# Extensive-Form Games 

Branislav Bošanský<br>Czech Technical University in Prague<br>branislav.bosansky@agents.fel.cvut.cz

November 10, 2015

Previously ... on multi-agent systems.

## Construct a Mathematical Program for Other Equilibria

## Correlated Equilibrium (2 players)

Given a 2-player game $G=(N, A, u)$, a Correlated Equilibrium is a tuple $(v, \pi, \sigma)$, where $v$ is a couple of random variables $v=\left(v_{1}, v_{2}\right)$ with respective domains $D=\left(D_{1}, D_{2}\right), \pi$ is a joint distribution over $v, \sigma=\left(\sigma_{1}, \sigma_{2}\right)$ is a vector of mappings $\sigma_{i}: D_{i} \rightarrow A_{i}$, and for each agent $i$ and every mapping $\sigma_{i}^{\prime}: D_{i} \rightarrow A_{i}$ it is the case that

$$
\sum_{d \in D} \pi(d) u_{i}\left(\sigma_{i}\left(d_{i}\right), \sigma_{-i}\left(d_{-i}\right)\right) \geq \sum_{d \in D} \pi(d) u_{i}\left(\sigma_{i}^{\prime}\left(d_{i}\right), \sigma_{-i}\left(d_{-i}\right)\right)
$$

## Construct a Mathematical Program for Other Equilibria

## Stackelberg Equilibrium (2 players)

Given a 2-player game $G=(N, A, u)$, a Stackelberg Equilibrium is a tuple $\left(s_{1}, s_{2}\right)$ such that

$$
\left(s_{1}, s_{2}\right)=\underset{s_{1}^{\prime} \in S_{1}, s_{2}^{\prime} \in B R_{2}\left(s_{1}^{\prime}\right)}{\arg \max } u_{1}\left(s_{1}^{\prime}, s_{2}^{\prime}\right)
$$

where $B R_{2}\left(s_{1}\right)$ is a set of pure strategies that are the best response of player 2 to strategy $s_{1}$.

And now ...

## Strategies and Induced Normal Form in Perfect Information EFGs



Find a Subgame Perfect Equilibrium
Transform this game into the normal form representation
Find all pure Nash Equilibria

## Strategies and Induced Normal Form in Games with Chance



Find a Subgame Perfect Equilibrium
Transform this game into the normal form representation Find all pure Nash Equilibria

## Strategies and Induced Normal Form in Imperfect Information EFGs



Transform this game into the normal form representation
Find all pure Nash Equilibria

## Construct a Game

## Kuhn Poker

Kuhn Poker is a zero-sum two player card game, a simplified variant of poker. The deck includes only three playing cards, a King, Queen, and Jack. One card is randomly dealt to each player and the betting round begins. The first player decides whether he is going to bet or pass the turn to the second player. If any player chooses to bet the opposing player must call the bet in order to stay in the round, or fold and lose the game. Players cannot bet more than once. After the betting round ends, the player with the highest card wins the pot.

Write down a game tree visualization of this game.

Write down a normal form representation of this game.

