Solving Normal-Form Games

Branislav Bošanský

Czech Technical University in Prague

branislav.bosansky@agents.fel.cvut.cz

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Previously ... on multi-agent systems.

Examples

Give an example of a general-sum game, with two pure NE such that when players play strategies from different equilibria, the expected outcome is worse for both players.

Can this situation happen in a zero-sum game?

Iterated Elimination of Dominated Strategies

Determine whether the process of iterated elimination of **strictly dominated** strategies yields a single possible outcome. If so, verify that this is the only NE of the game.

	X	**Y**	**Z**
A	$(1,0)$	$(3,0)$	$(2,1)$
B	$(3,1)$	$(0,1)$	$(1,2)$
C	$(2,1)$	$(1,6)$	$(0,2)$

	X	**Y**	**Z**
A	$(1,3)$	$(6,1)$	$(4,2)$
B	$(0,4)$	$(2,9)$	$(-1,3)$
C	$(2,-1)$	$(5,2)$	$(6,3)$

How would you prove that the elimination of strictly dominated strategies preserve Nash Equilibria?

And now ...

Computing Nash Equilibria in Mixed Strategies

	L	**R**
U	$(3, -3)$	$(-1, 1)$
D	$(-2, 2)$	$(1, -1)$

This game has no NE in pure strategies.

How can we find a NE in mixed strategies? Recall the definition … each player plays the best response to the strategy of the opponent.

Let s_L, s_R be the probabilities from a mixed strategy of the column player. What is the expected utility of the row player for playing action U and D respectively?

Computing Nash Equilibria in Mixed Strategies

	L	**R**
U	$(3,-3)$	$(-1,1)$
D	$(-2,2)$	$(1,-1)$

$\mathbb{E}[U] = 3s_L - s_R$, $\mathbb{E}[D] = -2s_L + s_R$

Now, the row player plays the best response. What is the expected utility? $\max(\mathbb{E}[U], \mathbb{E}[D]) = V^*$
What is to goal of the column player? $\min V^*$
Now we have everything for a linear program ...

Computing Nash Equilibria in Mixed Strategies

	L	**R**
U	$(3, -3)$	$(-1, 1)$
D	$(-2, 2)$	$(1, -1)$

$$\min_{s_L, s_R, V^*} V^* \qquad \text{s.t.}$$
$$s_L + s_R = 1, \quad 0 \leq s_L, s_R \leq 1$$
$$3s_L - s_R \leq V^*$$
$$-2s_L + s_R \leq V^*$$

Construct a Mathematical Program for the General-Sum Case

Key observations:

- you will need a multiplication (the problem cannot be formulated as an LP, it is PPAD-complete (or NP-complete for finding a specific NE))
- maximin and minmax do not equal any more – we need to consider utilities of each player separately
- if any strategy is played with a non-zero probability, it is a best response (yields maximal utility against the strategy of the opponent)
- rewrite the inequality constraints using slack variables $a \geq b \rightsquigarrow a - t = b, t \geq 0$

Construct a Mathematical Program for Other Equilibria

Correlated Equilibrium (2 players)

Given a 2-player game $G = (N, A, u)$, a *Correlated Equilibrium* is a tuple (v, π, σ), where v is a couple of random variables $v = (v_1, v_2)$ with respective domains $D = (D_1, D_2)$, π is a joint distribution over v, $\sigma = (\sigma_1, \sigma_2)$ is a vector of mappings $\sigma_i : D_i \to A_i$, and for each agent i and every mapping $\sigma'_i : D_i \to A_i$ it is the case that

$$\sum_{d \in D} \pi(d) u_i(\sigma_i(d_i), \sigma_{-i}(d_{-i})) \geq \sum_{d \in D} \pi(d) u_i(\sigma'_i(d_i), \sigma_{-i}(d_{-i}))$$

Construct a Mathematical Program for Other Equilibria

Stackelberg Equilibrium (2 players)

Given a 2-player game $G = (N, A, u)$, a *Stackelberg Equilibrium* is a tuple (s_1, s_2) such that

$$(s_1, s_2) = \underset{s'_1 \in S_1, s'_2 \in BR_2(s'_1)}{\arg\max} u_1(s'_1, s'_2)$$

where $BR_2(s_1)$ is a set of pure strategies that are the best response of player 2 to strategy s_1.