OTEVŘENÁ INFORMATIKA

Extensive-Form Games

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AE4M36MAS, Fall 2015

Finite Sequential Games

Different representations

- Normal-form Games
  - visually represented as matrix games

- Extensive-form Games
  - **finite sequential games that evolve in time**
  - the concept of time is implicitly integrated into the model
  - visually represented as finite trees
  - general enough to represent different types of uncertainty

Finite Sequential Games

Published Online July 19 2007
Science 14 September 2007:
Vol. 317 no. 5844 pp. 1518-1522
DOI: 10.1126/science.1144079

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RESEARCH ARTICLE

Checkers Is Solved

Jonathan Schaeffer[*], Neil Burch, Yngvi Björnsson[†], Akihiro Kishimoto[‡], Martin Müller, Robert Lake, Paul Lu, Steve Sutphen

RESEARCH

RESEARCH ARTICLE

COMPUTER SCIENCE

Heads-up limit hold'em poker is solved

Michael Bowling,[1*] Neil Burch,[1] Michael Johanson,[1] Oskari Tammelin[2]

Finite Sequential Games

Extensive-Form Games

Perfect-Information Games

Perfect-Information Games with Chance

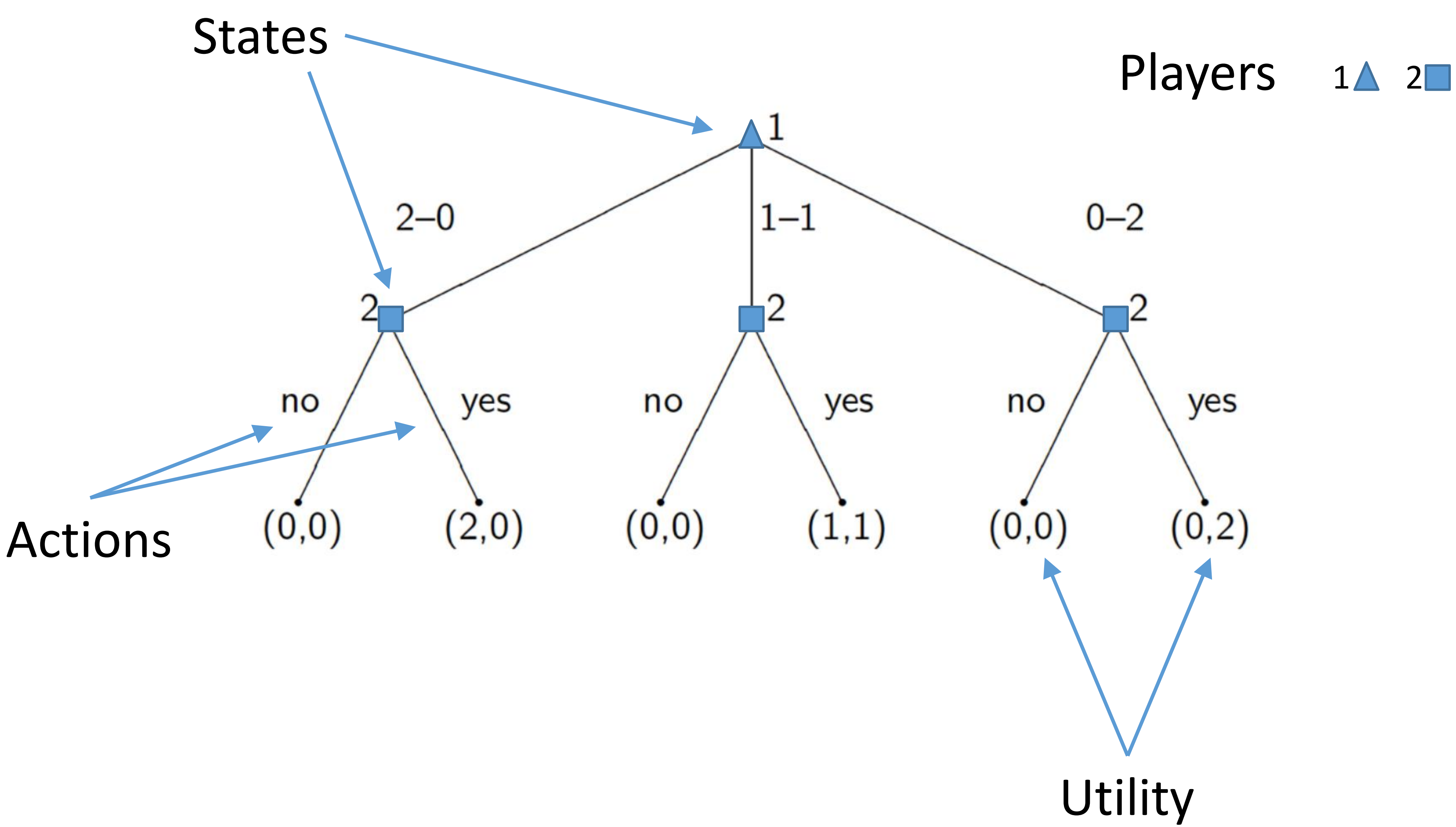
Imperfect-Information Games

Solving Zero-Sum Games

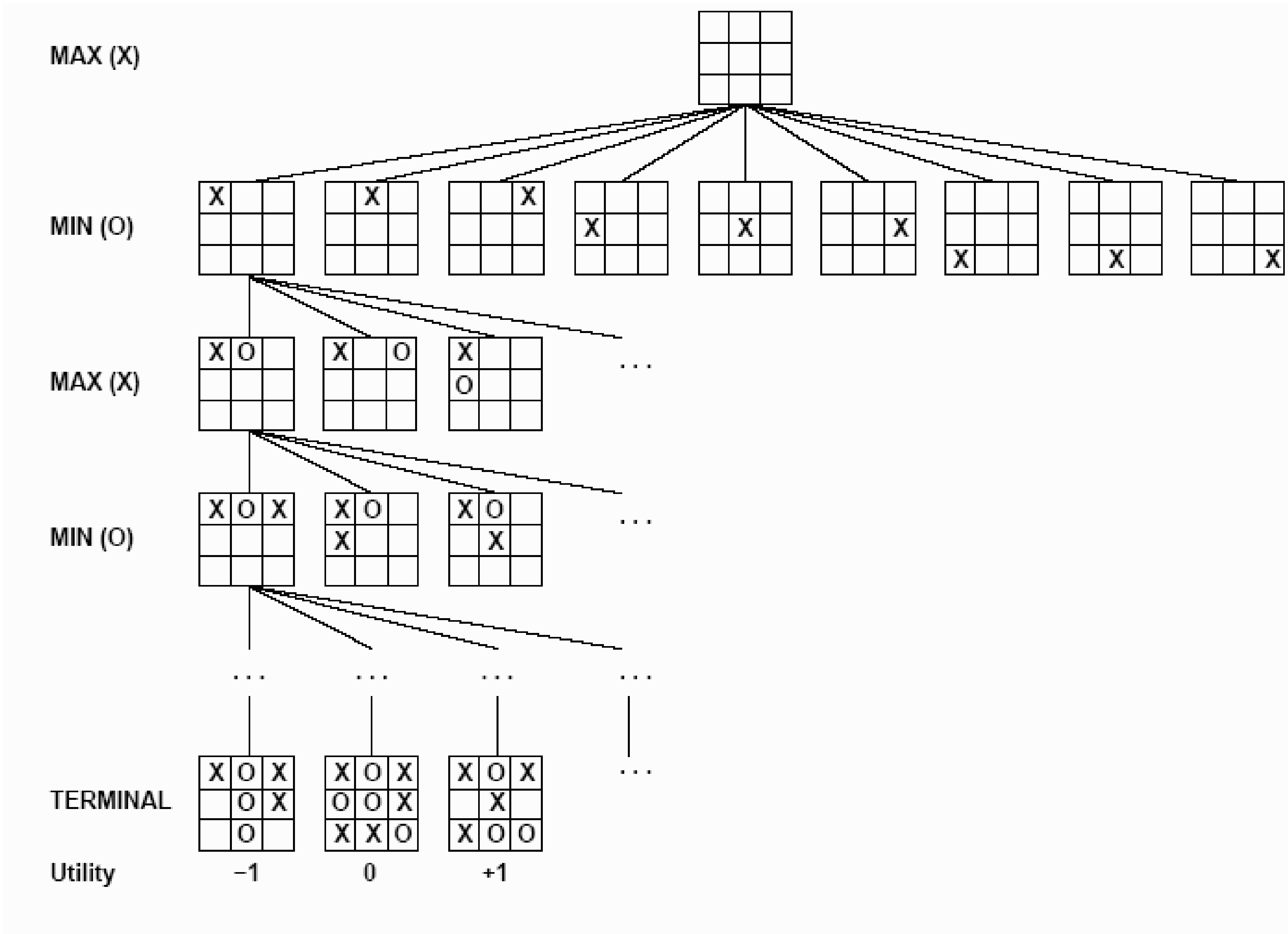
Solving General-Sum Games

Approximate Solutions to Large Zero-Sum Games

Game Theory: Extensive-Form Games



Game Theory: Extensive-Form Games



Game Theory: Extensive-Form Games

Definition:

A (finite) perfect-information game in the extensive form is defined as a tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

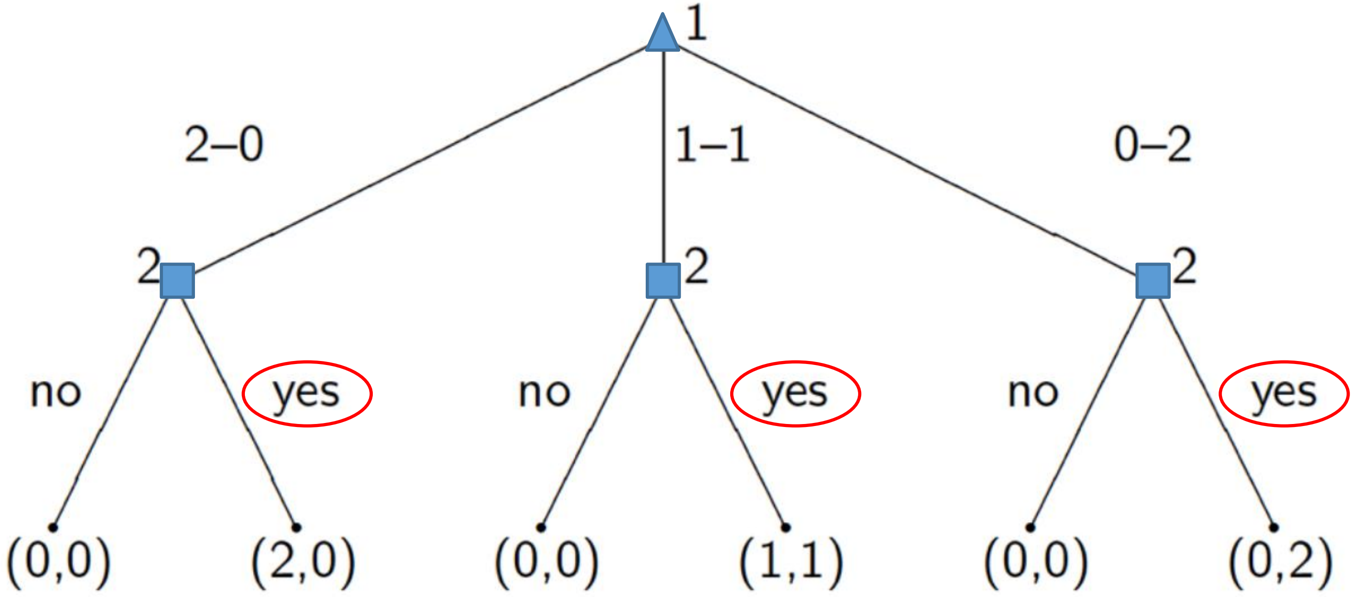
- Players $N = \{1, 2, \dots\}$
- Actions A
- Choice nodes and label for these nodes
  - Choice nodes: H
  - Action function: $\chi : H \rightarrow 2^A$
  - Player function: $\rho : H \rightarrow N$
- Terminal nodes Z
- Successor function $\sigma : H \times A \rightarrow H \cup Z$
- Utility function $u = (u_1, \dots, u_n)$; $u_i : Z \rightarrow \mathbb{R}$

EFGs: Actions and Strategies

Pure strategy: an assignment of an action for each state

$$S_i = \prod_{h \in H, \rho(h)=i} \chi(h)$$

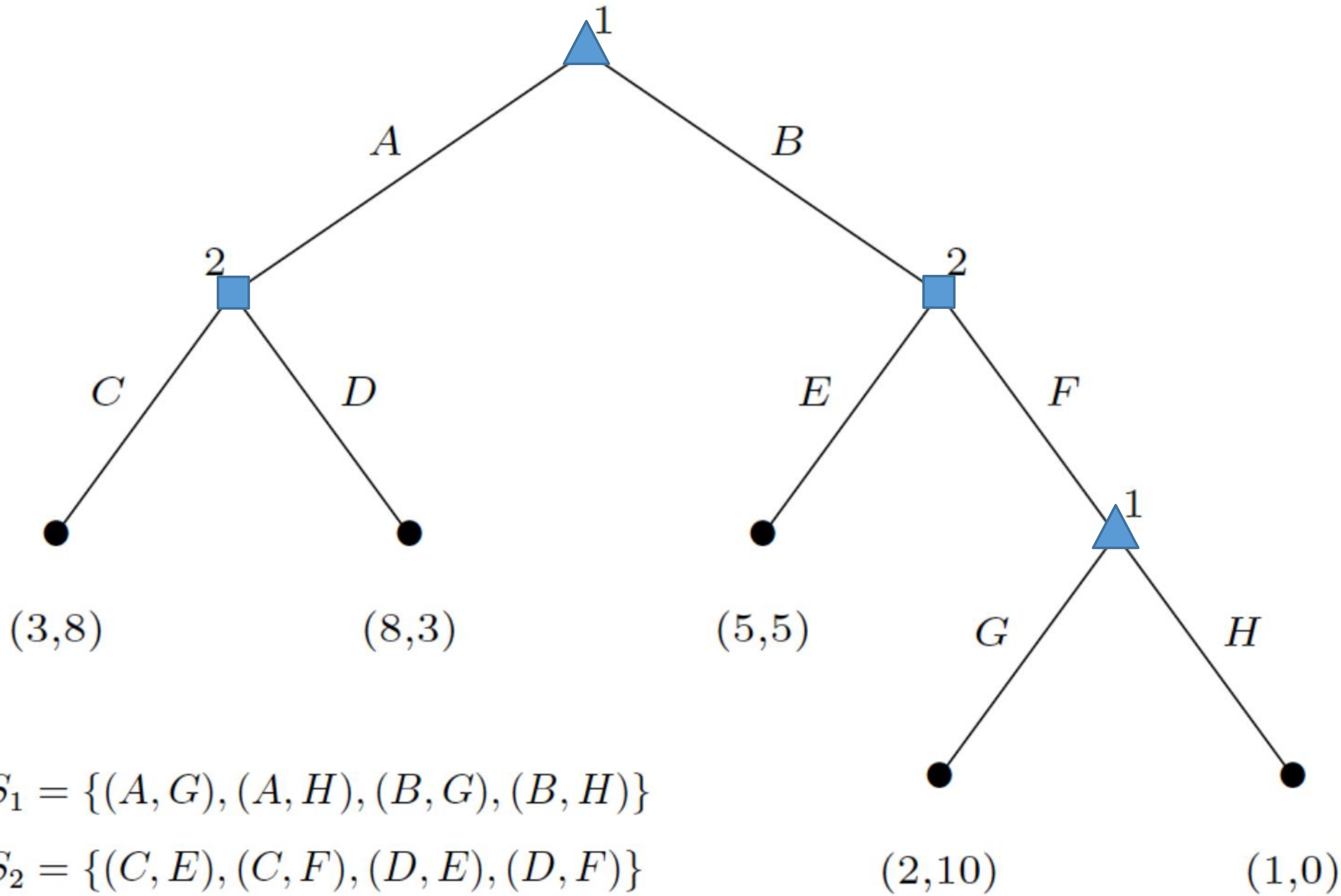
Action is uniquely identified by the state, in which it is taken.



