CZECH TECHNICAL UNIVERSITY IN PRAGUE

OTEVŘENÁ INFORMATIKA

Distributed Constraint Reasoning 1

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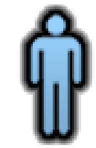
AE4M36MAS Autumn 2013 - Lecture 9

?

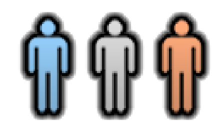
Where are We?



Agent architectures (inc. BDI architecture)

Logics for MAS

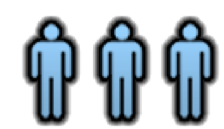
Non-cooperative game theory



Cooperative game theory

Auctions

Social choice

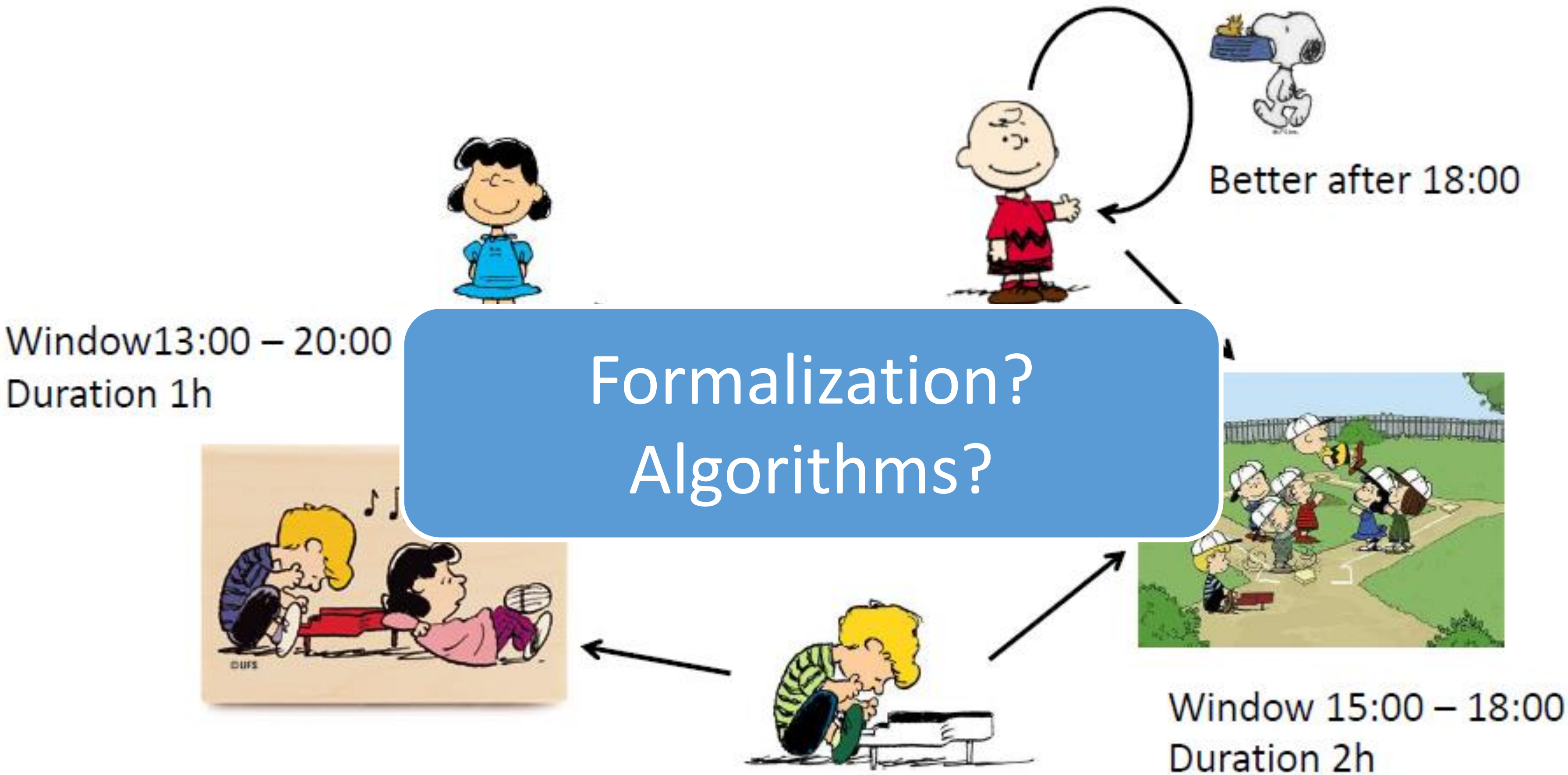


Distributed constraint reasoning

Introduction

Distributed Constraint Reasoning 1

Motivating Example: Meeting Scheduling



Formalization?
Algorithms?

Constraint Reasoning

Constraints pervade our lives (time, money, ...) and usually perceived as elements that **limit solutions** to the problems we face

From a **computational point of view**, they:

- **reduce the space** of possible solutions
- **encode knowledge** about the problem at hand
- are **key components** for efficiently solving hard problems

Hard computational problems can often be **made tractable** by carefully **considering the constraints** that define the **structure of the problem**.

- applies to planning and scheduling, operational research, automated reasoning and decision theory, computer vision... and multiagent systems

Constraint Reasoning in/for MAS

Focus on how **constraint reasoning** can be used to address **coordination and optimization** problems in MAS.

- Set of agents must come to some **agreement**, typically via some form of **negotiation**, about **which action** each agent should take in order to jointly obtain the **best solution** for the **whole system**.

We will consider **Distributed Constraint Reasoning Problems** where:

- Each agent **negotiates locally** with just a subset of other agents (usually called **neighbours**) that are those that can **directly influence** his/her behaviour.

Lecture Objectives

At the end of 2-lecture series you will

- will be able to express problems as distributed constraint reasoning problems
- will be able to use basic solution algorithms for constraint satisfaction and optimization

Lecture Outline

1. Introduction
2. Definitions and Examples
3. Solution Methods
4. Asynchronous Backtracking Algorithm
5. Summary and Outlook

Definitions

Distributed Constraint Reasoning 1

Constraint Network

A **constraint network** $\mathcal{N}$ is formally defined as a triple $\langle X, D, C \rangle$ where:

- $X = \{x_1, \dots, x_n\}$ is a set of **variables**;
- $D = \{D_1, \dots, D_n\}$ is a set of **variable domains**, which enumerate all possible values of the corresponding variables; and
- $C = \{C_1, \dots, C_m\}$ is a set of **constraints**; where a constraint C_i is defined on a subset of variables $S_i \subseteq X$ which comprise the **scope of the constraint**
  - $r_i = |S_i|$ is the **arity** of constraint i

Hard vs. Soft Constraints

Hard constraint C_i^h is a Boolean **predicate** P_i that defines **valid joint assignments** of variables in the scope

$$P_i: D_{i_1} \times \cdots \times D_{i_r} \rightarrow \{\mathrm{F}, \mathrm{T}\}$$

Soft constraint C_i^s is a **function** F_i that maps every possible joint assignment of all variables in the scope to a real value

$$F_i: D_{i_1} \times \cdots \times D_{i_r} \rightarrow \Re$$

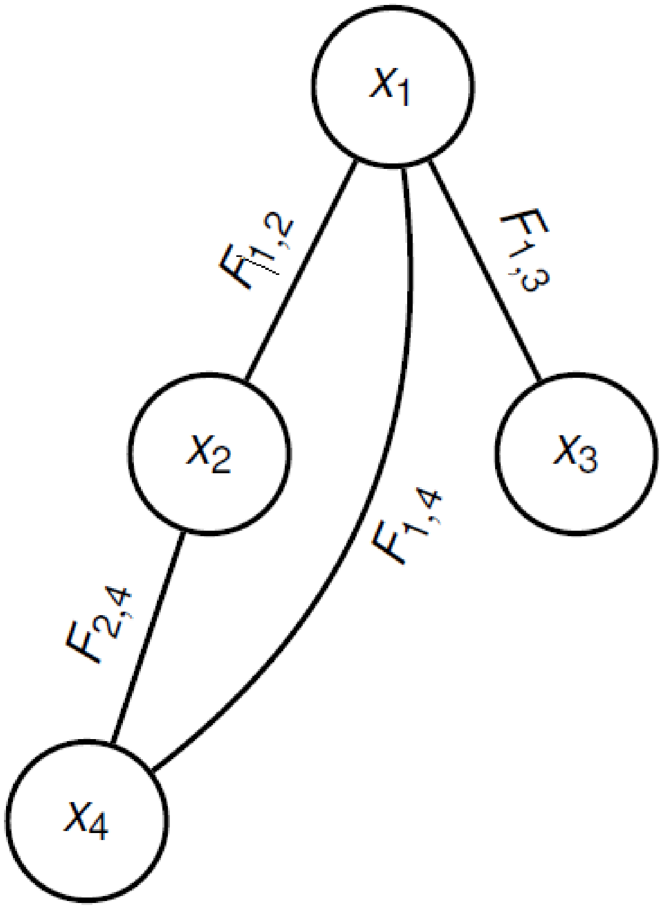
Binary Constraint Networks

Binary constraint networks are those where each **constraint** (soft or hard) is defined **over two variables**.

