DISTRIBUTED CONSTRAINTS SATISFACTION

AE4M36MAS - Multiagent systems

CENTRALIZED CASE

Constraint satisfaction problem

Find an assignment for variables that satisfy given constraints.

- $\mathcal{X} = \{x_1, \ldots, x_n\}$ — set of *variables* to assign
- $\mathcal{D} = \{D_1, \ldots, D_n\}$ — set of *domains* ($x_i \in D_i$)
- $\mathcal{C} = \{C_1, \ldots, C_m\}$ — set of *constraints*

$C_i \subseteq D_{i_1} \times \cdots \times D_{i_r}$ denotes a r-ary constraint over variables $x_{i_1}, \ldots, x_{i_r}$

Constraint satisfaction problem

Solution: n-tuple $(d_1, \cdots, d_n)$, such that:

- $d_i \in D_i$, for $1 \leq i \leq n$
- $(d_{i_1}, \ldots, d_{i_r}) \in C_k$ for every constraint $C_k \subseteq D_{i_1} \times \cdots \times D_{i_r}$

Centralized algorithm

Synchronized backtracking

$v_i \leftarrow$ value from D_i consistent with $(v_1, \ldots, v_{i-1})$;
if *No such* v_i *exists* **then**
    backtrack ;
else if $i = n$ **then**
    stop ;
else
    `ChooseValue(`x_{i+1}, $(v_1, \ldots, v_i)$`)` ;
end

Algorithm 1: `ChooseValue(`x_i, $(v_1, \ldots, v_{i-1})$`)`

Centralized algorithm

Enhancements?

- AC-3 algorithm? (arc consistency - e.g. combinatorial optimization)

DISTRIBUTED CASE

Distributed case

- $\mathcal{X} = \{x_1, \ldots, x_n\}$ — set of *variables* to assign
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- $\mathcal{A} = \{A_1, \ldots, A_k\}$ — set of *agents*

Every variable **must be** assigned to one of the agents.

$\rightarrow$ otherwise the DCSP problem is **not fully defined**

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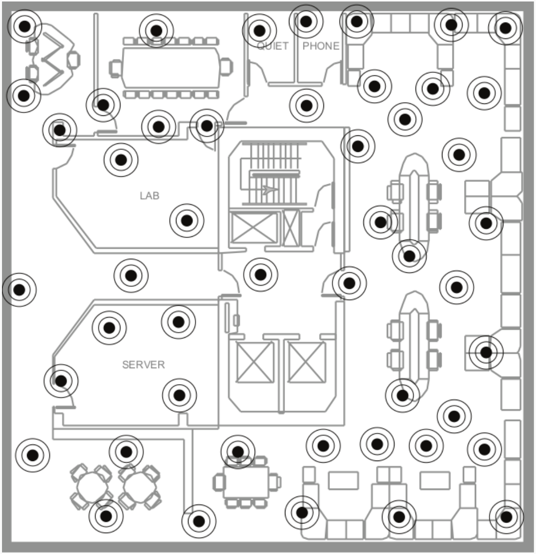
Example



ASYNCHRONOUS BACKTRACKING

What do we assume?

- Every agent controls a single variable
- Constraints are binary

- Messages are delivered in a finite time (but this time may vary randomly)
- Messages are delivered in the order they were sent
  $\rightarrow$ imagine an unreliable TCP/IP network

What does an agent initially knows?

- Total ordering of agents (priorities)
- Constraints he is involved in
- Domain of a variable controlled by himself

Data structures

- Current assignment
- Set of outgoing links ($\sim$ who needs to know my assignment)
- **Agent view** — agent's idea about current assignment of other agents

  $\rightarrow$ May be out of sync!
- **Nogood store** — justification of forbidden values in the domain

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Minimalistic example — meeting scheduling

- John needs to arrange a meeting with Bob and Alice
- As all agents, he is a busy guy — both meetings must happen in a single day
- Bob doesn't know about Alice's meeting and vice versa

Minimalistic example — meeting scheduling

$\mathcal{A} = \{\text{Alice}, \text{Bob}, \text{John}\}$

$\mathcal{X} = \{x_{\text{Alice}}, x_{\text{Bob}}, x_{\text{John}}\}$
Agent i controls variable x_i.

$\mathcal{D} = \{D_{\text{Alice}}, D_{\text{Bob}}, D_{\text{John}}\}$
$D_{\text{Alice}} = \{\text{Mon}, \text{Thu}\}$
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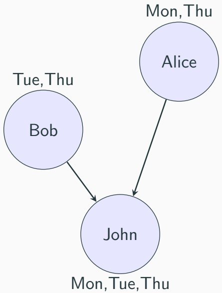


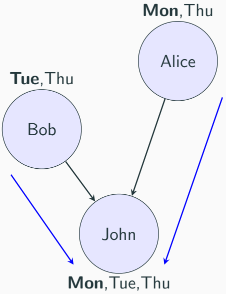
Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\varnothing$ John: $\varnothing$

Let's all propose a date and see what happens!

