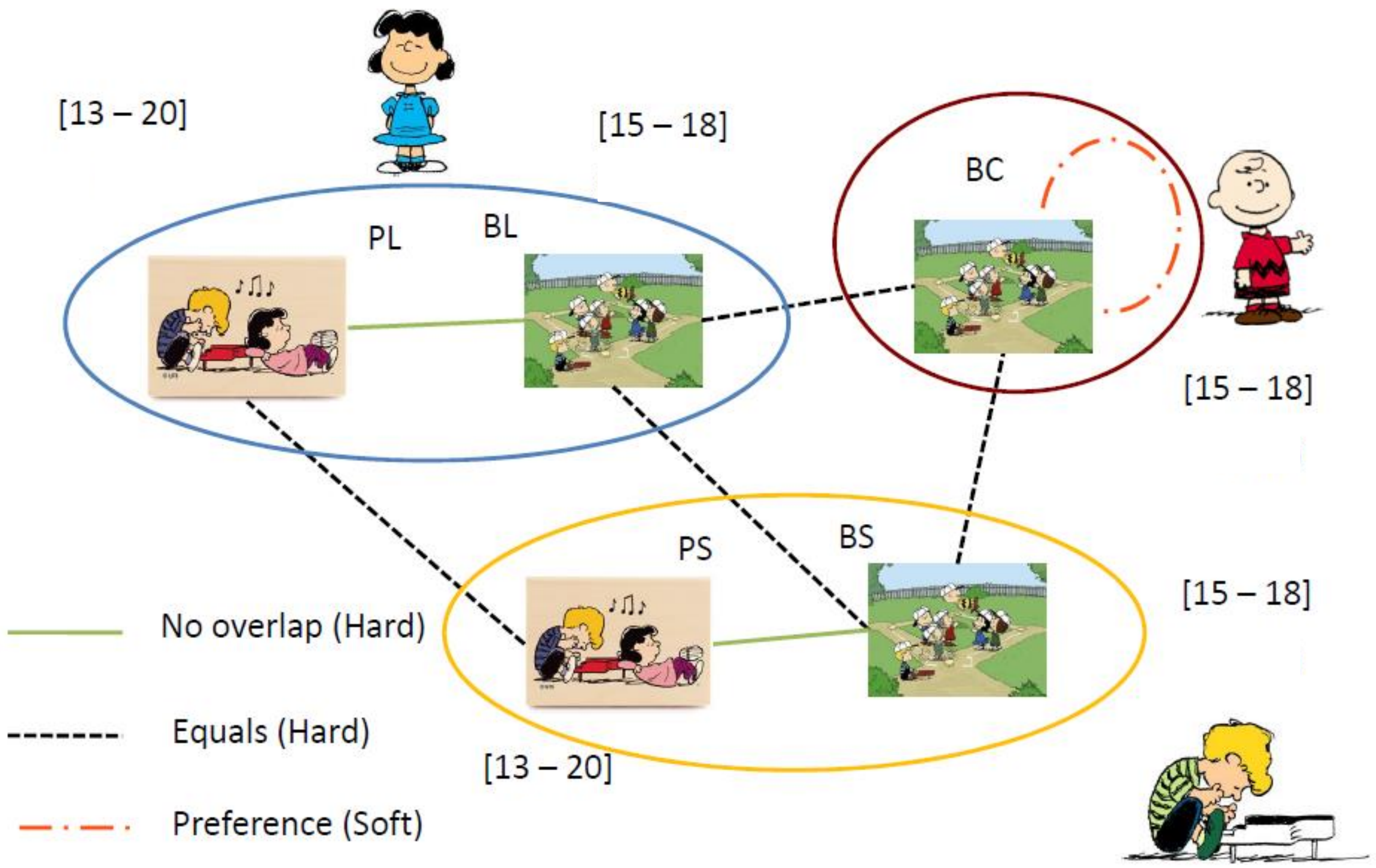
<u>**Objective**</u>: Find a **channel allocation** where **no overlapping sensors** use the **same** channel.