$A_1 = \{2-0, 1-1, 0-2\}$ $A_2 = \{no_{\{2-0\}}, yes_{\{2-0\}}, no_{\{1-1\}}, yes_{\{1-1\}}, no_{\{0-2\}}, yes_{\{0-2\}}\}$

$S_1 = \{2-0, 1-1, 0-2\}$ $S_2 = \{\{no_{\{2-0\}}, no_{\{1-1\}}, no_{\{0-2\}}\}, \{no_{\{2-0\}}, no_{\{1-1\}}, yes_{\{0-2\}}\}, \dots\}$

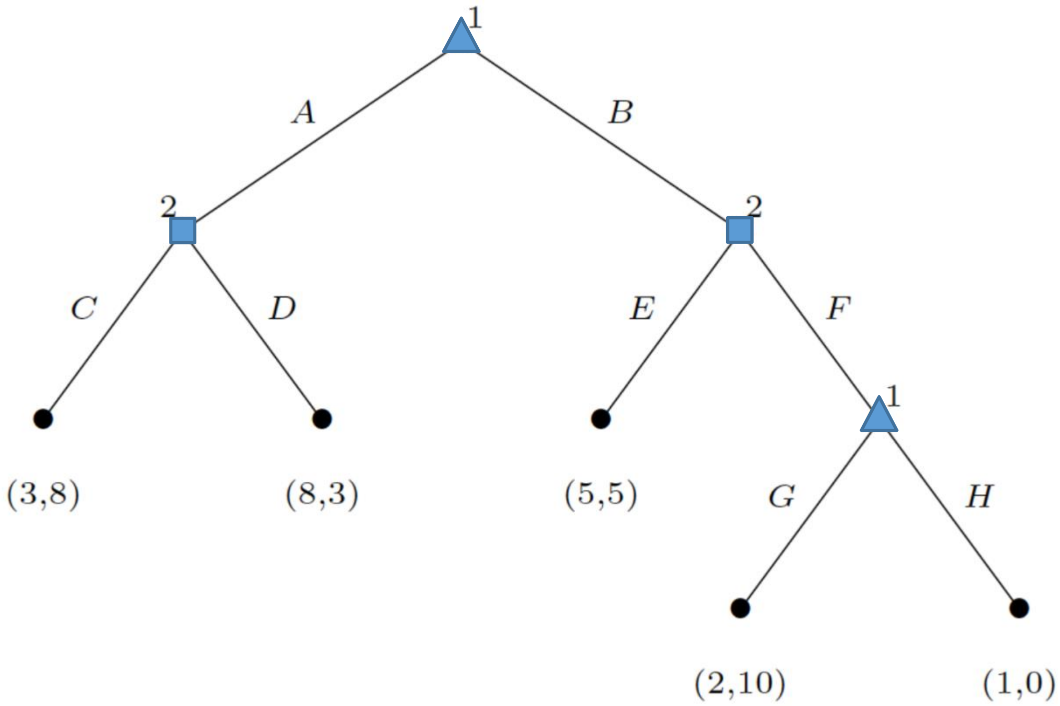
Pure Strategies in EFGs



$S_1 = \{(A,G),(A,H),(B,G),(B,H)\}$

$S_2 = \{(C,E),(C,F),(D,E),(D,F)\}$

Induced Normal Form

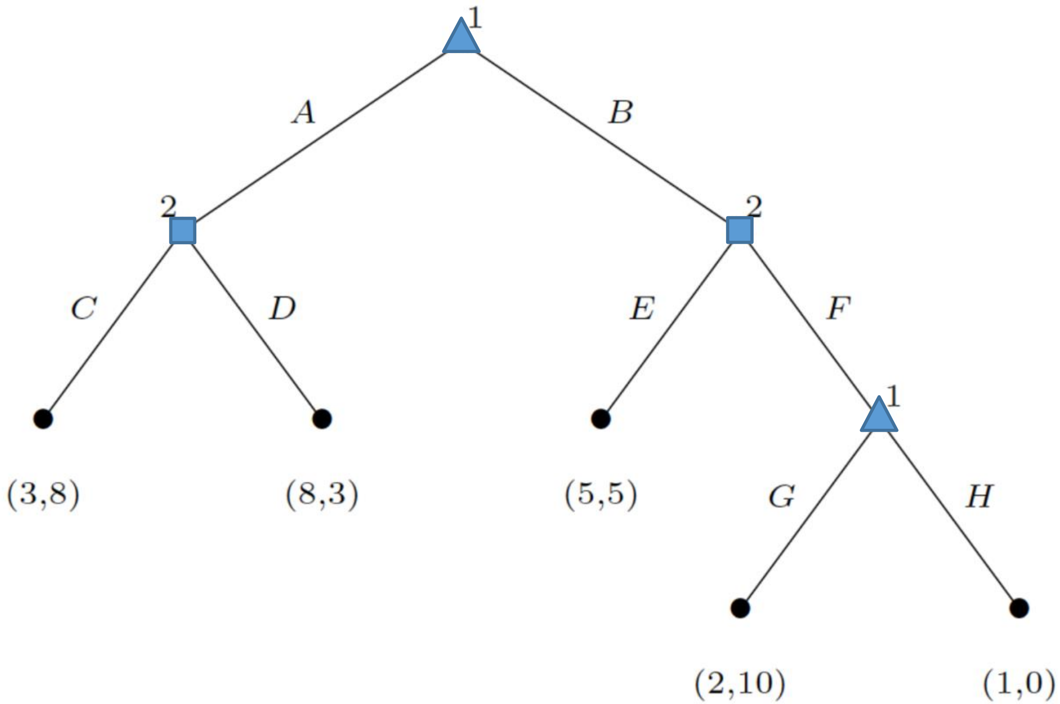


$S_1 = \{(A, G), (A, H), (B, G), (B, H)\}$

$S_2 = \{(C, E), (C, F), (D, E), (D, F)\}$

	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3, 8	3, 8	8, 3	8, 3
(A,H)	3, 8	3, 8	8, 3	8, 3
(B,G)	5, 5	2, 10	5, 5	2, 10
(B,H)	5, 5	1, 0	5, 5	1, 0