Bob $\rightarrow$ John:
Ok?(Bob $\rightarrow$ Tue)

Alice $\rightarrow$ John:
Ok?(Alice $\rightarrow$ Mon)



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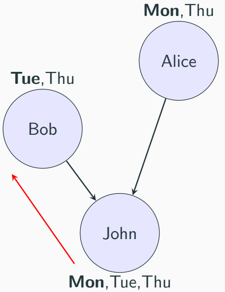
John: Argh, I wanted to have both meetings in one day :-(Let's make them change their minds...

John $\rightarrow$ Bob:
Nogood($\{\text{Bob} \rightarrow \text{Tue}, \text{Alice} \rightarrow \text{Mon}\}$)



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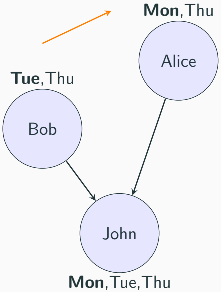
Alice: $\varnothing$ Bob: $\{\text{Alice} \rightarrow \text{Mon}\}$ John: $\{\text{Alice} \rightarrow \text{Mon}\}$

Bob: Who is that Alice? I've never heard of her.

Bob $\rightarrow$ Alice:
AddLink(Alice $\rightarrow$ Bob)



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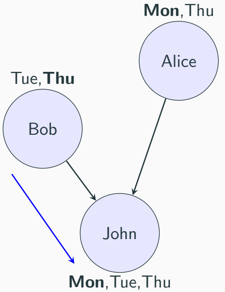
Minimalistic example — meeting scheduling

Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Mon}\}$ John: $\{\text{Alice} \to \text{Mon}\}$

Bob: John told me that the meeting cannot happen on Tuesday if Alice opts for Monday. Let's try Thursday then...

Bob $\to$ John:

Ok?(Bob $\to$ Thu)



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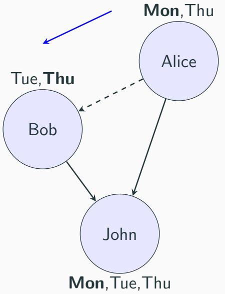
Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Mon}\}$ John: $\{\text{Alice} \to \text{Mon}, \text{Bob} \to \text{Thu}\}$

Alice: Bob, why are you so curious?

Alice $\to$ Bob:

Ok?(Alice $\to$ Mon)



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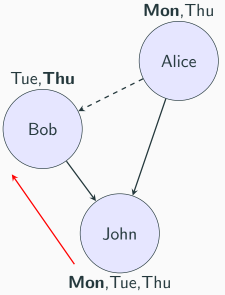
John: They tried it again. Alright, one more try...

John $\to$ Bob:
Nogood($\{\mathsf{Bob} \to \mathsf{Thu}, \mathsf{Alice} \to \mathsf{Mon}\}$)



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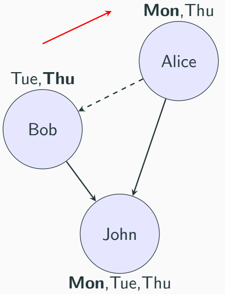
Bob: I have run out of options. It's up to Alice now...

Bob $\to$ Alice:
Nogood($\{\text{Alice} \to \text{Mon}\}$)



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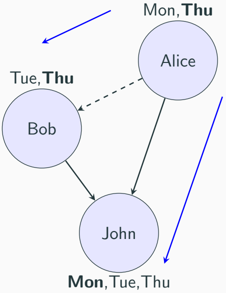
Alice: I have one more option, let's try Thursday.

Alice $\to$ Bob, John:
$\text{Ok?}(\{\text{Alice} \to \text{Thu}\})$



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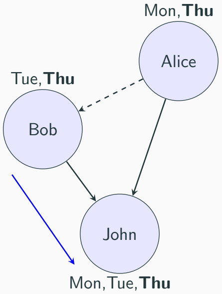
John: Finally. Thursday seems like a viable option.



Alice: $\varnothing$ Bob: $\{\text{Alice} \to \text{Thu}\}$ John: $\{\text{Alice} \to \text{Thu}, \text{Bob} \to \text{Thu}\}$