Example: Meeting Scheduling



from http://cs.smu.ca/~pawan/wi07/petcu.pdf

Meeting Scheduling Formalization



Solution Approach: DCSP

Distributed Constraint Reasoning 1

Requirements on a Good Algorithm

Soundness/Correctness: the solution returned is valid

Termination: in a finite number of steps

Completeness: finds an (optimal) solution if it exists

- DCSP Solution approaches
  - Top-Down
    - Prunning (e.g. Filtering, Hyper-resolution)
      rarely produce a solution
      → used for preprocessing
    - Search (e.g. Asynchronous backtracking)
      complete but not scalable
  - Bottom-Up (e.g. Distributed breakout)
    approximate but scalable

Distributed Algorithms

Synchronous: agents take steps following some fixed order (or computing steps are done simultaneously, following some external clock).

Asynchronous: agents take steps in arbitrary order, at arbitrary relative speeds.

Partially synchronous: there are some restrictions in the relative timing of events

Synchronous vs Asynchronous

Synchronous

- A few agents are active, most are **waiting**
- Active agents take decisions with **up-to-date** information
- Low degree of concurrency
- Poor robustness
- Algorithms: direct extensions of centralized ones

Asynchronous

- All agents are active **simultaneously**
- Information is less updated, **obsolescence** appears
- High degree of **concurrency**
- High **robustness**
- Algorithms: new approaches

Asynchronous Backtracking Algorithm (ABT)

Distributed Constraint Reasoning 1

Asynchronous Backtracking: Assumptions

1. Agents communicate by **sending messages**
2. An agent can send messages to others, iff it knows their identifiers (**directed communication** / no broadcasting)
3. The **delay** transmitting a message is **finite** but random
4. For any pair of agents, messages are **delivered in the order** they were sent
5. Agents **know the constraints in which they are involved**, but not the other constraints
6. Each agent owns a **single variable** (agents = variables)
7. Constraints are **binary** (2 variables involved)

not essential, can be lifted

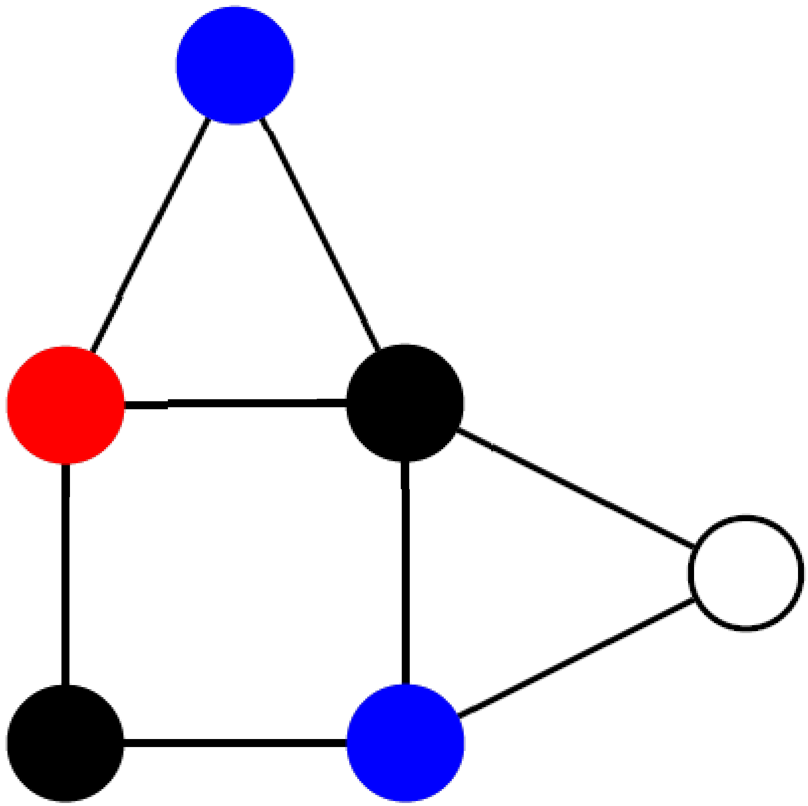
Synchronous Backtracking

Agents agree on an variable order and repeat:

1. send partial solution up to X_{k-1} to k-th agent.
2. k-th agent generates the next extension to this partial solution.
3. if the solution cannot be extended consistently: $k \leftarrow k-1$ (backtrack control to previous agent).
4. if solution can be extended consistently, $k \leftarrow k+1$ (pass control to the next agent)
5. if $k < 1$: stop → unsolvable.
6. if $k > n$: stop → assignment = solution

***Problem**: Only one agent working at a time => very inefficient*

Backtracking Illustration



Asynchronous Backtracking (ABT)

Revolutionary idea in 1998

Fully asynchronous algorithm

- all agents active, take a value and inform
- no agent has to wait for other agents

Total order among agents (to avoid cycles) → **priorities**

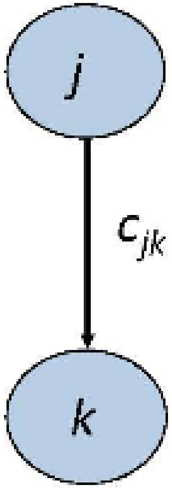
Constraints are directed: from higher-priority to lower-priority agents

ABT plays in asynchronous distributed context the same role as backtracking in centralized

ABT: Core Principles

High-priority agents **decide** on assignment, **lower-priority** have to **accommodate** or say they cannot.

1. Higher-priority agent (j) informs a lower-priority agent (k) of its assignment
2. Lower-priority agent (k) evaluates the shared c_{jk} constraint with its own assignment
   - If permitted → no action
   - else → look for a value consistent with j
     - If it exists → k takes that value
     - else → the agent view of k is a ***nogood*** → distributed backtrack



More communication needed in asynchronous case to compensate for the lack of shared execution state

ABT: NoGoods

Nogood: conjunction of (variable, value) pairs of higher-priority agents which removes a value of the current (lower-priority) agent.

Example: $x \neq y, D_x = D_y = \{a, b\}$, x higher-priority than y:

- when x assumes a and a message $[x \leftarrow a]$ arrives to y, the agent y generates the nogood $x = a \implies y \neq a$ that removes value a of D_y
- if x changes value, when $[x \leftarrow b]$ arrives to y, the no good $x = a \implies y \neq a$ is eliminated, value a is available again and a new nogood removing b is generated

*Nogoods are required to ensure **systematic traversal** of search space **in asynchronous, distributed context***

ABT: NoGood Resolution

When all values of variable y are removed, the **conjunction** of the left-hand sides of its nogoods is **also a nogood**.

Resolution: the process of generating a new nogood that is a logical **consequence** of existing ones.

<u>Example:</u>

$x \neq y,\ z \neq y, D_x = D_y = D_z = \{a, b\},\ x, z$ higher priority than y

assume: $x = a \Rightarrow y \neq a;\ z = b \Rightarrow y \neq b;$ i.e., all values for y ruled out

then: $x = a\ \wedge\ z = b$ is a nogood

i.e. in a directed form: $x = a \Rightarrow z \neq b$ (assuming x higher-priority than z)

(escalating the problem from y)

How ABT Works

Asynchronous action; spontaneous assignment

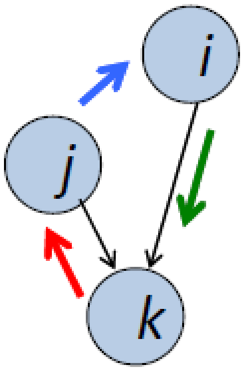
Four operations:

- **Assignment**: j takes value a: j informs lower priority agents
- **Backtrack (no good)**: k has no consistent values with higher-priority agents: k resolves nogoods and sends a a backtrack (nogood) message
- **New links**: j receives a nogood mentioning i, unconnected with j: j asks i to set up a link
- **Stop**: “no solution” (empty nogood) detected by an agent: stop

Solution: when agents are silent for a while (*quiescence*), every constraint is satisfied => solution;

- detected by specialized algorithms outside ABT

ABT: **Messages**



Ok?$(i \rightarrow k, a)$: higher-priority agent i informs lower-priority agent k that it takes value a

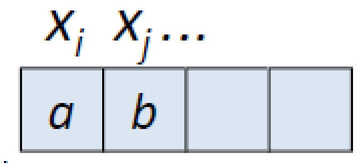
NoGood$(k \rightarrow j, i = a \Rightarrow j \neq b)$: when all k's values are forbidden:

- k requests j (the nearest higher-priority agent in the nogood) to *backtrack*
- then: k forgets j's value, k takes some value
- j may detect obsolescence of the NoGood message

AddLink$(j \rightarrow i)$: set a link from i to j, to know i's value

Stop: there is no solution

ABT: Data Structures

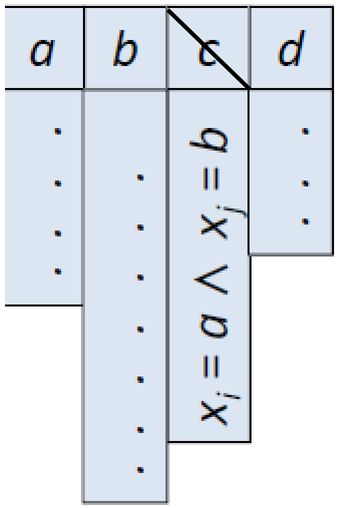


Current context / agent view: values of higher-priority constrained agents

NoGood store: each removed value has a *justifying* nogood

$$x_i = a \wedge x_j = b \Rightarrow x_k \neq c$$

- Stored nogoods must be **active**: left-hand side of the nogood satisfied in the current context
- If a nogood is no longer active, it is removed (and the value is available again)



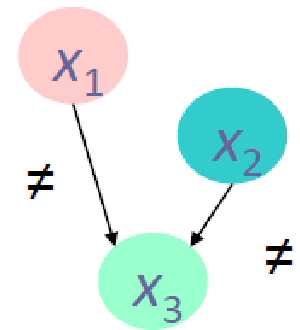
ABT: Graph Coloring Example

Variables x_1, x_2, x_3; $D_1 = \{b, a\}, D_2 = \{a\}, D_3 = \{a, b\}$

3 agents, lex ordered:

2 difference constraints: c_{13} and c_{23}

Constraint graph:

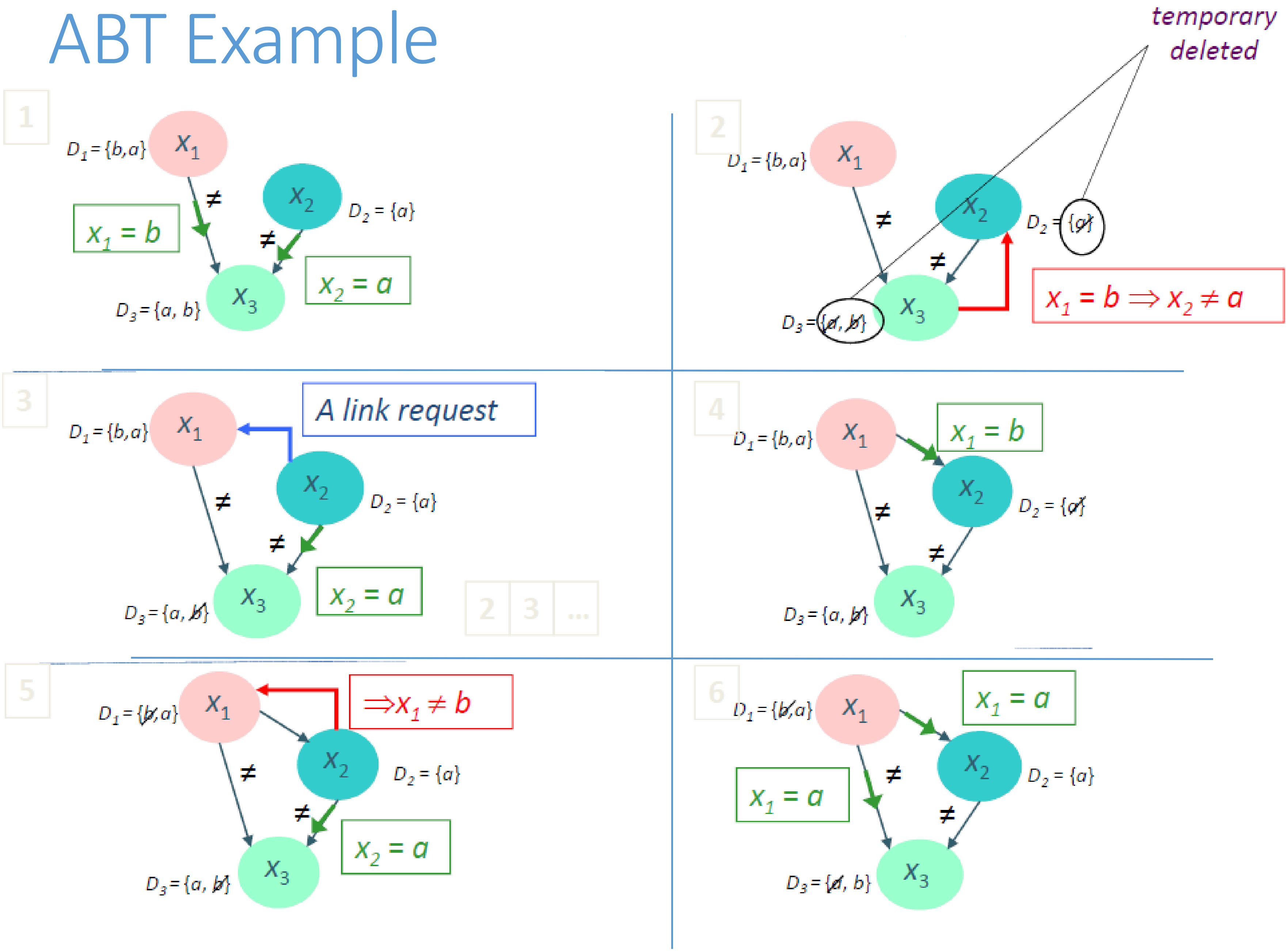


Value-sending agents: x_1 and x_2

Constraint-evaluating agent: x_3

Each agent *checks* constraints of incoming links: $Agent_1$ and $Agent_2$ check nothing, $Agent_3$ checks c_{13} and c_{23}