Nash Equilibria in EFGs



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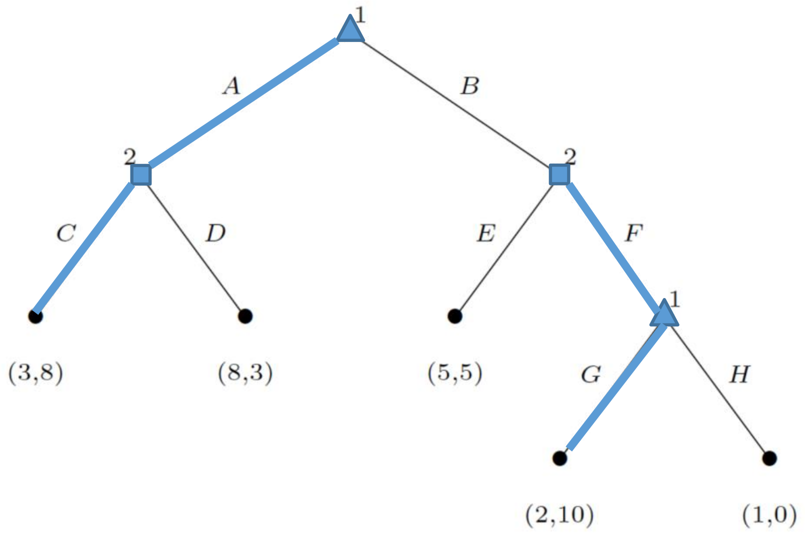
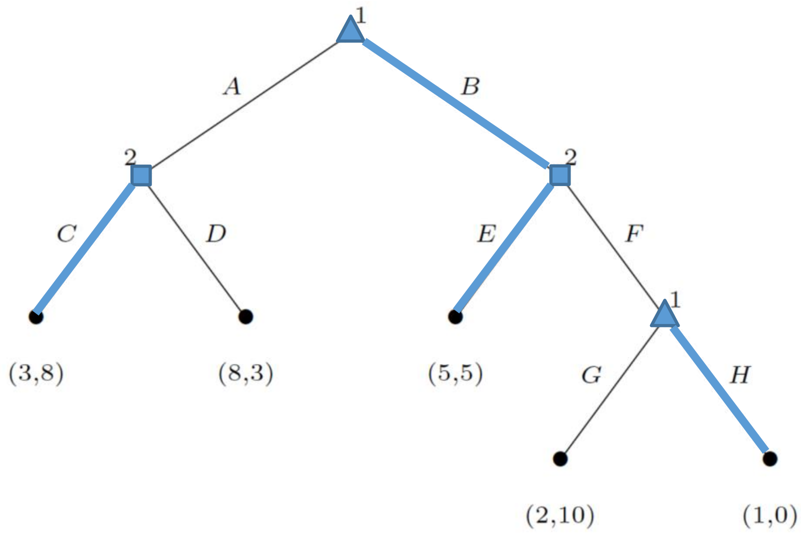
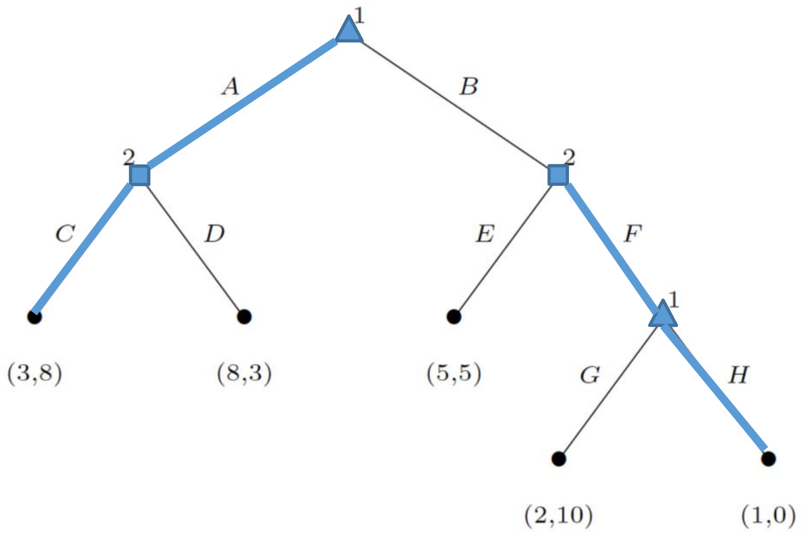
Nash Equilibria in EFGs







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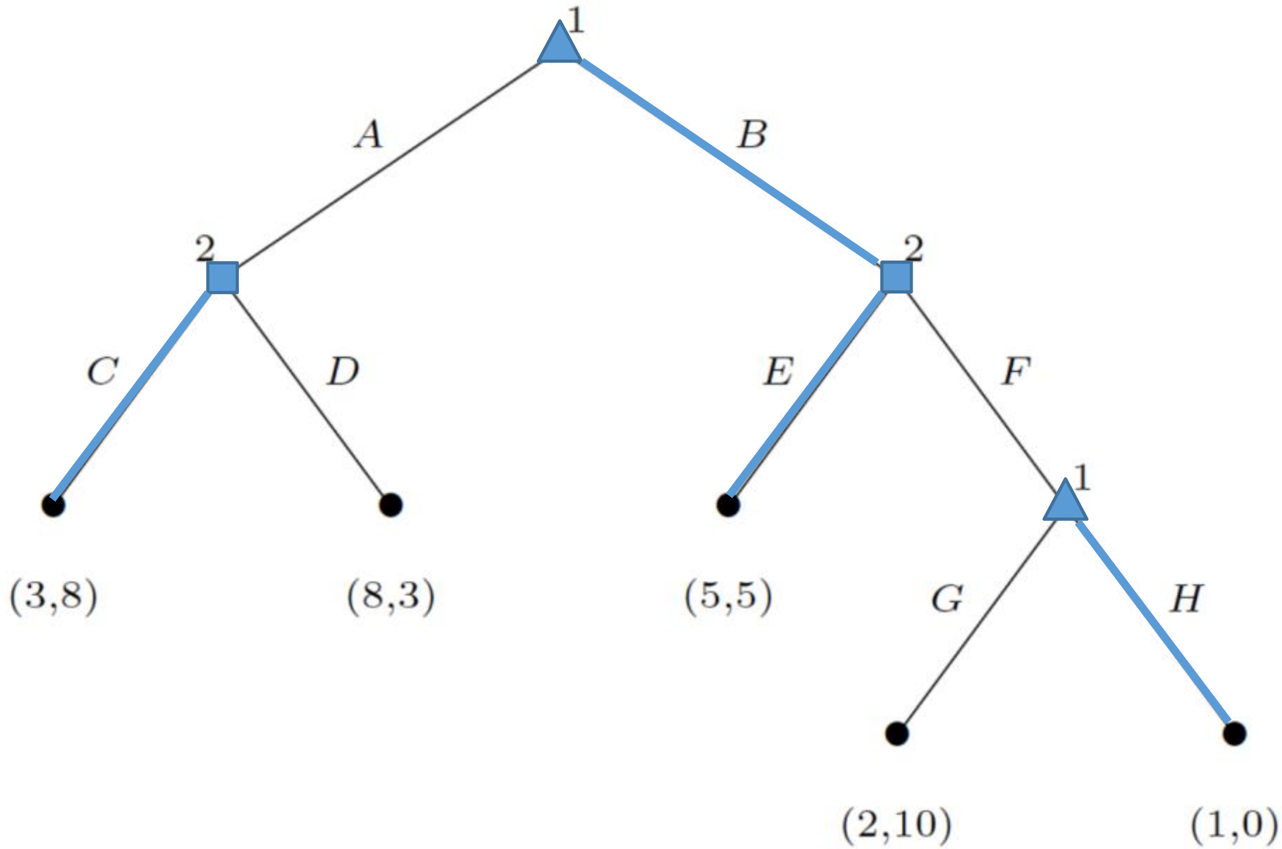
Nash Equilibria in EFGs - threats



	(C,E)	(C,F)	(D,E)	(D,F)
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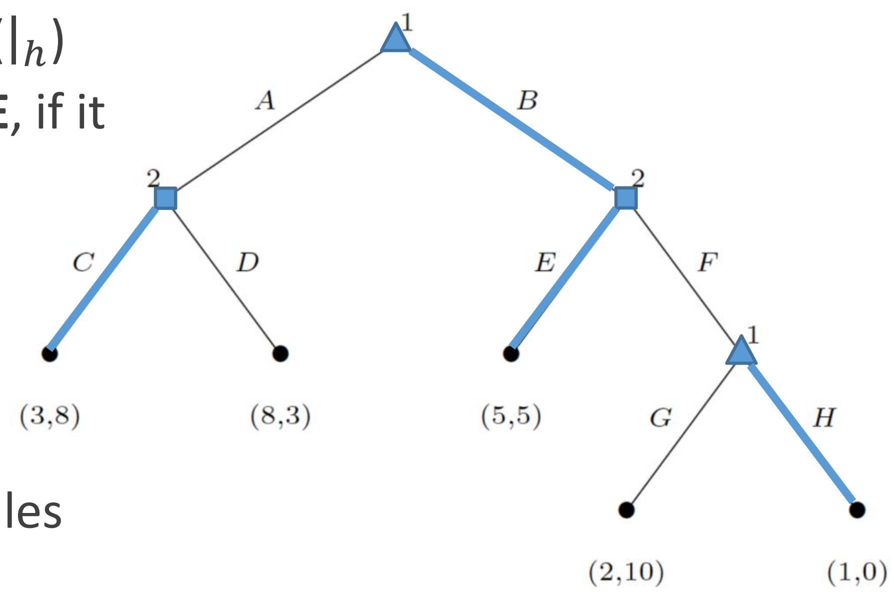
Solution Concepts in EFGs