Asynchronous backtracking

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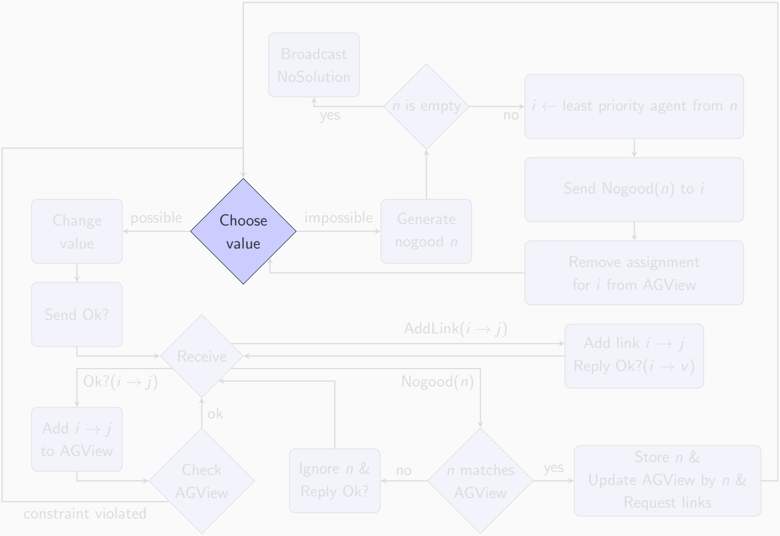

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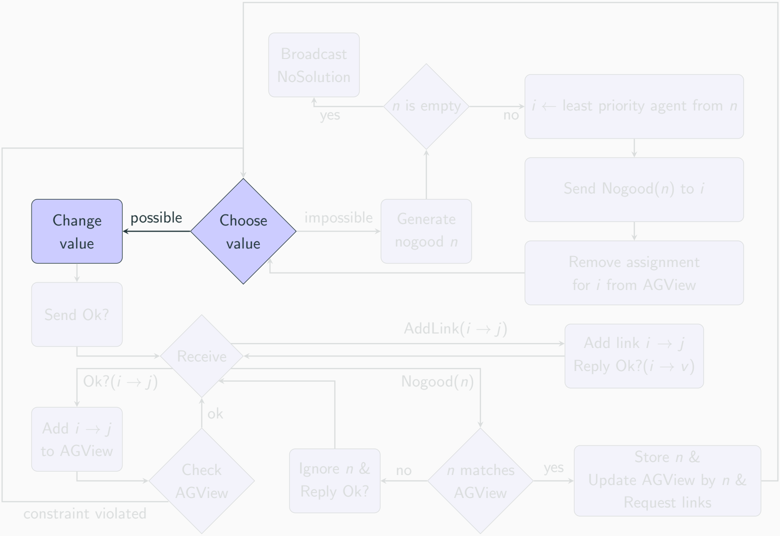

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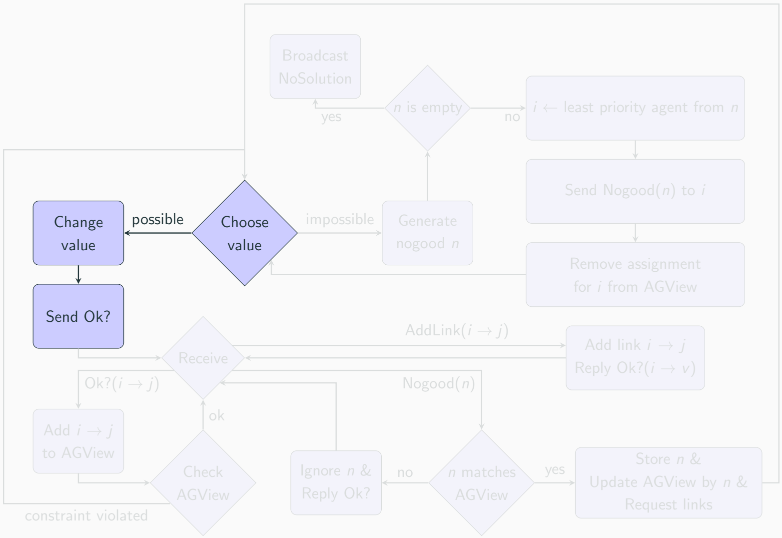

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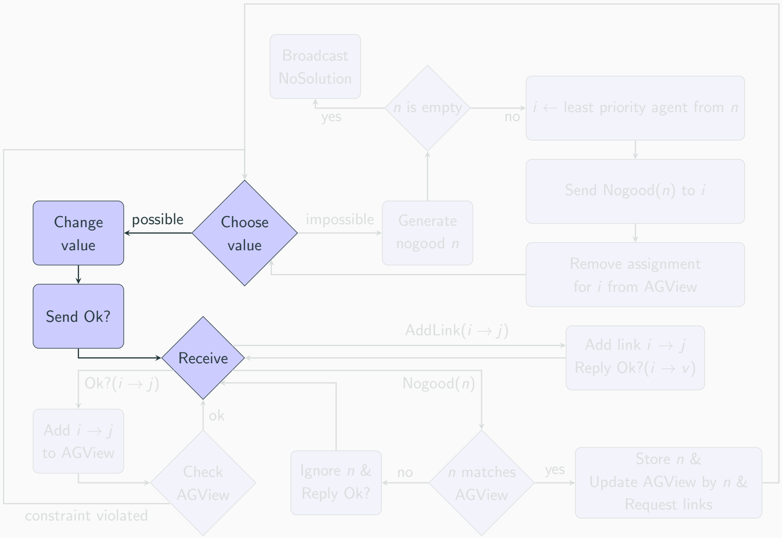