Every constraint network can be **mapped to a binary** constraint network

- requires the addition of variables and constraints
- may add complexity to the model

Binary constraint networks can be represented by a **constraint graph**



Types of Constraint Reasoning Problems

Constraint Satisfaction Problem (CSP)

- **Objective**: find an **assignment** for all the variables in the network that **satisfies all constraints**.

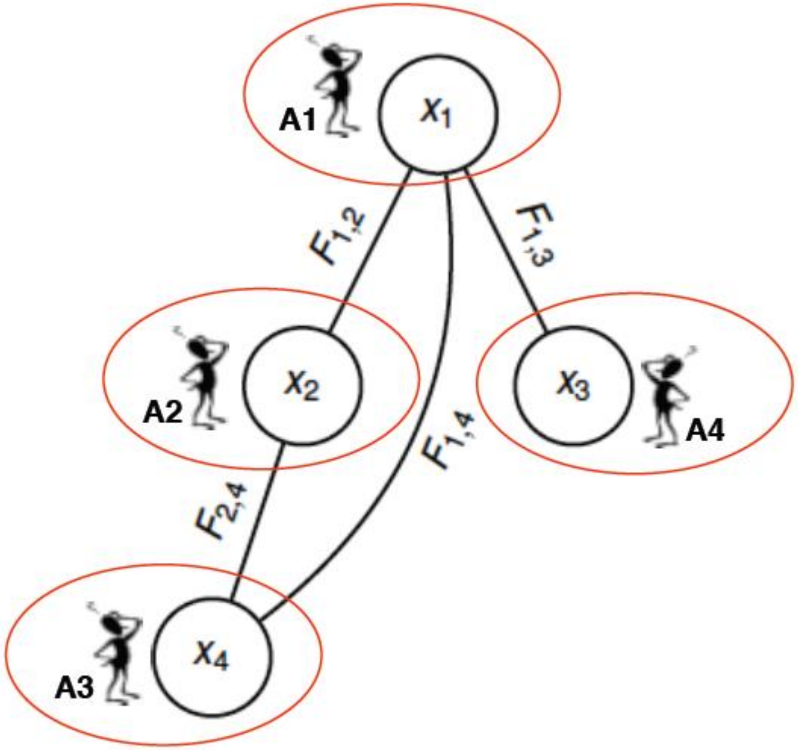
Constraint Optimization Problem (COP)

- **Objective**: find an **assignment** for all the variables in the network that **satisfies all constraints** and **optimizes a global function**.
- **Global function** = aggregation (typically sum) of constrain functions, i.e., $F = \sum F_i$

Distributed Constraint Reasoning

When operating in a **decentralized context**:

- a set of **agents control variables**
- agents **interact to find a solution** to the constraint network



Distributed Constraint Reasoning Problem

A distributed constraint reasoning problem consists of a **constraint network** $\langle X, D, C \rangle$ and a **set of agents** $A = \{A_1, \ldots, A_k\}$ where each agent:

- **controls a subset** of the variables $X_i \subseteq \mathrm{X}$
- is only **aware of constraints** that involve variable it controls
- communicates only with its **neighbours**

Types of DCR Problems

1. Distributed CSP (DCSP)
2. Distributed COP (DCOP)

Examples / Applications

Distributed Constraint Reasoning

Real World Applications

Many standard benchmark problems in computer science can be modeled using the DCOP framework:

- graph coloring

As can many real world applications:

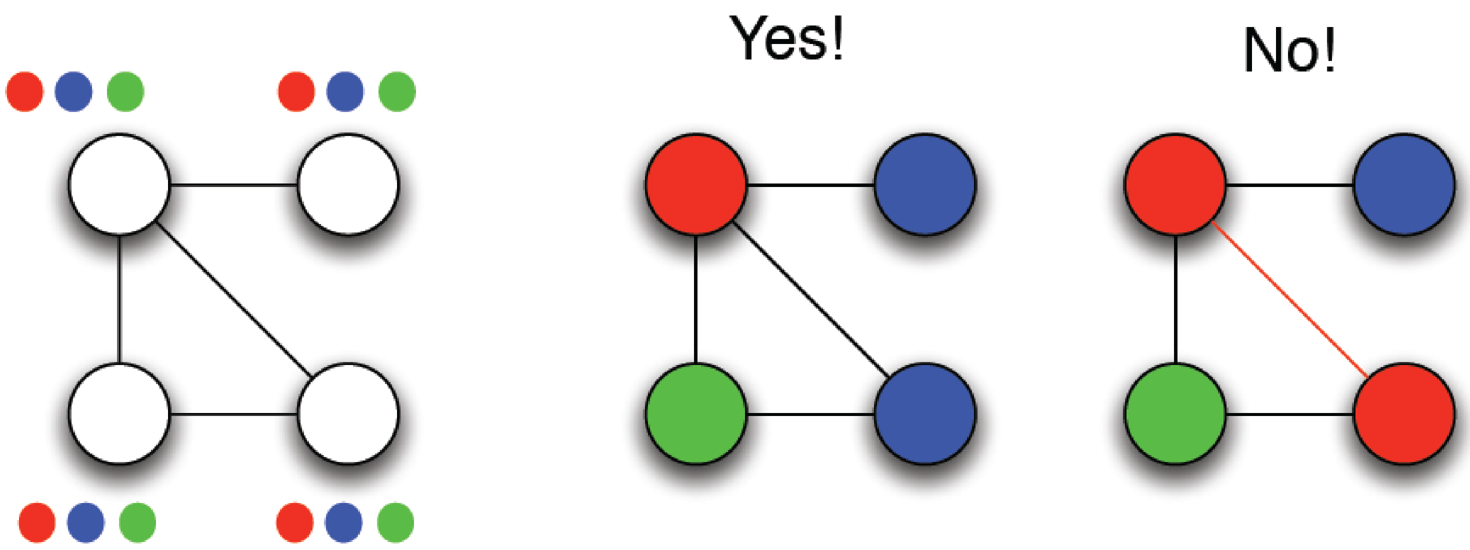
- human-agent organizations (e.g. meeting scheduling)
- sensor networks and robotics (e.g. target tracking)

Graph Colouring

- Popular benchmark
- Simple formulation
- Complexity controlled with few parameters:
  - Number of available colors
  - Number of nodes
  - Density ($\#nodes/\#constraints$)
- Many versions of the problem:
  - CSP, MaxCSP, COP