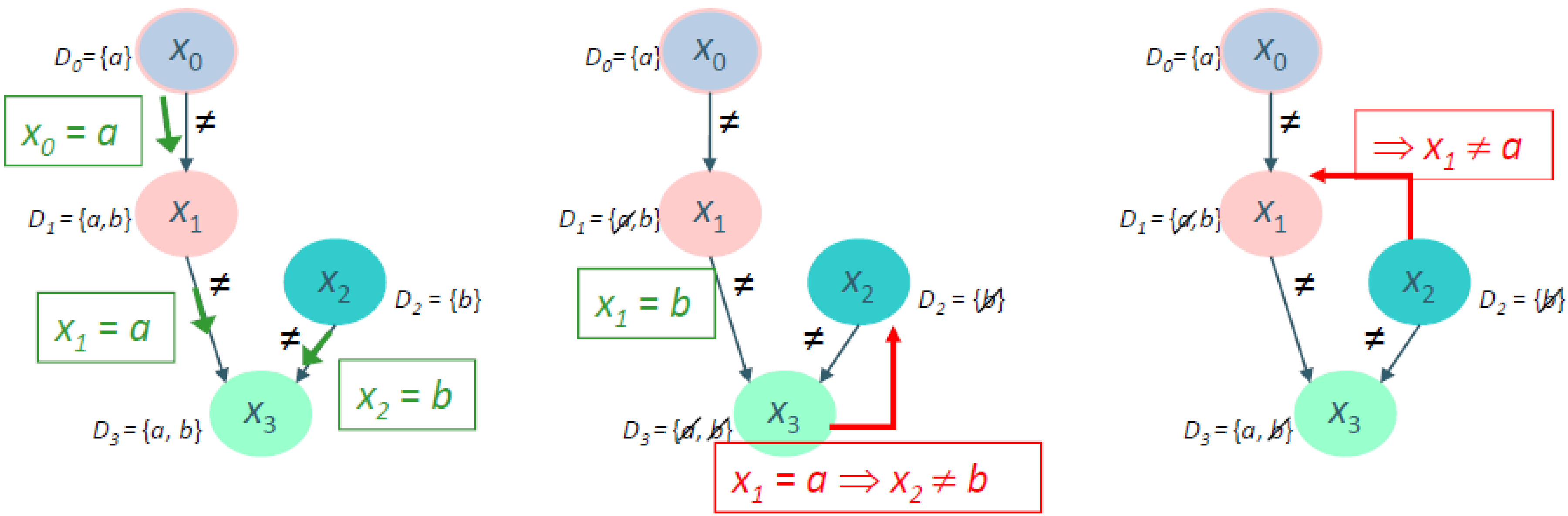
ABT Example



temporary
deleted
1
$D_1 = \{b,a\}$
X_1
X_2
$D_2 = \{a\}$
$x_1 = b$
$x_2 = a$
X_3
$D_3 = \{a, b\}$
2
$D_1 = \{b,a\}$
$D_2 = \{\cancel{a}\}$
$D_3 = \{\cancel{a}, \cancel{b}\}$
$x_1 = b \Rightarrow x_2 \neq a$
3
A link request
$D_1 = \{b,a\}$
$D_2 = \{a\}$
$x_2 = a$
$D_3 = \{a, \cancel{b}\}$
2
3
...
4
$D_1 = \{b,a\}$
$x_1 = b$
$D_2 = \{\cancel{a}\}$
$D_3 = \{a, \cancel{b}\}$
5
$D_1 = \{\cancel{b},a\}$
$\Rightarrow x_1 \neq b$
$D_2 = \{a\}$
$x_2 = a$
$D_3 = \{a, \cancel{b}\}$
6
$D_1 = \{\cancel{b},a\}$
$x_1 = a$
$x_1 = a$
$D_2 = \{a\}$
$D_3 = \{\cancel{a}, b\}$

ABT: Why AddLink?

Imagine ABT without AddLink message:



x_2 rejects Nogood message as obsolete (because it does not know the value of x_1), x_3 keeps on sending it => infinite loop!!