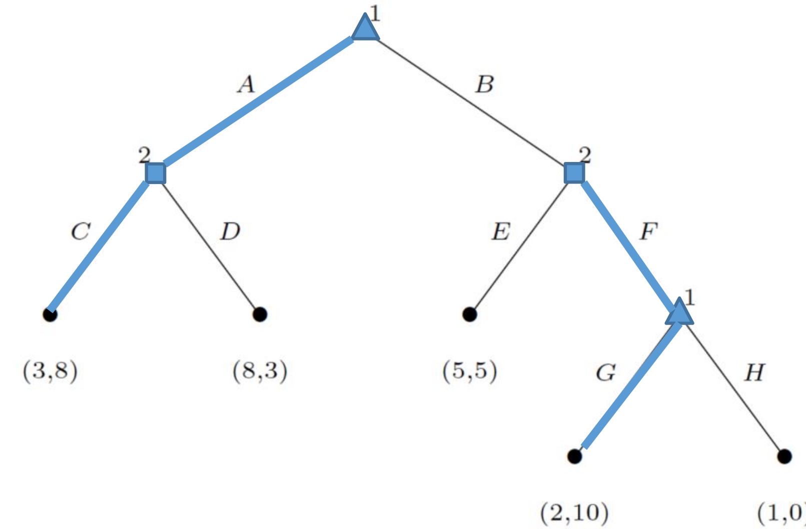
- Sub-game perfect (SPE)
  - sub-game of G rooted at some node h is the restriction of G to the descendants of h ($|_h$)
  - Strategy profile is a **sub-game perfect NE**, if it is a NE for every sub-game of G
  - Every SPE is NE



- Refinements of NE
  - further assumptions on NE strategy profiles
  - Sub-game perfect equilibria
  - Sequential equilibria
  - Quasi-perfect equilibria
    - E. van Damme. Stability and Perfection of Nash Equilibria (1991)

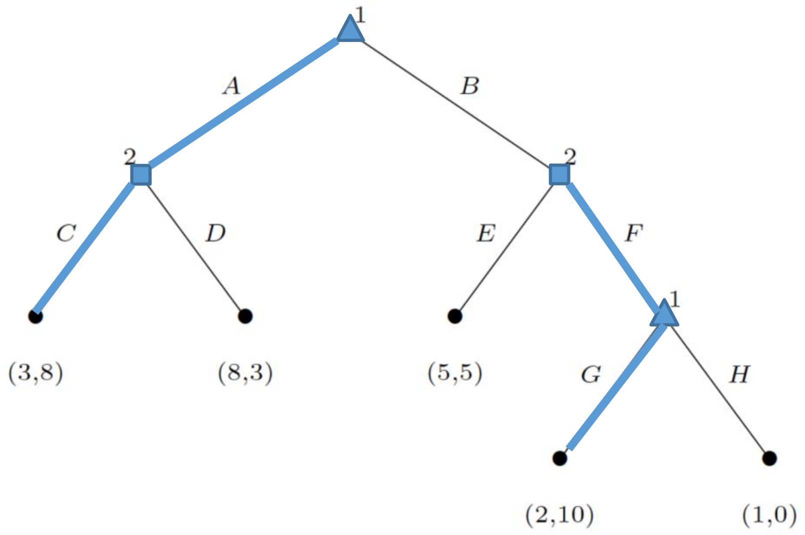
Computing SPE: Backward Induction

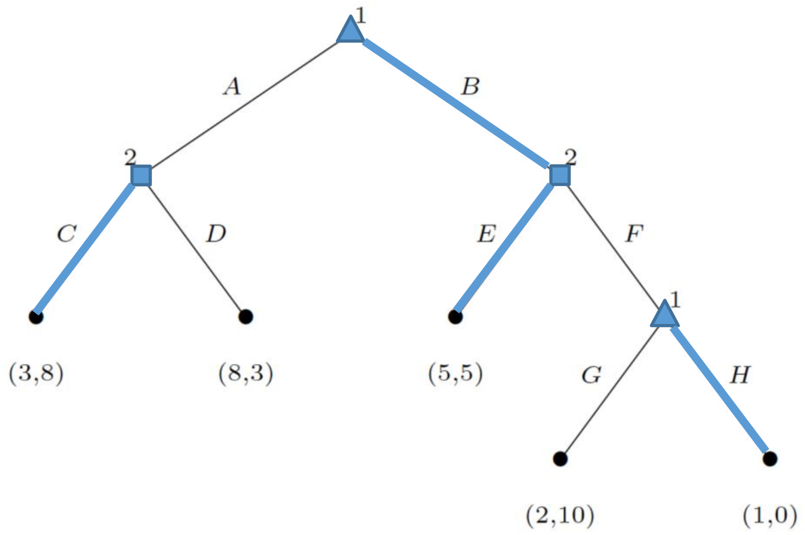
function BackwardInduction (node h) **returns** $u(h)$
if $h \in Z$ **then**
  return $u(h)$
$best\_util \leftarrow -\infty$
forall $a \in \chi(h)$ **do**
  $util\_at\_child \leftarrow$ BackwardInduction$(\sigma(h, a))$
  if $util\_at\_child_{\rho(h)} > best\_util_{\rho(h)}$ **then**
    $best\_util \leftarrow util\_at\_child$
return $best\_util$

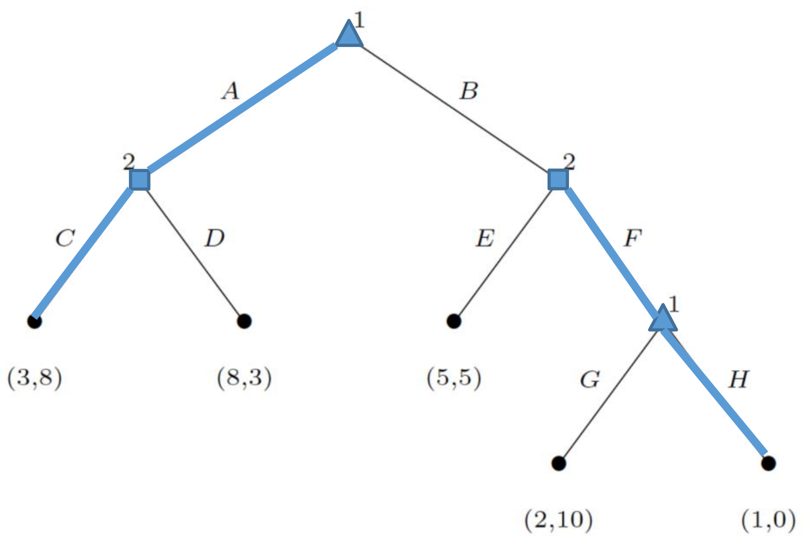


- works for general-sum n-players games with perfect information
- computes pure strategy subgame-perfect equilibrium
- every perfect-information EFG has a pure (subgame-perfect) Nash equilibrium

Nash Equilibrium in EFGs – SPE?