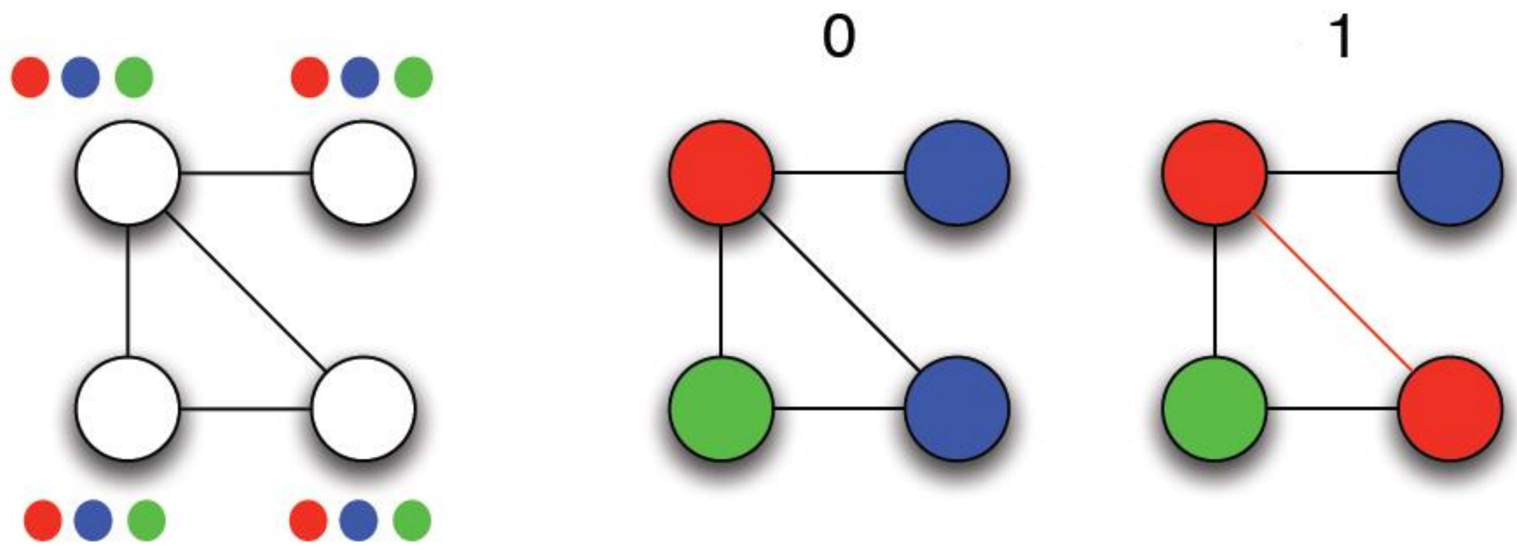
Graph Colouring: CSP

- Nodes can take k colors
- Any two adjacent nodes should have different colors
  - If it happens this is a conflict



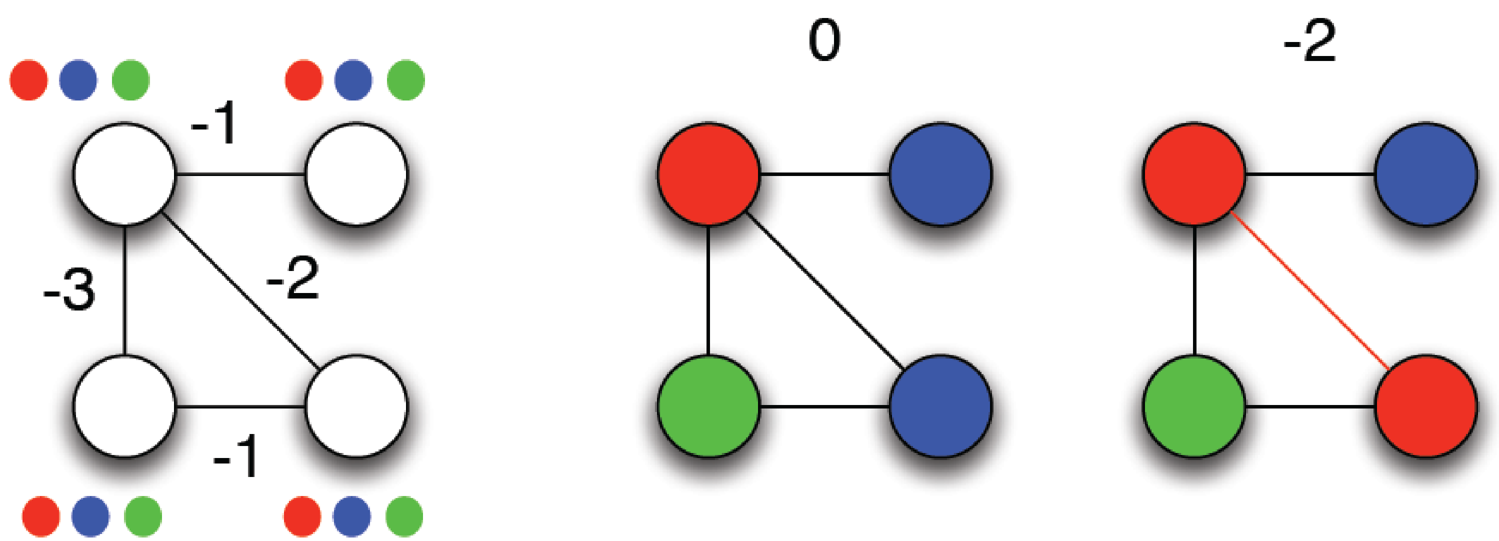
Graph Colouring: Max CSP

- Minimize the number of conflicts



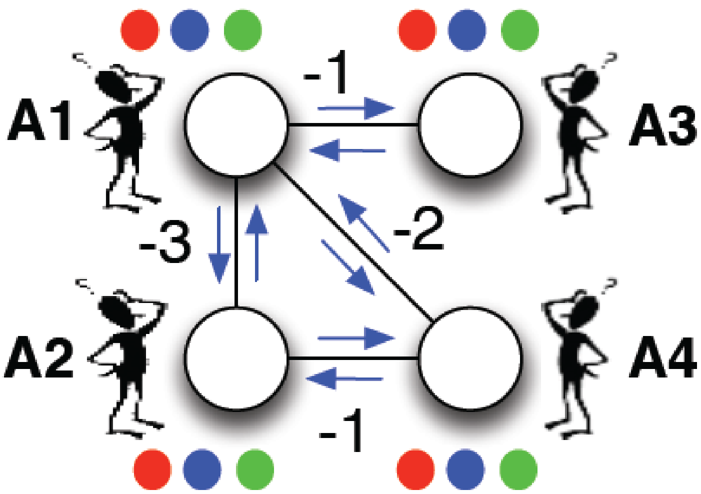
Graph Colouring: COP

- Different weights to violated constraints
- Preferences for different colors



Graph Colouring: DCOP

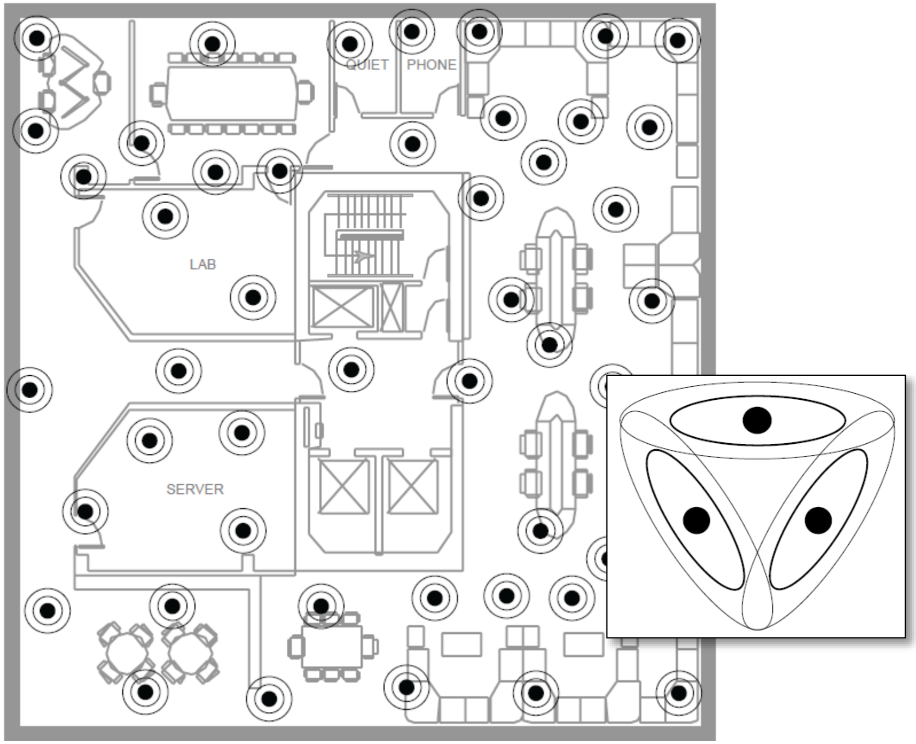
- Each node:
  - controlled by one agent
- Each agent:
  - Preferences for different colors
  - Communicates with its direct neighbours in the graph