AddLink avoids it: **obsolete** info is **removed** in finite time

ABT Propoerties

Soundness/Correctness

- silent network <=> all constraints are satisfied

Completeness

- ABT performs an **exhaustive traversal** of the search space
- Parts not searched: those eliminated by nogoods
- Nogoods are legal: **logical consequences** of constraints
- Therefore, either there is no solution => ABT generates the empty nogood, or it finds a solution if it exists

Termination

- there is no infinite loop (by induction in the depth of the agent)

Asynchronous Weak-Commitment Search (AWC)

ABT problem: highly constraint variables can be assigned very late

Solution: Use **dynamic priorities**

Change **ok**? messages to include agent's current priority

Use **min-conflict heuristic**: choose assignment minimizing the number of violations

Distributed Breakout Algorithm

ABT Issues

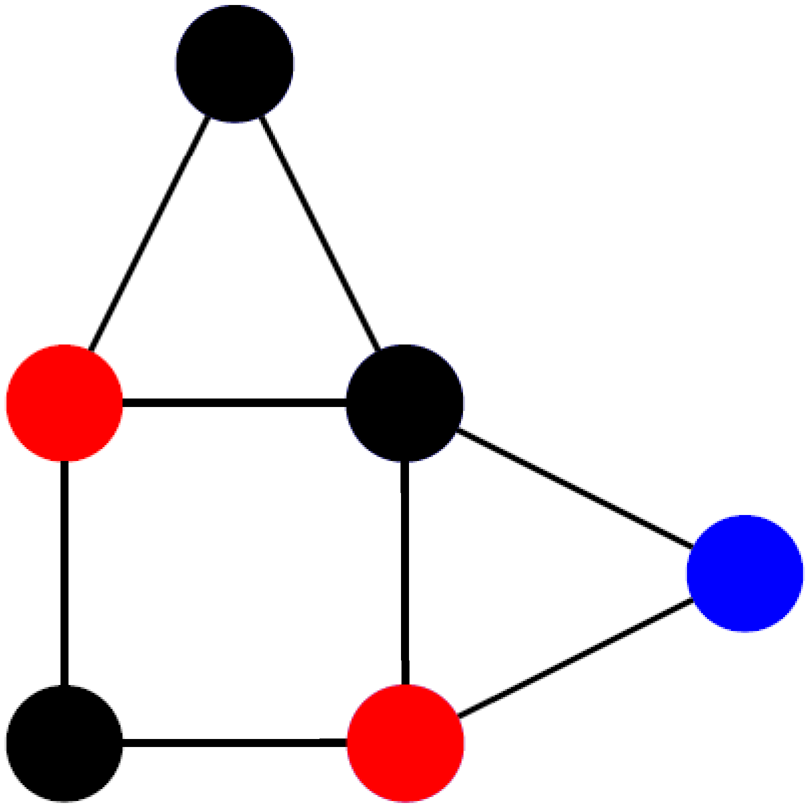
Uneven division of labor: lowest-priority agents do most of the work

Generating nogoods is complex and computationally expensive operation (also for AWC)

Cannot scale to large problems (100's of variables at most)

➔ *What if we sacrifice completeness?*

Hill Climbing



Hill Climbing

Agents asynchronously change their assignments so that they reduce the number of their violated constraints.

Can get **stuck in local optima** ➔ use techniques to escape local optima.

But: **detection** of local optima **expensive** in a distributed system.

Quasi-Local Minimum

<u>Definition (Quasi-local minimum)</u>
An agent is in a quasi-local minimum if it is violating some constraint and neither it nor any of its neighbors can make a change that results in lower cost for all.

Quasi-local minimum can be **detected locally**

Distributed Breakout Algorithm

Key idea: If in a quasi-local minimum, increase the weight of violated constraints

Messages:

- HANDLE-OK?($i \rightarrow j$, x_i) where i is the agent and x_i is its current value
- HANDLE-IMPROVE(i, *improve*) where *improve* is the maximum i could gain by changing to some other color

HANDLE-OK?(j, x_j)

received-ok$[j] \leftarrow$ TRUE

agent-view $\leftarrow$ *agent-view* $+ (j, x_j)$

if $\forall_{k \in neighbors}$ *received-ok*$[k] =$ TRUE

    then SEND-IMPROVE$()$

        $\forall_{k \in neighbors}$ *received-ok*$[k] \leftarrow$ FALSE

SEND-IMPROVE()