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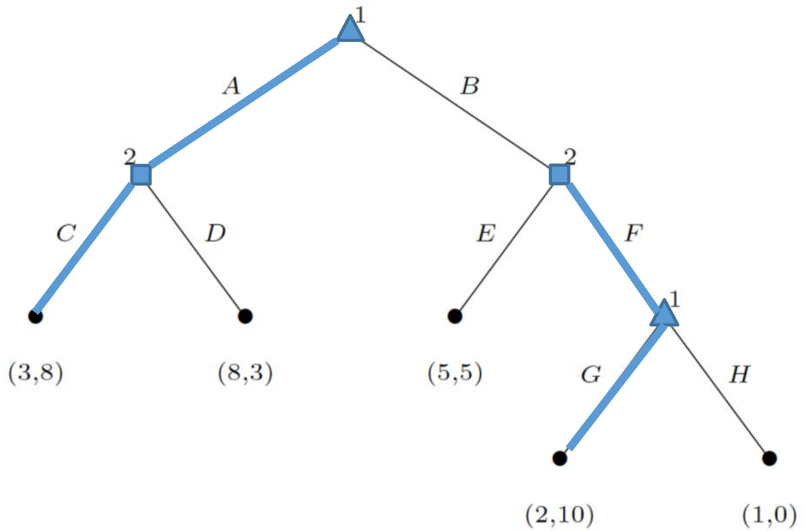
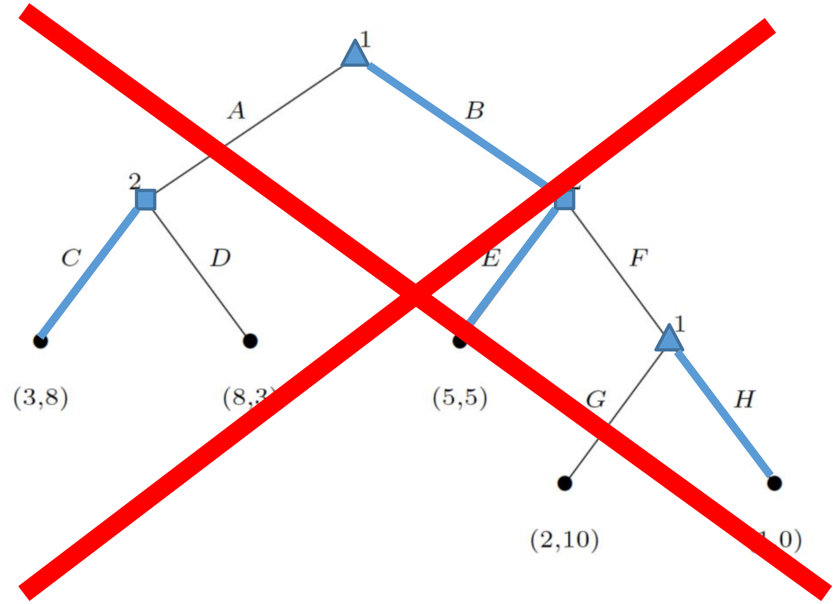
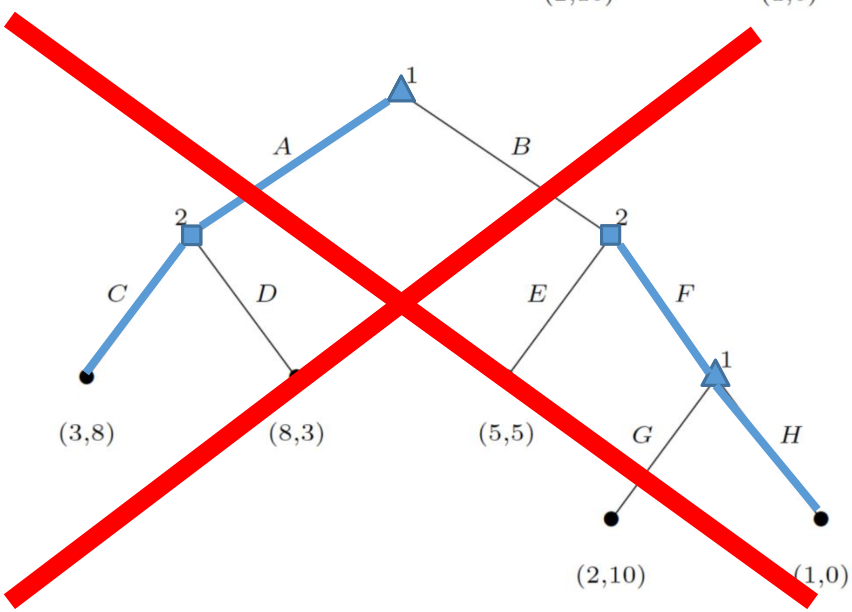
Nash Equilibrium in EFGs – SPE?







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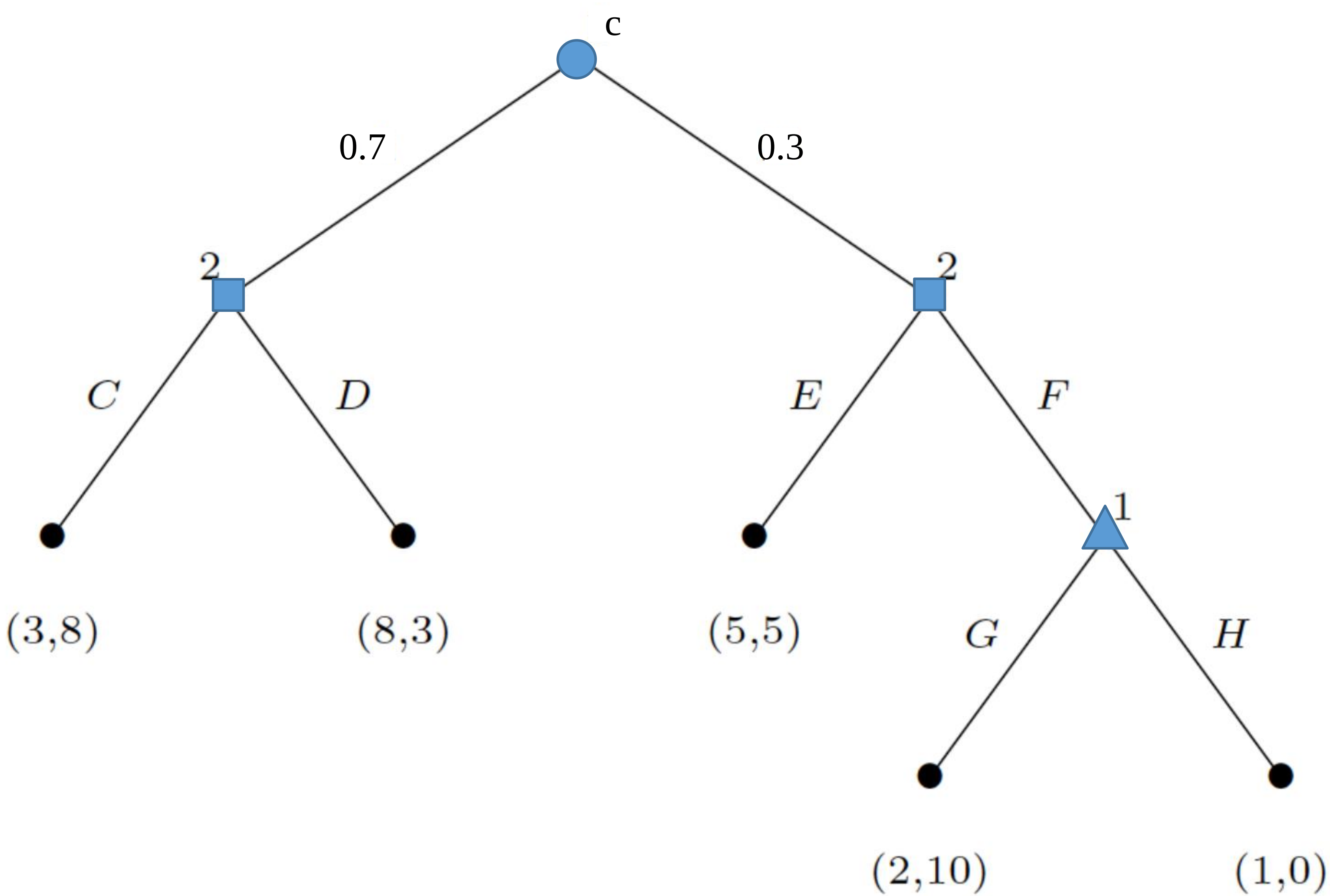
Extensive-Form Games

- Perfect-Information Games
- Perfect-Information Games with Chance
- Imperfect-Information Games
- Solving Zero-Sum Games
- Solving General-Sum Games
- Approximate Solutions to Large Zero-Sum Games

EFGs with Chance Nodes



Extensive-Form Games with Chance

Definition:

A (finite) perfect-information game in the extensive form is defined as a tuple $(N, A, H, Z, \chi, \rho, \gamma, \sigma, u)$, where:

- Players $N = \{1, 2, \ldots\} \cup \{c\}$
- Actions A
- Choice nodes and label for these nodes
  - Choice nodes: H
  - Action function: $\chi : H \to 2^A$
  - Player function: $\rho : H \to N$
  - Stochastic transitions: $\gamma : \{\chi(h) \mid h \in H, \rho(h) = c\} \to [0,1]$,
    $\sum_{a \in \chi(h)} \gamma(a) = 1 \quad (\forall h \in H, \rho(h) = c)$
- Terminal nodes Z
- Successor function $\sigma : H \times A \to H \cup Z$
- Utility function $u = (u_1, \ldots, u_n)$; $u_i : Z \to \mathbb{R}$

Extensive-Form Games

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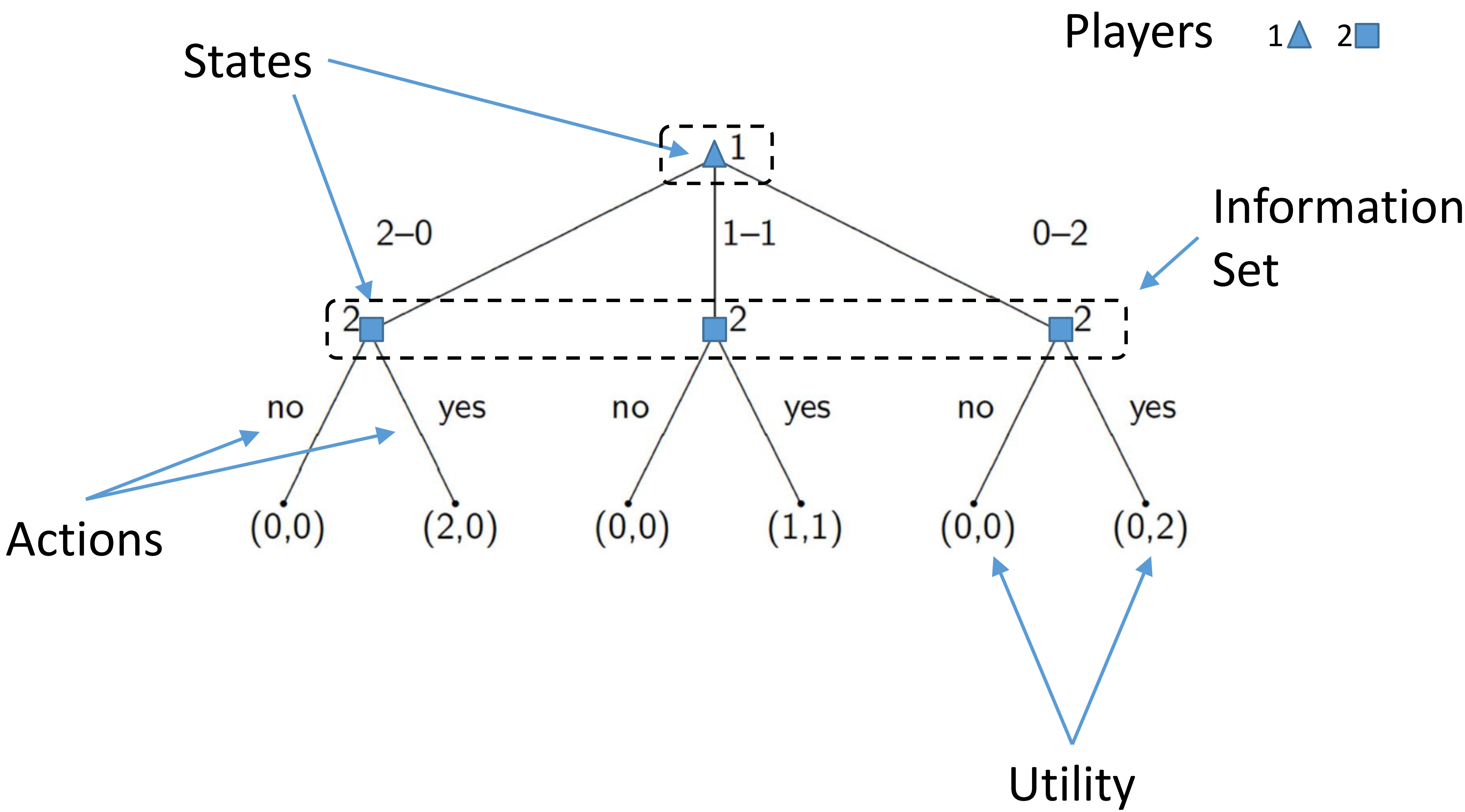
Imperfect Information EFGs



Imperfect Information EFGs

Definition:

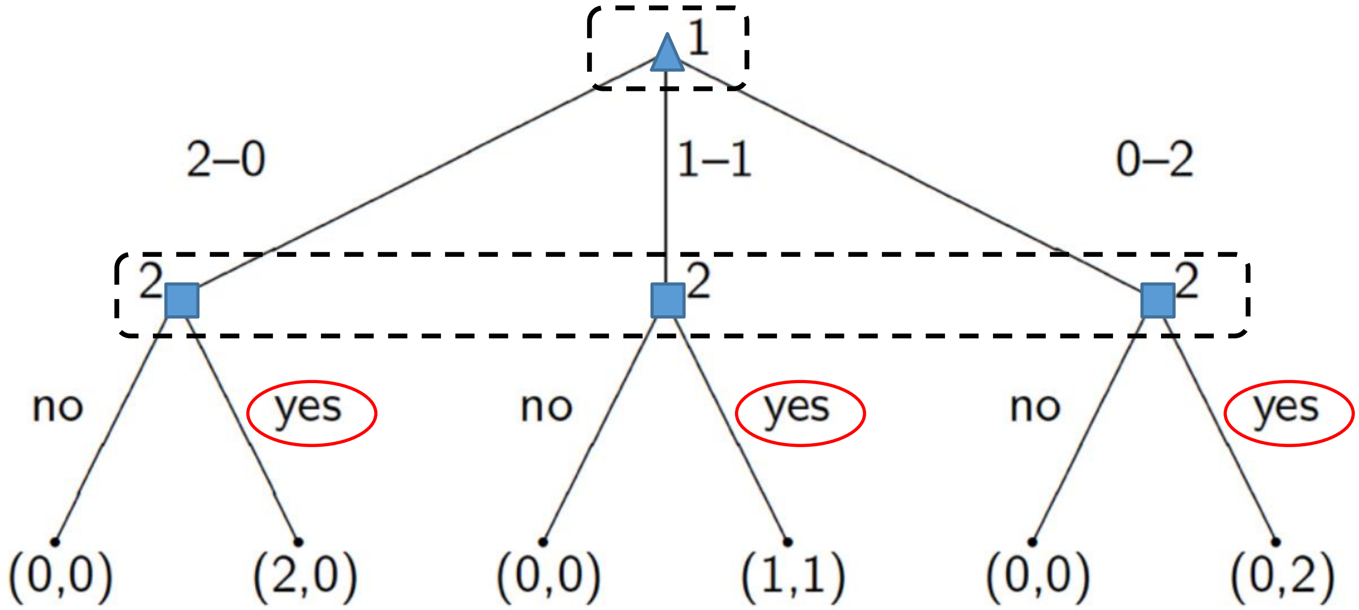
A (finite) imperfect-information game in the extensive form is defined as a tuple $(N, A, H, Z, \chi, \rho, \sigma, u, I)$, where:

- tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$ is a perfect-information extensive-form game
- $I = (I_1, \dots, I_n)$, where $I_i = (I_{i,1}, \dots, I_{i,k})$ is a set of equivalence classes on (i.e., a partition of) choice nodes of a player i with the property that $\chi(h) = \chi(h')$ and $\rho(h) = \rho(h')$, whenever $h \in I_{i,j}$ and $h' \in I_{i,j}$ for some j
- we use $\chi(I_{i,j})$ instead of $\chi(h)$ for some $h \in I_{i,j}$

II EFGs: Actions and Strategies

Pure strategy: an assignment of an action for each **information set**

Action is uniquely identified by the **information set**, in which it is taken.