- A1 and A2 exchange preferences and conflicts
- A3 and A4 do not communicate

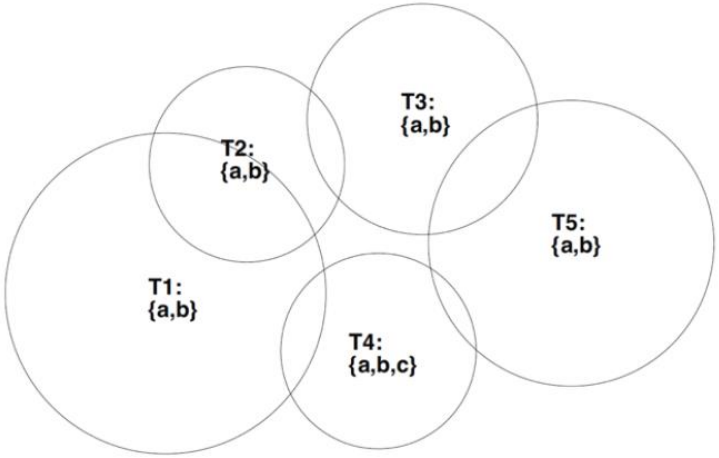
DCOP formalization

Channel Allocation in Sensor Networks



Find a **non-conflicting assignment** of communication channels assuming **local communication only**

DCSP Formalization of Channel Allocation



Agents $A = \{A_1, \ldots, A_n\}$ correspond to **sensors**.

Variables $X = \{X_1, \ldots X_n\}$ correspond to **selected** broadcast **channels**: X_i is the channel on which the sensor A_i broadcasts.

Domains $D = \{D_1, \ldots, D_n\}$ correspond to **available channels**.

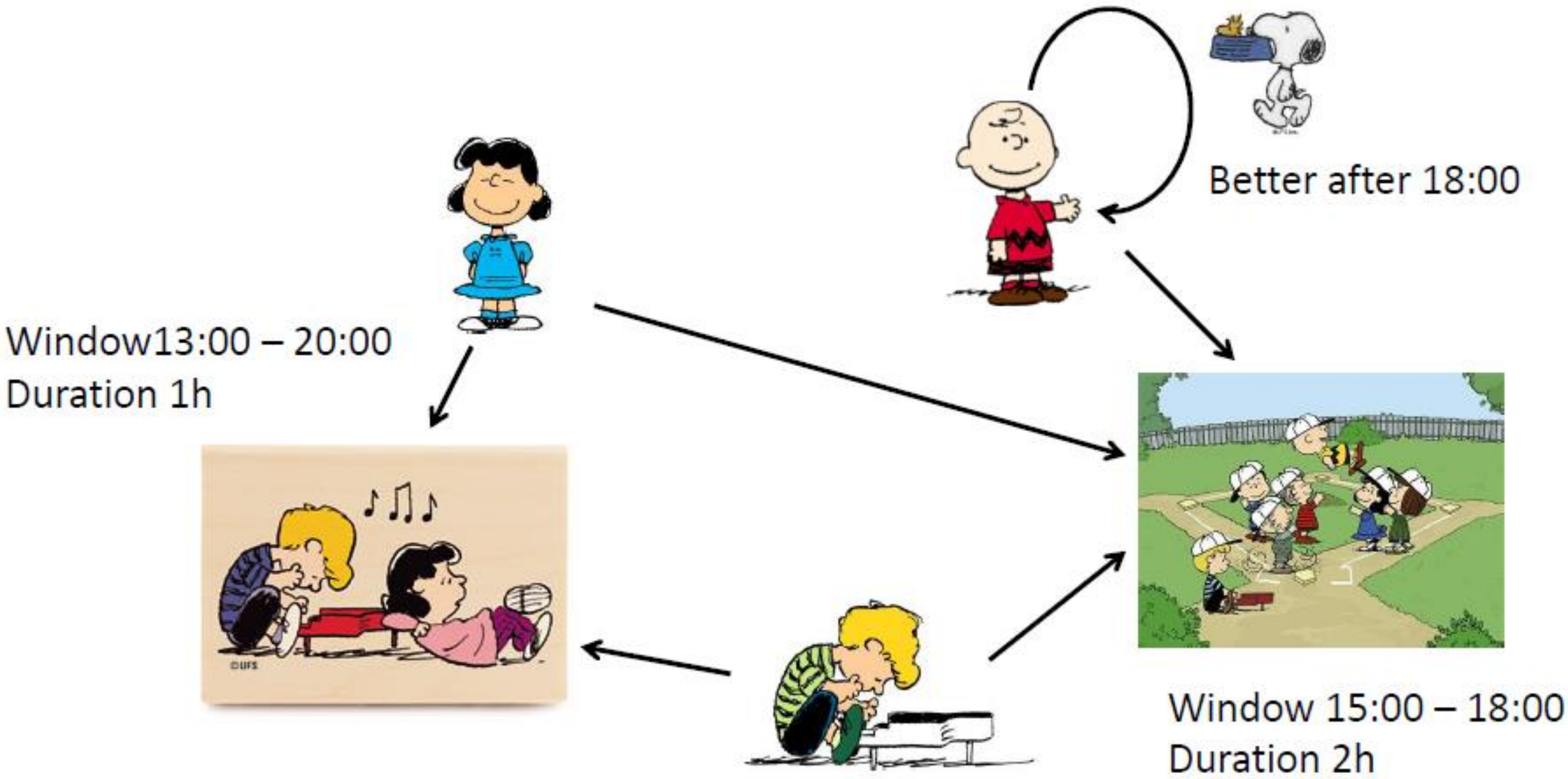
For each pair of sensors i, j that have **overlapping** broadcast **ranges**, there is a corresponding Boolean constraint $P_{i,j}$ so that

$$P_{i,j}(X_i, X_j) = T \text{ iff } X_i \neq X_j$$

i.e. sensors will **overlapping** ranges must use **different** channels.

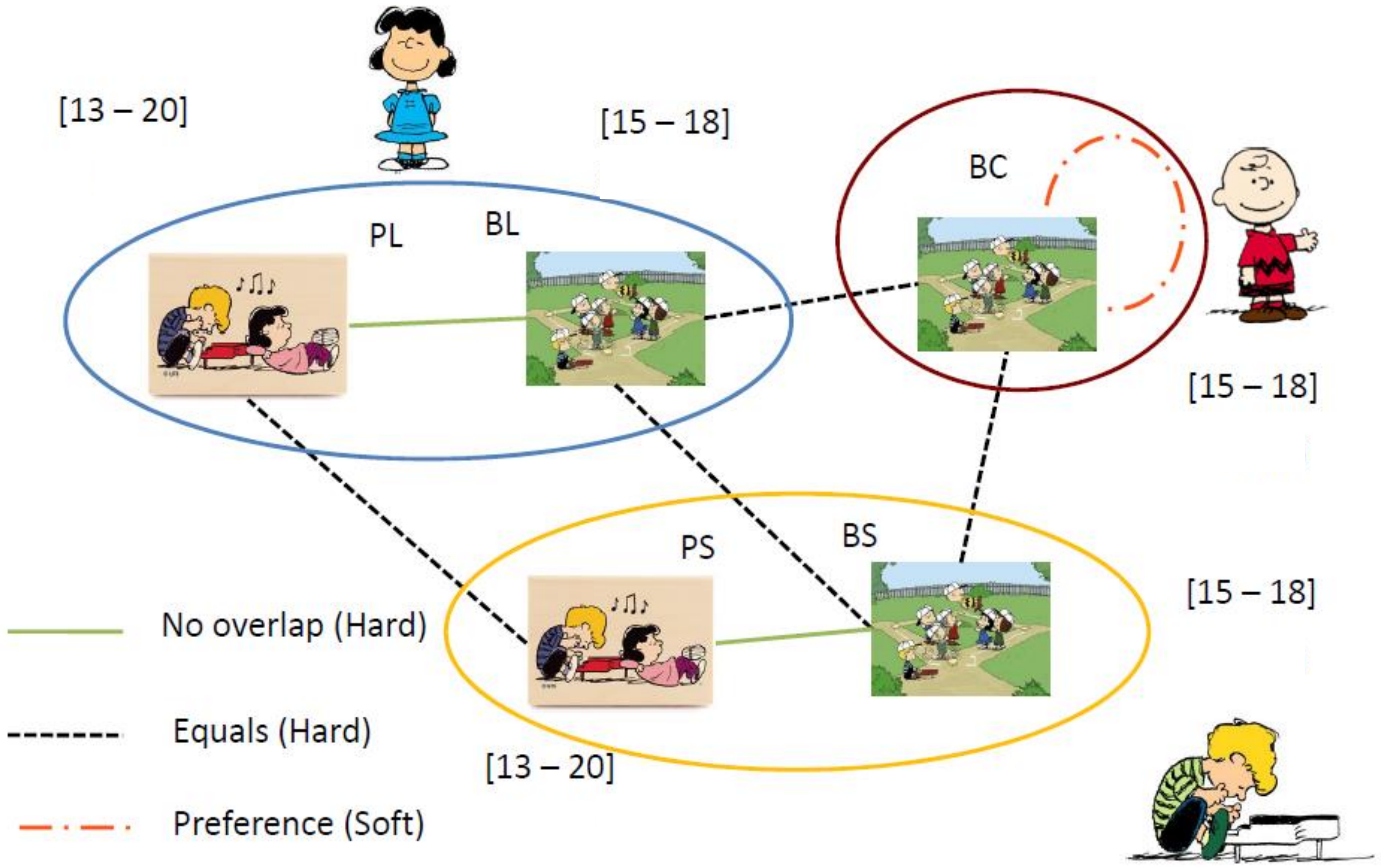
<u>**Objective**</u>: Find a **channel allocation** where **no overlapping sensors** use the **same** channel.

