## Auctions

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January 12, 2016

## Simple Auction Example

Consider a second-price, sealed-bid auction with two bidders who have independent, private values $v_{i}$ which are either 1 or 3 . For each bidder, the probabilities of 1 and 3 are both 0.5 .

■ What is the seller's expected revenue?
■ Now let's suppose that there are three bidders who have independent, private values $v_{i}$ which are either 1 or 3 . For each bidder, the probabilities of 1 and 3 are both 0.5 . What is the sellers expected revenue in this case?

## Simple Auction Example

A seller runs a second-price, sealed-bid auction for an object. There are two bidders, $a$ and $b$, who have independent, private values $v_{i}$ which are either 0 or 1 . For both bidders the probabilities of $v_{i}=0$ and $v_{i}=1$ are 0.5 each. Both bidders understand the auction, but bidder $b$ sometimes makes a mistake about his value for the object. Half of the time his value is 1 and he is aware that it is 1 the other half of the time his value is 0 but occasionally he mistakenly believes that his value is 1 . Lets suppose that when $b$ 's value is 0 he acts as if it is 1 with probability 0.5 and as if it is 0 with probability 0.5 . So in effect bidder $b$ sees value 0 with probability 0.25 and value 1 with 0.75 probability. Bidder $a$ never makes mistakes about his value for the object, but he is aware of the mistakes that bidder $b$ makes. Assume that if there is a tie at a bid of $x$ for the highest bid the winner is selected at random from among the highest bidders and the price is $x$.

- Assume bidder $b$ is not aware of his mistake and bids optimally given the perceptions of the value of the object. Is bidding his true value still a dominant strategy for bidder $a$ ?


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- Assume that $b$ is aware of his mistake and bids must be integers. What are the optimal strategies?


## Simple Auction Example

Consider a first-price, sealed-bid auction with two bidders who have independent, private values $v_{i}$ which are independent and uniformly distributed over the set $\{0,1,2\}$. The bids in the auction must be nonnegative integers. Assume that ties are broken randomly.

- Find all equilibria.


## Overview of the Multi-Agent Systems Course

- A collection of formal models, algorithms, and perspectives when modeling situations with multiple (typically rational entities).
■ Still fairly new, less settled than other areas (e.g., compared to planning, theory of algorithms, complexity theory, ...)
- Useful in your future industrial/academic career.
- Reactive Planning:
- simulations of large systems (production systems, computer games, transport simulations, weather, disease spreading, ...)
- robotics
- internet of things (a truly open multi-agent system - e.g., a really smart fridge)


## Overview

- Game Theory (direct, equilibrium computation):
- security applications (designing/implementing security protocols)
- robust optimization (zero-sum games)
- computer games (board, puzzle - game-playing, creating a competitive opponent for players)
- bounded rationality (quantal response equilibrium, ...)

■ Game Theory (indirect, mechanism design):

- designing rules (e.g., spectrum auctions, how to motivate users to do something)
- resource allocation (e.g., computation time, cpu, mem; different from scheduling - you want the rational agents to participate, the allocation must be fair)


## Overview

- Game Theory (cooperative):
- cost/utility sharing (how to distribute costs/utility among teams in a fair way)
- creating optimal teams (logistics, transportation, energy, finding and grouping users for sales)
- Social Choice
- Crowdsourcing, finding the ground truth based on votes
- design rules for voting (e.g., you want to give the opportunity to the users to improve your app)
- DCSP/DCOP
- task completion by a collection of robots (drones, nanobots, ...)
- example of nice distributed algorithms