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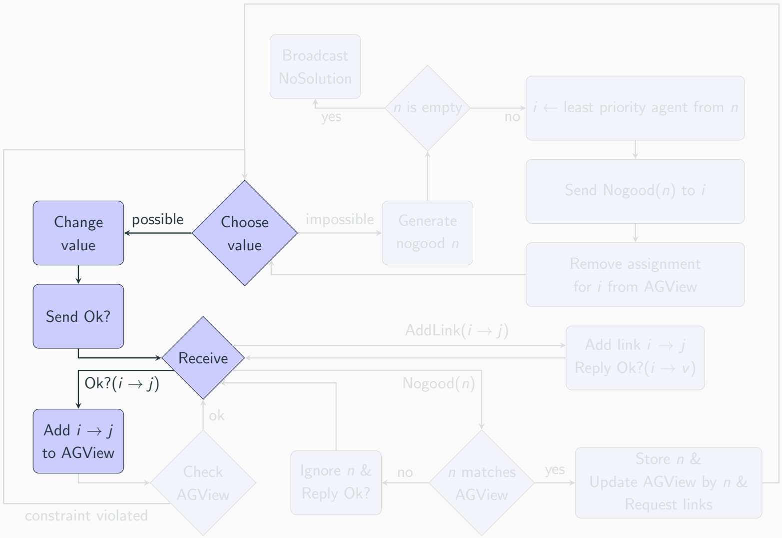

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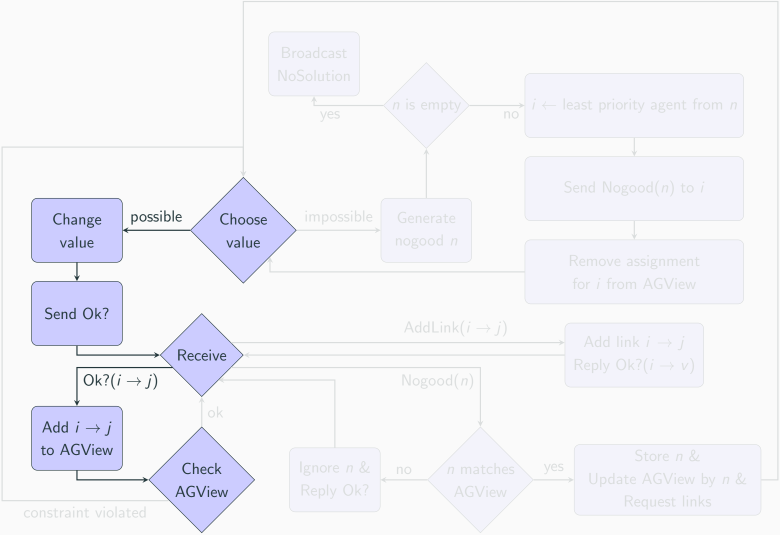

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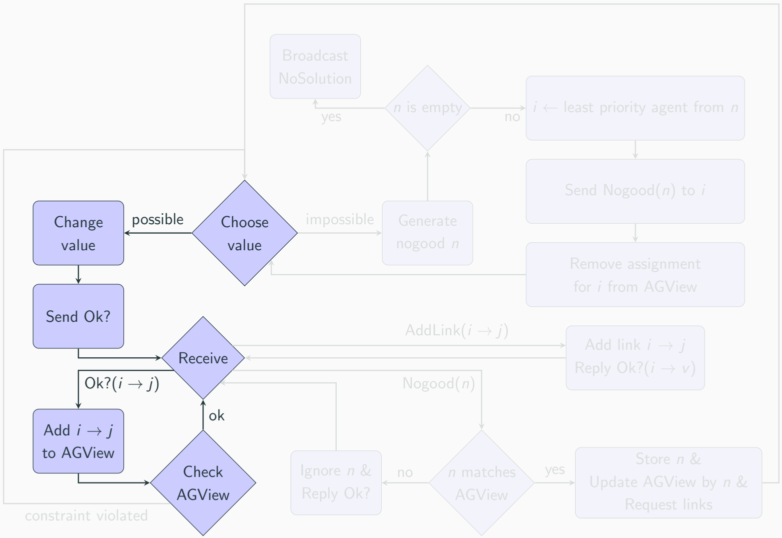