Example: Meeting Scheduling



from http://cs.smu.ca/~pawan/wi07/petcu.pdf

Meeting Scheduling Formalization



Why to Apply DCSP/DCOP?

Hard-to bound problems

No agreement on a **common model**

No **trusted** third party / **Privacy** concerns

Resilience / Robustness

Dynamism

Efficiency *typically not the reason!*

Solution Approach: DCSP

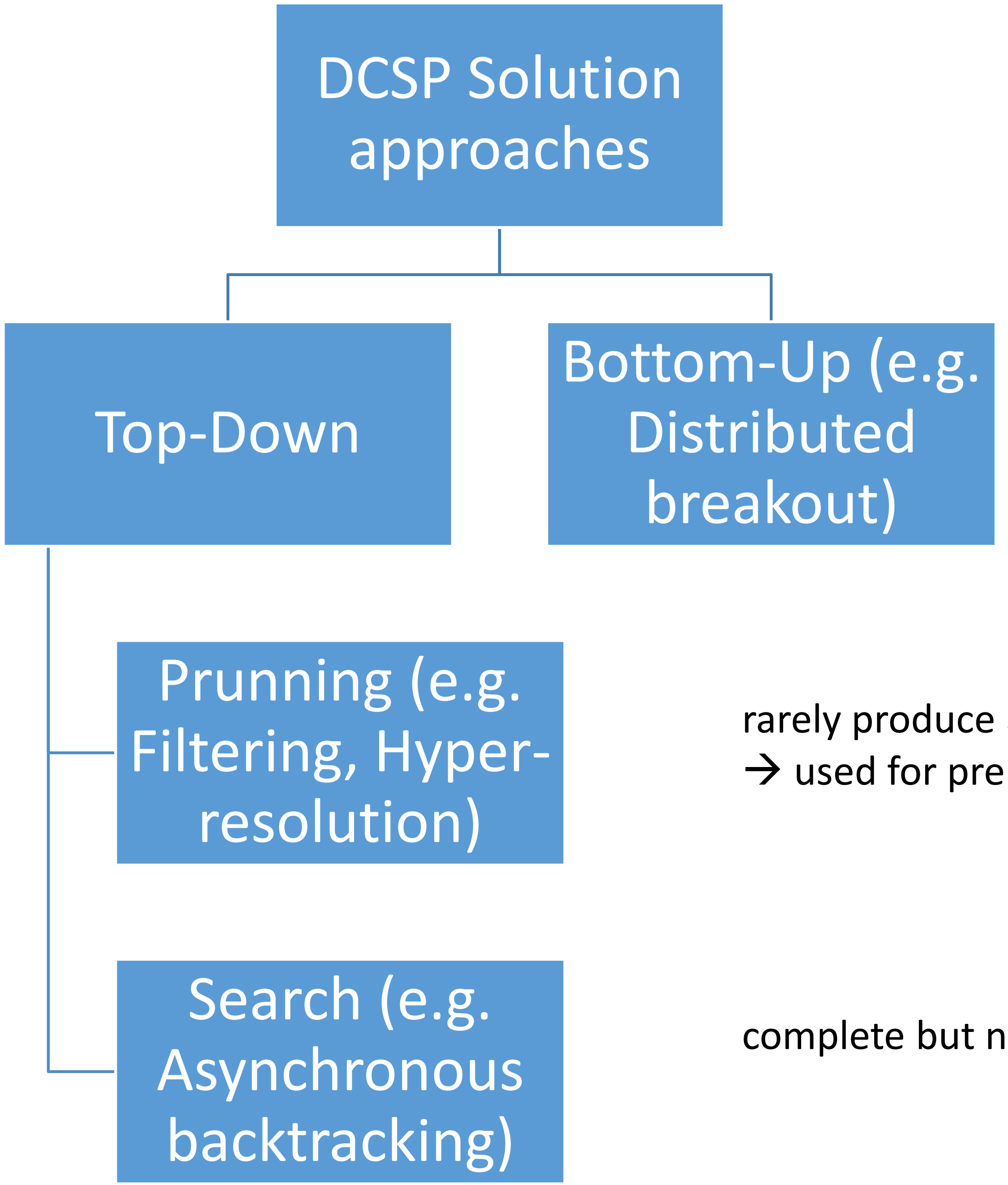
Distributed Constraint Reasoning 1

Requirements on a Good Algorithm

Soundness/Correctness: the solution returned is valid

Termination: in a finite number of steps

Completeness: finds an (optimal) solution if it exists



approximate but scalable

rarely produce a solution
→ used for preprocessing

complete but not scalable

Distributed Algorithms

Synchronous: agents take steps following some fixed order (or computing steps are done simultaneously, following some external clock).

Asynchronous: agents take steps in arbitrary order, at arbitrary relative speeds.

Partially synchronous: there are some restrictions in the relative timing of events

Synchronous vs Asynchronous

Synchronous

- A few agents are active, most are waiting
- Active agents take decisions with updated information
- Low degree of concurrency / poor robustness
- Algorithms: direct extensions of centralized ones

Asynchronous

- All agents are active simultaneously
- Information is less updated, obsolescence appears
- High degree of concurrency / robust approaches
- Algorithms: new approaches

Asynchronous Backtracking: Assumptions

1. Agents communicate by **sending messages**
2. An agent can send messages to others, iff it knows their identifiers (**directed communication** / no broadcasting)
3. The **delay** transmitting a message is **finite** but random
4. For any pair of agents, messages are **delivered in the order** they were sent
5. Agents **know the constraints in which they are involved**, but not the other constraints
6. Each agent owns a **single variable** (agents = variables)
7. Constraints are **binary** (2 variables involved)

not essential, can be lifted

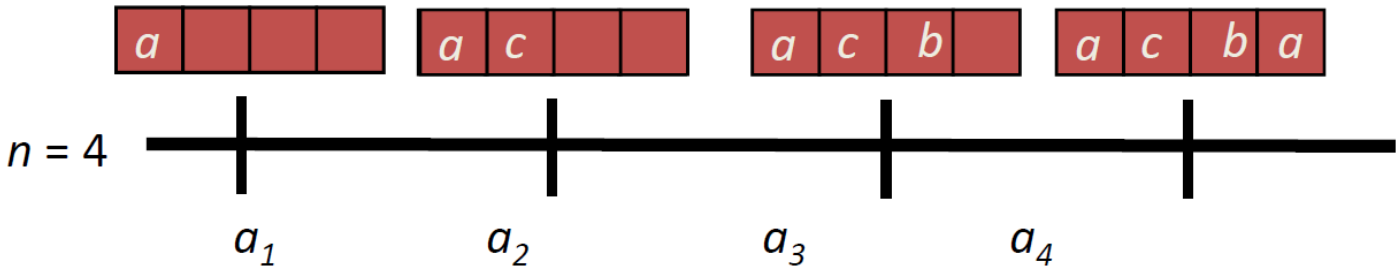
Asynchronous Backtracking Algorithm (ABT)