$cost \leftarrow$ evaluation of x_i given current weights and values.
$\textit{my-improve} \leftarrow$ possible maximal improvement
$\textit{new-value} \leftarrow$ value that gives maximal improvement
$\forall_{k \in neighbors}\ k$.HANDLE-IMPROVE$(i, \textit{my-improve}, cost)$

HANDLE-IMPROVE$(j, improve, eval)$

$received\text{-}improve[j] \leftarrow improve$

if $\forall_{k \in neighbors}\ received\text{-}improve[k] \neq$ NONE

  then SEND-OK

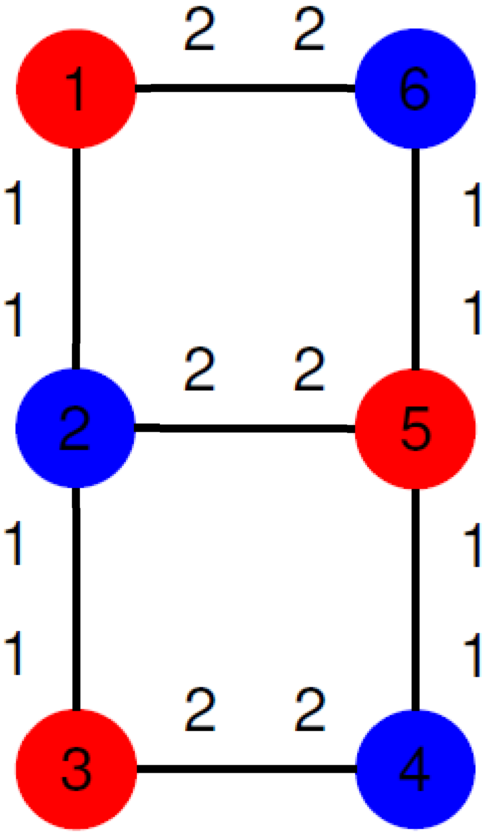
    $agent\text{-}view \leftarrow \emptyset$

    $\forall_{k \in neighbors}\ received\text{-}improve[k] \leftarrow$ NONE

$\text{SEND-OK}()$

```
if ∀_{k∈neighbors} my-improve ≥ received-improve[k]
    then x_i ← new-value
if cost > 0 ∧ ∀_{k∈neighbors} received-improve[k] ≤ 0 ▷ quasi-local opt.
    then increase weight of constraint violations
∀_{k∈neighbors} k.HANDLE-OK?(i, x_i)
```

Distributed Breakout Example



Properties

Theorem (Distributed Breakout is not Complete)
Distributed breakout can get stuck in local minimum. Therefore, there are cases where a solution exists and it cannot find it.

Why to use DCOPs?

Well-defined problem

- Clear formulation that captures most important aspects
- Many solution techniques
  - Optimal: ABT, ADOPT, DPOP, ...
  - Approximate: DSA, MGM, Max-Sum, ...

Solution techniques that can handle large problems

- approximate

When to Apply DCSP/DCOP?

Hard-to bound problems

No agreement on a **common model**

No **trusted** third party / **Privacy** concerns

Resilience / Robustness

Limited **communication**

High **dynamism**

Efficiency *typically not the reason!*

Conclusions

(Distributed) **constraint satisfaction** (CSP) is a **general**, widely applicable framework to model problems in terms of Boolean constraints over variables

Distributed CSP is required if there are **constraints** on **communication** or disclosure of **private** information, problem is difficult to formalize centrally or the system needs to be resilient

Top-down and **bottom-up** techniques exist

- top-down are complete but computationally more intensive on most problems
- bottom-up are faster but can get stuck in local minima

Very active areas of research with a lot of progress – new algorithms emerging frequently.

Reading: [Vidal] – Chapter 2, [Shoham] – Chapter 1, IJCAI 2011 Optimization in Multi-Agent Systems tutorial, Part 2, 0-35min, prof. Faltings lecture