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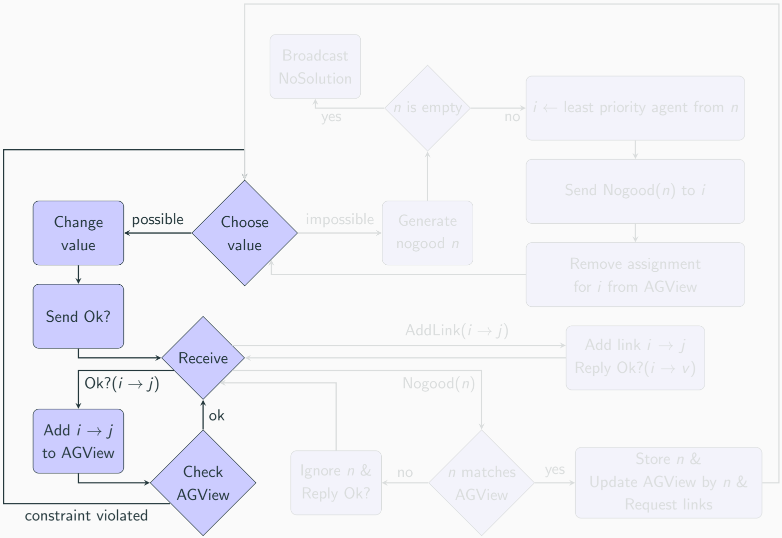

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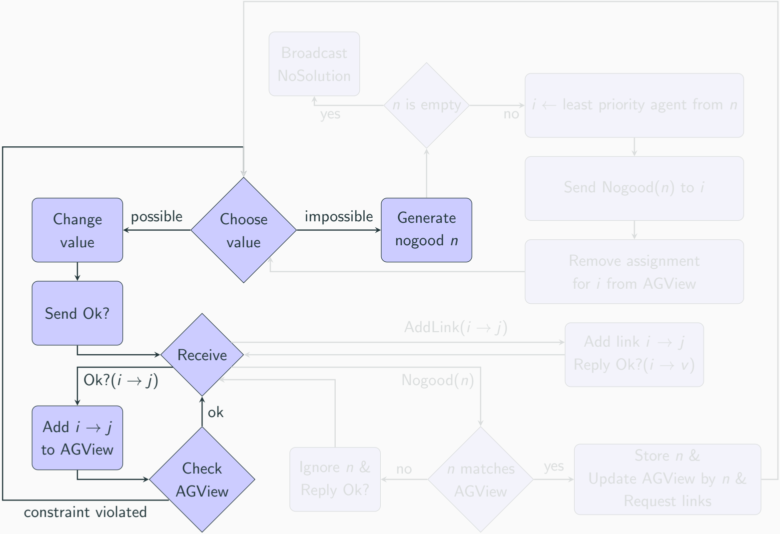

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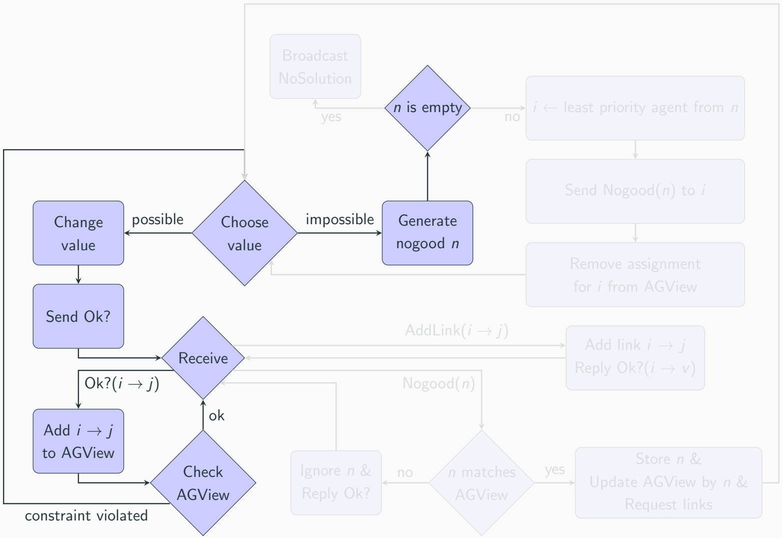

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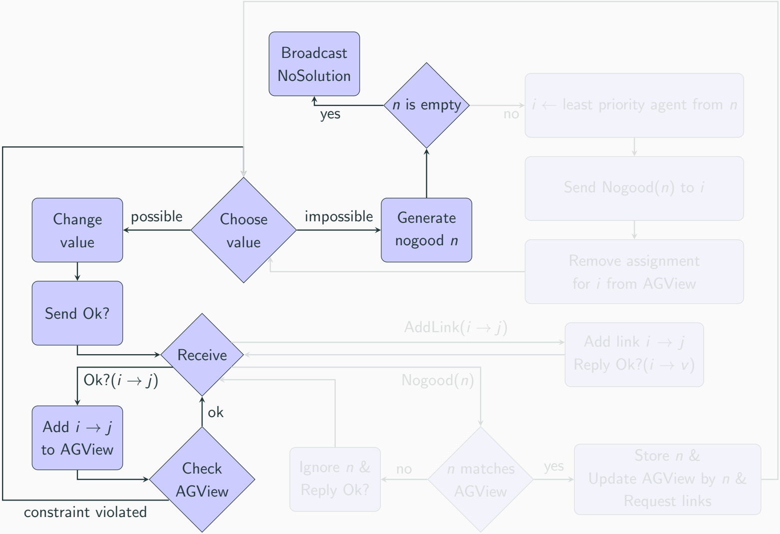