Distributed Constraint Reasoning 1

Synchronous Backtracking

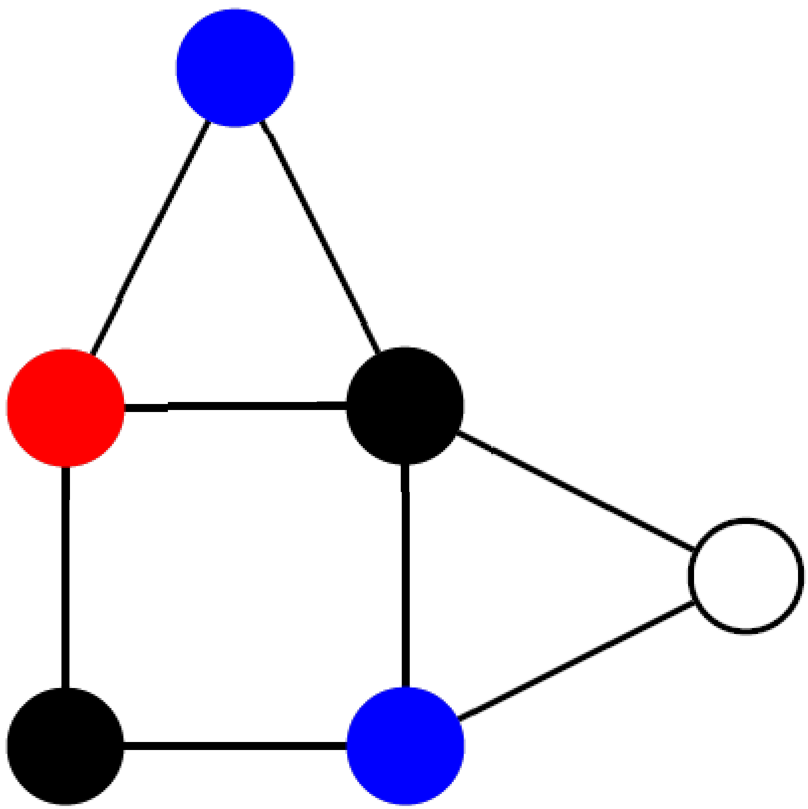
Agents agree on an variable order and repeat:

1. send partial solution up to X_{k-1} to k-th agent.
2. k-th agent generates the next extension to this partial solution.
3. if the solution cannot be extended consistently: $k \leftarrow k - 1$ (backtrack control to previous agent).
4. if solution can be extended consistently, $k \leftarrow k + 1$ (pass control to the next agent)
5. if $k < 1$: stop → unsolvable.
6. if $k > n$: stop → assignment = solution



Problem: *Only one agent working at a time => very inefficient*

Backtracking Illustration



Asynchronous Backtracking (ABT)

Revolutionary idea in 1998

Fully asynchronous algorithm

- all agents active, take a value and inform
- no agent has to wait for other agents

Total order among agents (to avoid cycles) → priorities

Constraints are directed: from higher-priority to lower-priority agents

ABT plays in asynchronous distributed context the same role as backtracking in centralized

ABT: Core Principles

High-priority agents decide on assignment, lower-priority have to **accommodate** or say they cannot.

Higher-priority agent (j) informs a lower-priority agent (k) of its assignment

Lower-priority agent (k) evaluates the constraint with its own assignment

- If permitted → no action
- else → look for a value consistent with j
  - If it exists → k takes that value
  - else → the agent view of k is a ***nogood*** → distributed backtrack

ABT: NoGoods

Nogood: conjunction of (variable, value) pairs of higher priority agents, which removes a value of the current one

Example: $x \neq y, D_x = D_y = \{a, b\}$, x higher-priority than y:

- when x assumes a and a message $[x \leftarrow a]$ arrives to y, the agent y generates the nogood $x = a \;\; => \;\; y \neq a$ that removes value a of D_y.
- if x changes value, when $[x \leftarrow b]$ arrives to y, the no good $x = a \;\; =>$ $y \neq a$ is eliminated, value a is available again and a new nogood removing b is generated

*Nogoods are required to ensure **systematic traversal** of search space **in asynchronous, distributed context***

ABT: NoGood Resolution

When all values of variable y are removed, the **conjunction** of the left-hand sides of its nogoods is **also a nogood**.

Resolution: the process of generating a new nogood that is a logical **consequence** of existing ones.

<u>Example:</u>

$x \neq y, \;\; z \neq y, D_x = D_y = D_z = \{a, b\}, \; x, z$ higher priority than y

assume: $x = a \Rightarrow y \neq a$; $z = b \Rightarrow y \neq b$; i.e., all values for y ruled out

then: $x = a \; \wedge \; z = b$ is a nogood

i.e. in a directed form: $x = a \Rightarrow z \neq b$ (assuming x higher-priority than z)

How ABT Works

Asynchronous action; spontaneous assignment

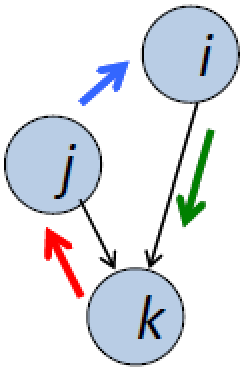
Four operations:

- **Assignment**: j takes value a: j informs lower priority agents
- **Backtrack**: k has no consistent values with higher-priority agents: k resolves nogoods and sends a a backtrack (nogood) message
- **New links**: j receives a nogood mentioning i, unconnected with j: j asks i to set up a link
- **Stop**: "no solution" (empty nogood) detected by an agent: stop

Solution: when agents are silent for a while (quiescence), every constraint is satisfied => solution;

- detected by specialized algorithms outside ABT

ABT: Messages



Ok?$(i \rightarrow k, a)$: higher-priority agent i informs lower-priority agent k that it takes value a

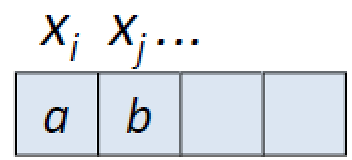
NoGood$(k \rightarrow j, i = a \Rightarrow j \neq b)$:

- when all k*'s* values are forbidden:
- k requests j (the nearest higher-priority agent in the nogood) to *backtrack*
- then: k forgets j's value, k takes some value
- j may detect obsolescence of the NoGood message

AddLink$(j \rightarrow i)$: set a link from i to j, to know i value

Stop: there is no solution

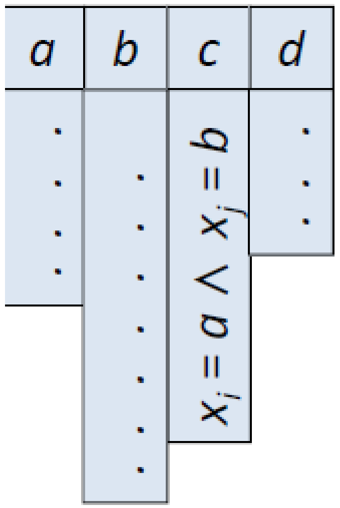
ABT: Data Structures



Current context / agent view: values of higher-priority constrained agents

NoGood store: each removed value has a *justifying* nogood

$$x_i = a \wedge x_j = b \Rightarrow x_k \neq c$$



- Stored nogoods must be **active**: left-hand side of the nogood satisfied in the current context
- If a nogood is no longer active, it is removed (and the value is available again)

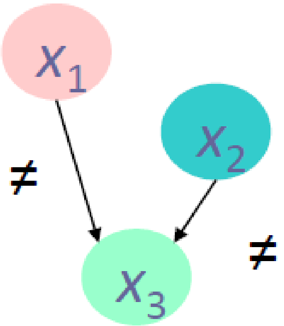
ABT: Graph Coloring Example

Variables x_1, x_2, x_3; $D_1 = \{b, a\}, D_2 = \{a\}, D_3 = \{a, b\}$

3 agents, lex ordered:

2 difference constraints: c_{13} and c_{23}

Constraint graph:

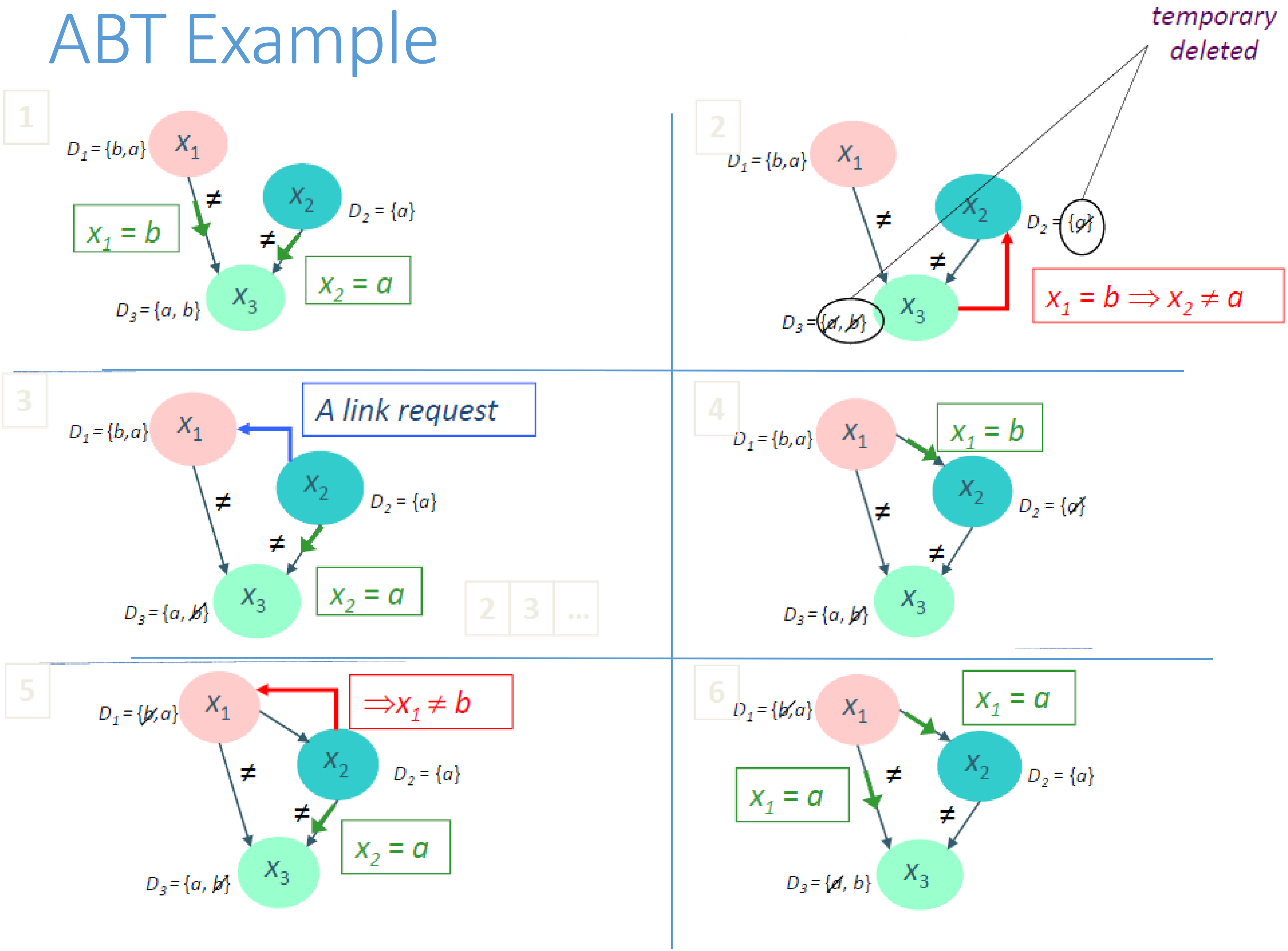


Value-sending agents: x_1 and x_2

Constraint-evaluating agent: x_3

Each agent *checks* constraints of incoming links: $Agent_1$ and $Agent_2$ check nothing, $Agent_3$ checks c_{13} and c_{23}

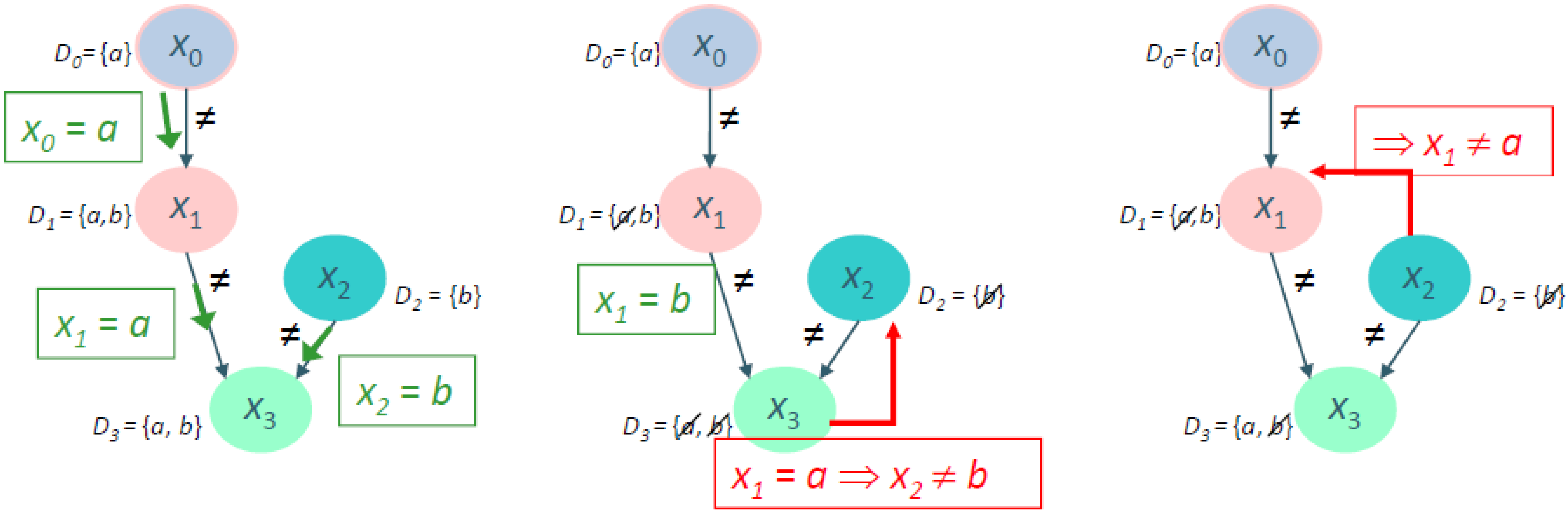
ABT Example



temporary
deleted
1
$D_1 = \{b,a\}$
x_1
$x_1 = b$
x_2
$D_2 = \{a\}$
$x_2 = a$
x_3
$D_3 = \{a, b\}$
2
$D_1 = \{b,a\}$
x_1
x_2
$D_2 = \{\cancel{a}\}$
$x_1 = b \Rightarrow x_2 \neq a$
x_3
$D_3 = \{\cancel{a}, \cancel{b}\}$
3
$D_1 = \{b,a\}$
x_1
A link request
x_2
$D_2 = \{a\}$
$x_2 = a$
x_3
$D_3 = \{a, \cancel{b}\}$
2 3 ...
4
$D_1 = \{b,a\}$
x_1
$x_1 = b$
x_2
$D_2 = \{\cancel{a}\}$
x_3
$D_3 = \{a, \cancel{b}\}$
5
$D_1 = \{\cancel{b},a\}$
x_1
$\Rightarrow x_1 \neq b$
x_2
$D_2 = \{a\}$
$x_2 = a$
x_3
$D_3 = \{a, \cancel{b}\}$
6
$D_1 = \{\cancel{b},a\}$
x_1
$x_1 = a$
x_2
$D_2 = \{a\}$
$x_1 = a$
x_3
$D_3 = \{\cancel{a}, b\}$

ABT: Why AddLink?