$A_1 = \{2-0, 1-1, 0-2\}$ $A_2 = \{no, yes\}$

$S_1 = \{2-0, 1-1, 0-2\}$ $S_2 = \{no, yes\}$

Strategies in EFGs

Existence of a pure NE is no longer guaranteed for imperfect-information EFGs

Mixed strategies

- Probabilistic distribution over pure strategies

Behavioral Strategies

- Probabilistic distribution over actions to play for each information set

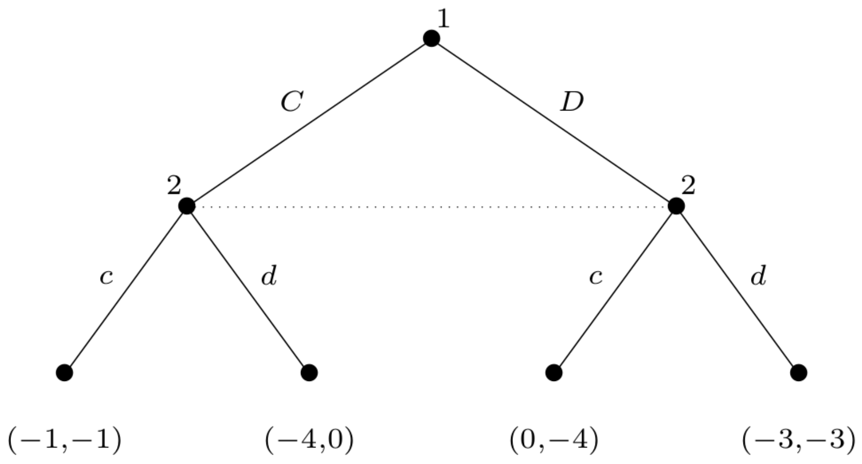
There is a broad class of imperfect-information games in which the expressive power of mixed and behavioral strategies coincides. This is the class of games of **perfect recall**. Intuitively speaking, in these games no player forgets any information she previously knew.

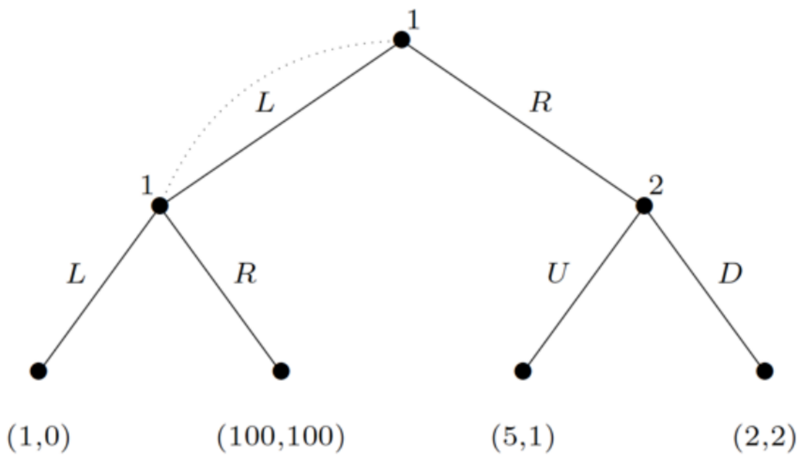
Perfect Recall in EFGs

Player i has **perfect recall** in an imperfect-information game G if for any two nodes h, h' that are in the same information set for player i,for any path $h_0, a_0, h_1, a_1, h_2, \ldots, h_n, a_n, h$ from the root of the game tree to h (where the h_j are decision nodes and the a_j are actions) and for any path $h_0, a'_0, h'_1, a'_1, h'_2, \ldots, h'_m, a'_m, h'$ from the root to h' it must be the case that:

1. $n = m$
2. for all $0 \le j \le n$, h_j and h'_j are in the same equivalence class for player i
3. for all $0 \le j \le n$, if $\rho(h_j) = i$ (i.e., h_j is a decision node of player i), then $a_j = a'_j$.

Perfect vs. Imperfect Recall





Remembering all information induces very large strategies

Easier to solve

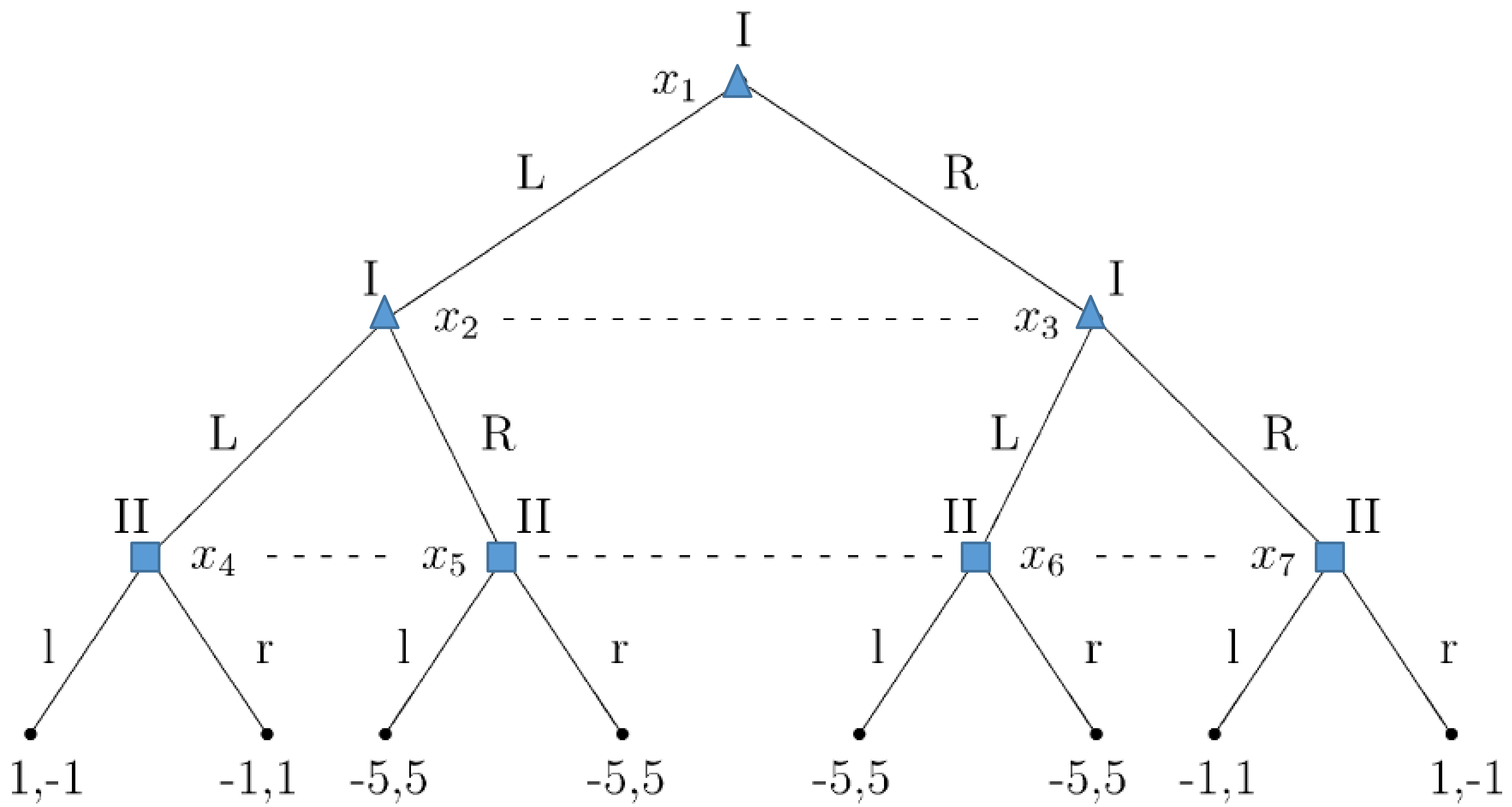
Strategies can be compactly represented

Smaller trees, unnecessary information can be forgotten

Much harder to solve

Equilibrium in behavior strategies might not exist

NE in Imperfect Recall Games



Wichardt. Existence of Nash equilibria in finite extensive form games with imperfect recall: A counterexample (2008)