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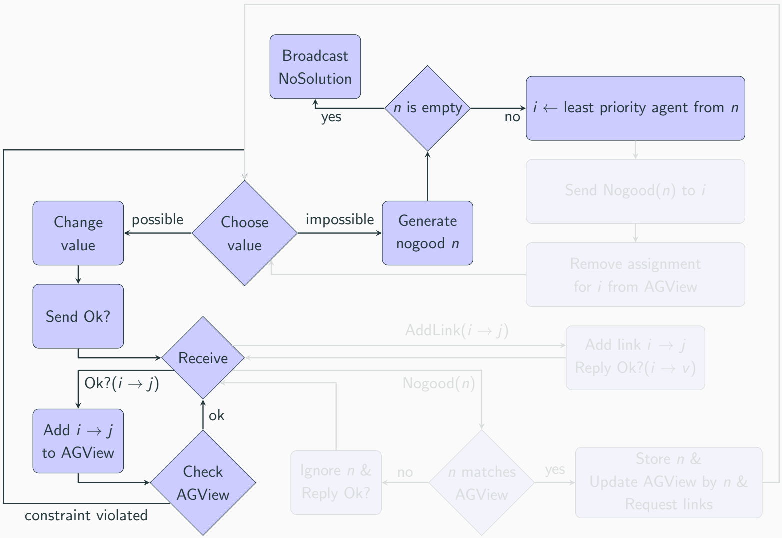

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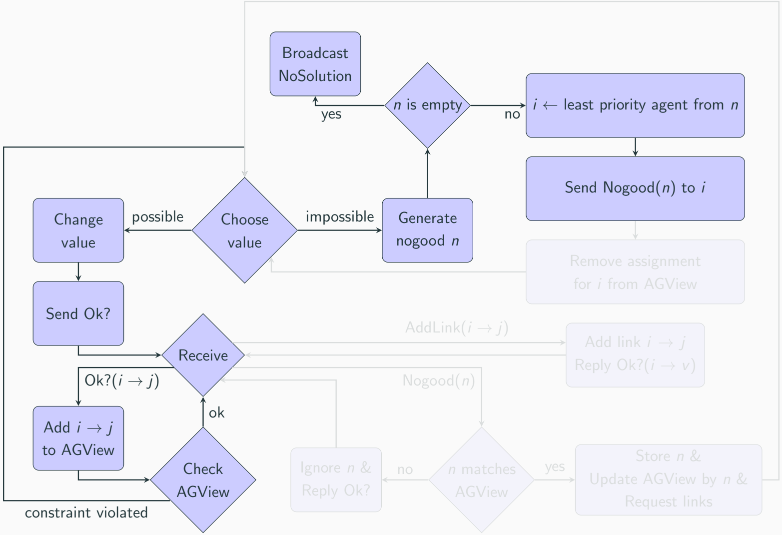

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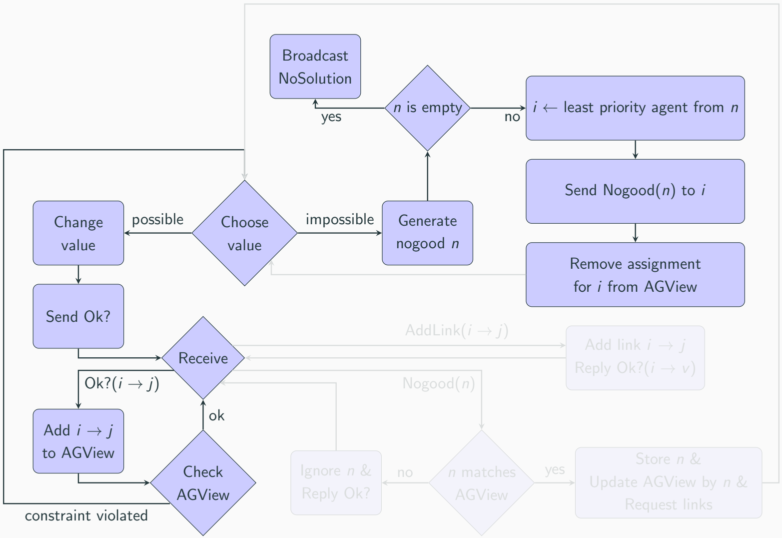