Imagine ABT without AddLink message:



x_2 rejects Nogood message as obsolete (because it does not know the value of x_1), x_3 keeps on sending it => infinite loop!!

AddLink avoids it: obsolete info is removed in finite time

ABT Propoerties

Soundness/Correctness

- silent network <=> all constraints are satisfied

Completeness

- ABT performs an **exhaustive traversal** of the search space
- Parts not searched: those eliminated by nogoods
- Nogoods are legal: **logical consequences** of constraints
- Therefore, either there is no solution => ABT generates the empty nogood, or it finds a solution if it exists

Termination

- there is no infinite loop (by induction in the depth of the agent)

Asynchronous Weak-Commitment Search (AWC)

ABT problem: highly constraint variables can be assigned very late

Solution: Use **dynamic priorities**

Change **ok**? messages to include agent’s current priority

Use **min-conflict heuristic**: choose assignment minimizing the number of violations

Distributed Breakout Algorithm

ABT Issues

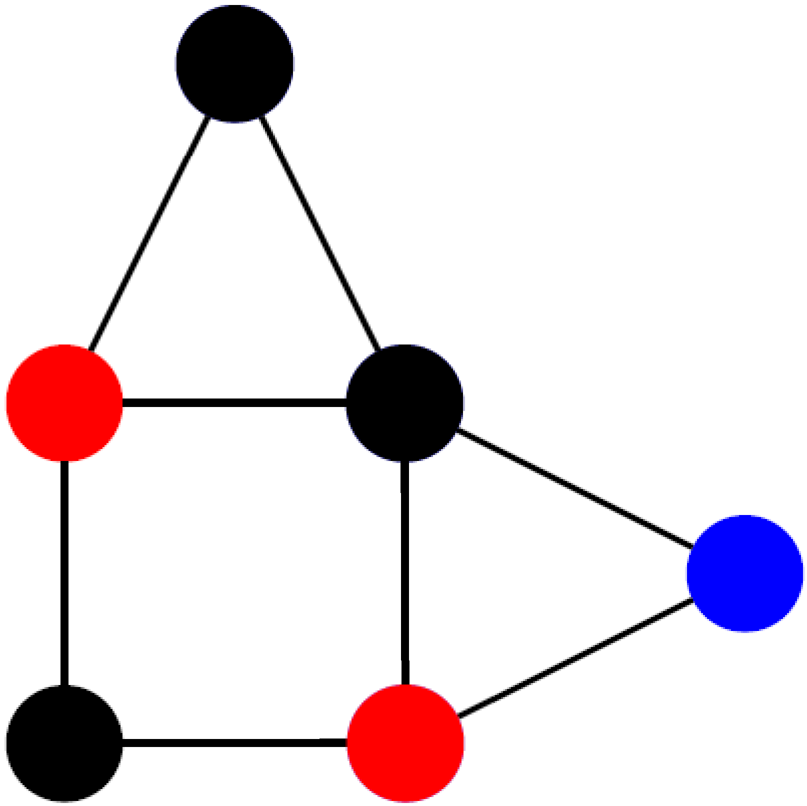
Uneven division of labor: lowest-priority agents do most of the work

Generating nogoods is complex and computationally expensive operation (also for AWC)

Cannot scale to large problems (100’s of variables at most)

➔ *What if we sacrifice completeness?*

Hill Climbing



Hill Climbing

Agents asynchronously change their assignments so that they reduce the number of their violated constraints.

Can get **stuck in local optima** → use techniques to escape local optima.

But: **detection** of local optima **expensive** in a distributed system.

Quasi-Local Minimum

Definition (Quasi-local minimum)
An agent is in a quasi-local minimum if it is violating some constraint and neither it nor any of its neighbors can make a change that results in lower cost for all.

Quasi-local minimum can be **detected locally**

Distributed Breakout Algorithm

Key idea: If in a quasi-local minimum, increase the weight of violated constrainst

Messages:

- HANDLE-OK?(*i* → *j*, *x_i*) where *i* is the agent and *x_i* is its current value
- HANDLE-IMPROVE(*i, improve*) where *improve* is the maximum *i* could gain by changing to some other color

HANDLE-OK?(j, x_j)

$\textit{received-ok}[j] \leftarrow$ TRUE

$\textit{agent-view} \leftarrow \textit{agent-view} + (j, x_j)$

if $\forall_{k \in neighbors}\ \textit{received-ok}[k] =$ TRUE

    then SEND-IMPROVE$()$

        $\forall_{k \in neighbors}\ \textit{received-ok}[k] \leftarrow$ FALSE

SEND-IMPROVE()

cost ← evaluation of x_i given current weights and values.
my-improve ← possible maximal improvement
new-value ← value that gives maximal improvement
$\forall_{k \in neighbors}$ k.HANDLE-IMPROVE(i, *my-improve*, *cost*)

HANDLE-IMPROVE$(j, improve, eval)$

$received\text{-}improve[j] \leftarrow improve$

if $\forall_{k \in neighbors}\ received\text{-}improve[k] \neq$ NONE

  then SEND-OK

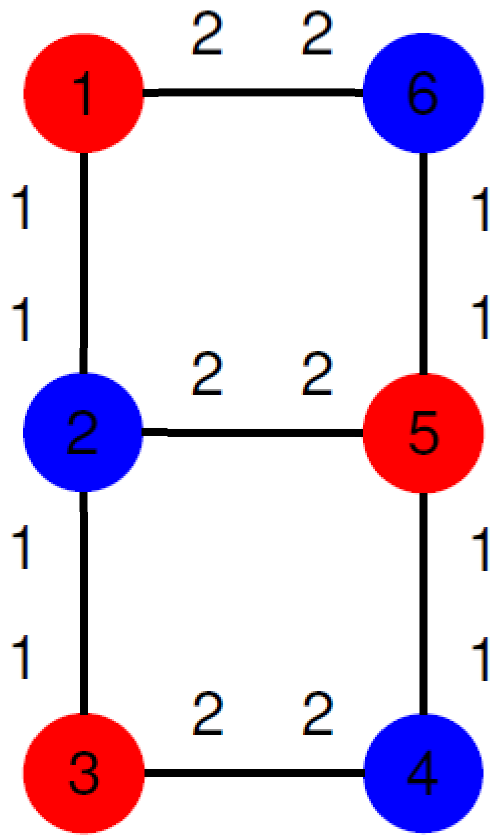
    $agent\text{-}view \leftarrow \emptyset$

    $\forall_{k \in neighbors}\ received\text{-}improve[k] \leftarrow$ NONE

SEND-OK()

if $\forall_{k \in neighbors}\ \mathit{my\text{-}improve} \geq \mathit{received\text{-}improve}[k]$
    then $x_i \leftarrow \mathit{new\text{-}value}$
if $\mathit{cost} > 0 \wedge \forall_{k \in neighbors}\ \mathit{received\text{-}improve}[k] \leq 0$ ▷ quasi-local opt.
    then increase weight of constraint violations
$\forall_{k \in neighbors}\ k$.HANDLE-OK?$(i, x_i)$

Distributed Breakout Example



Properties

Theorem (Distributed Breakout is not Complete)
Distributed breakout can get stuck in local minimum. Therefore, there are cases where a solution exists and it cannot find it.

Why to use DCOPs?

Well-defined problem

- Clear formulation that captures most important aspects
- Many solution techniques
  - Optimal: ABT, ADOPT, DPOP, ...
  - Approximate: DSA, MGM, Max-Sum, ...

Solution techniques that can handle large problems

- approximate

Conclusions

(Distributed) **constraint satisfaction** (CSP) is a **general**, widely applicable framework to model problems in terms of Boolean constraints over variables

Distributed CSP is required if there are **constraints** on **communication** or disclosure of **private** information, problem is difficult to formalize centrally or the system needs to be resilient

Top-down and **bottom-up** techniques exist

- top-down are complete but computationally more intensive on most problems
- bottom-up are faster but can get stuck in local minima

Very active areas of research with a lot of progress – new algorithms emerging frequently.

Reading: [Vidal] – Chapter 2, [Shoham] – Chapter 1, IJCAI 2011 Optimization in Multi-Agent Systems tutorial, Part 2, 0-35min, prof. Faltings lecture