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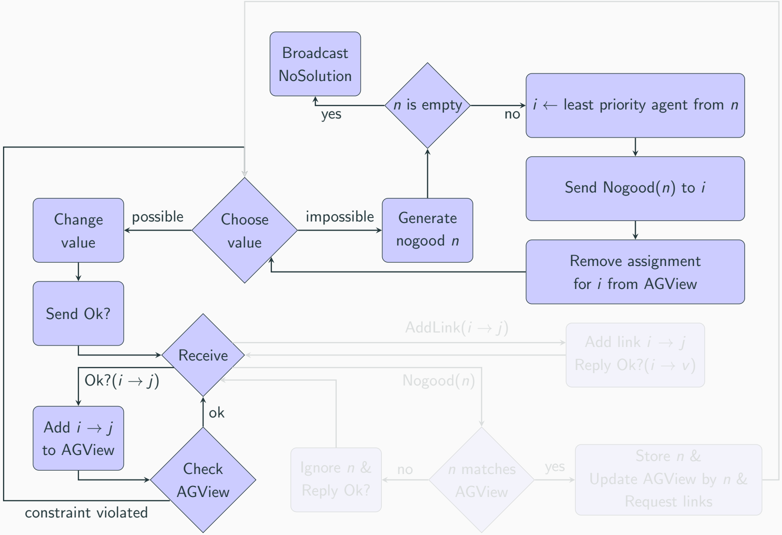

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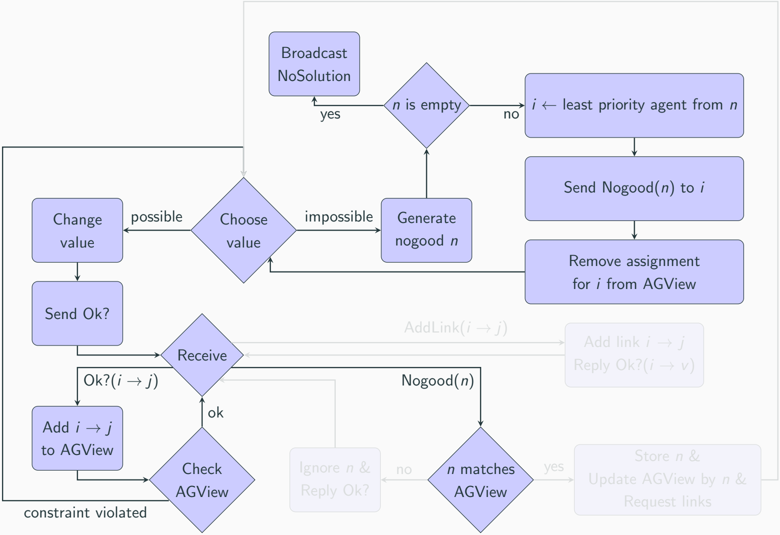

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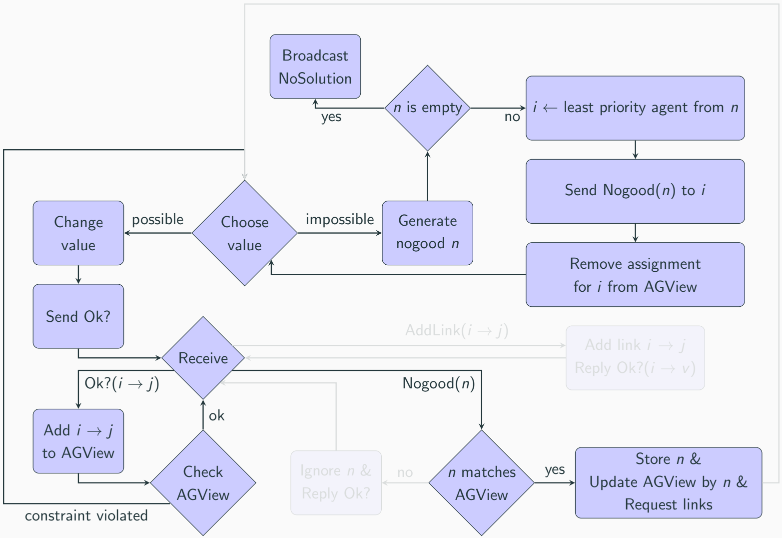

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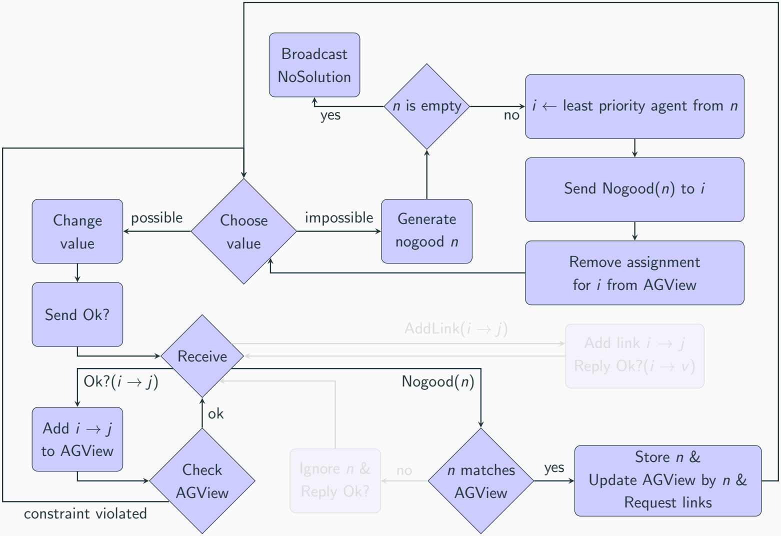

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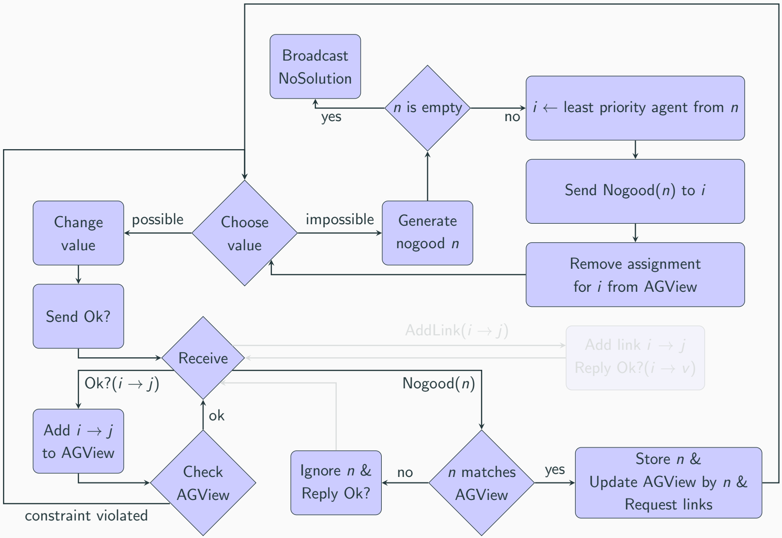

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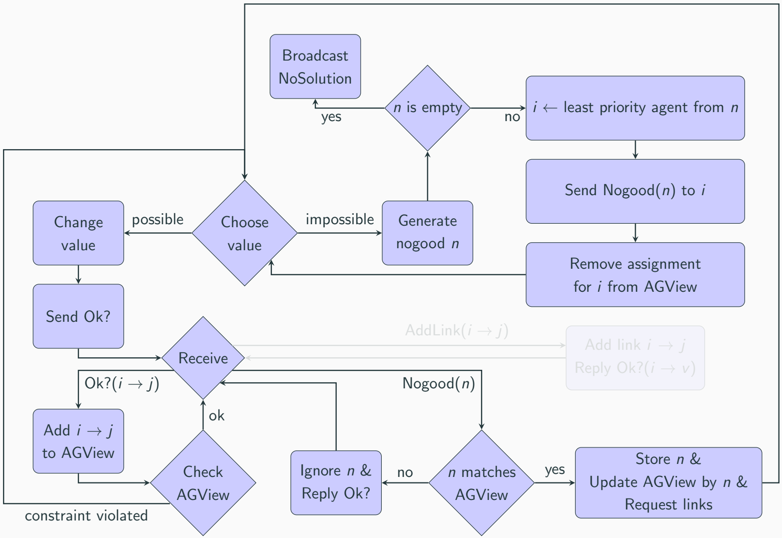

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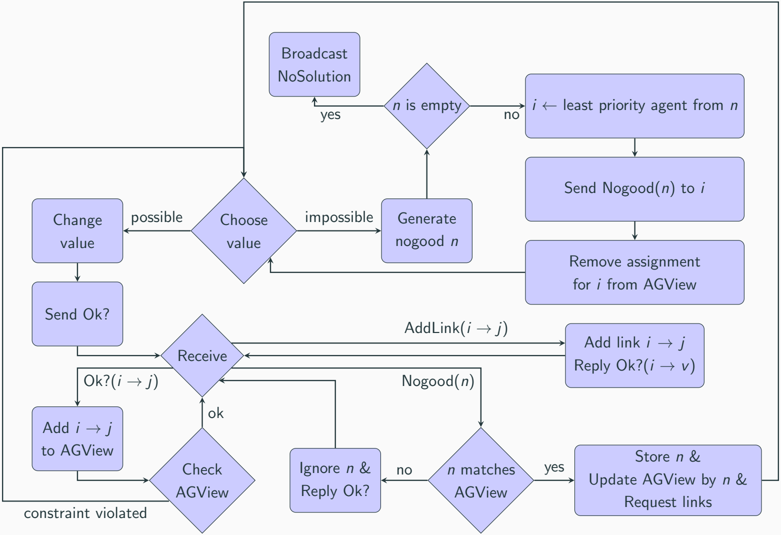

EXAMPLE