Auctions

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Auctions introduction

- Mechanism for allocating resources between agents
- An auction is a protocol that allows agents (=bidders) to indicate their interests in one or more resources and that uses these indications of interest to determine both an allocation of resources and a set of payments by the agents.

Why auctions

- Selling an item with unknown value, the value is assessed by the market
- Any object type can be allocated
- Easy automation

Auction rules

- Bidding rules: How offers are made
  - By whom
  - When
  - Content
- Clearing rules: Who gets which good, what money changes hands
- Information rules: What information is revealed, when and to whom

Valuation rules

- Common value: the good has the same value to all agents
- Private value: an agent A's valuation of the good is independent from other agent's valuation of the good
- Correlated value: valuations of the good are related
- Agent's payoff from participating in an auction

Agent's payoff

- if winner: item valuation - price paid
- if not: 0

Auction types

- Single-item auctions
- Multi-item auctions
- Reverse auctions

Single-item auctions

- English
  - Bidders shout their bids, once bidders stop shouting the highest bidder gets the good (minimum increments)
- Japanese
  - All bidders start standing, the auctioneer shouts ascending prices, bidders who are not willing to pay the announced price sit down
  - Once a bidder sits down, he cannot get back up
  - Last bidder standing gets the good
- Dutch
  - Prices announced with a decreasing clock, first bidder who shouts "mine" gets the good
- First price sealed
  - Bets in sealed envelopes
  - Highest bidder wins and pays his price
- Second price sealed
  - Bets in sealed envelopes
  - Highest bidder wins and pays the second highest price

Thinking about auctions

- Auctions seem suspiciously like games
- However, so far we have assumed that every player knows what game he is playing (number of players, actions available to players, utilities)
- In auctions this does not hold (unknown valuation of agents)
- Bayesian games
  - A set of games
  - The strategy space and number of players are the same for all games in this set
  - Payoffs differ across these games
  - Agents start with a common prior, telling them probability distribution over games (probability of playing this game)
  - Information sets over games for players (example)

Thinking about auctions

- How to behave in a given auction?
- How to design an auction?

Optimal behavior in Auctions

- Dutch/First price
  - Strategically equivalent
  - No dominant strategy, trade-off between probability of winning and amount paid upon winning
  - Individually optimal strategies depend upon assumption about others
- Second price sealed bid
  - Dominant strategy is truth telling
- English and Japanese
  - Complicated strategy space
  - Bids can be conditioned by actions of others (extensive form games)
  - Under independent private values model, it is dominant strategy for bidders to bid up to their valuations (how to bid?)

Optimal design of auctions

- Seller (designer) gets the money the winner paid (sellers revenue)
- Maximization of sellers revenue

Revenue equivalence

- Assume that each of n risk-neutral agents has an independent private valuation for a single good at auction, drawn from a common cumulative distribution $F(v)$ that is strictly increasing and atomless on $[v_{min}, v_{max}]$. Then any auction mechanism in which the good will be allocated to the agent with the highest valuation and any agent with valuation v_{min} has an expected utility of zero yields the same expected revenue, and hence results in any bidder with valuation v making the same expected payment.

Example

- Consider a second-price, sealed-bid auction with two bidders who have independent, private values v_i which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both $\frac{1}{2}$
  - What is the seller's expected revenue?
  - Now let's suppose that there are three bidders who have independent, private values v_i which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both $\frac{1}{2}$. What is the seller's expected revenue in this case?
  - Briefly explain why changing the number of bidders affects the seller's expected revenue.

Example

- A seller will run a second-price, sealed-bid auction for an object. There are two bidders, a and b, who have independent, private values v_i which are either 0 or 1. For both bidders the probabilities of $v_i = 0$ and $v_i = 1$ are each $\frac{1}{2}$. Both bidders understand the auction, but bidder b sometimes makes a mistake about his value for the object. Half of the time his value is 1 and he is aware that it is 1; the other half of the time his value is 0 but occasionally he mistakenly believes that his value is 1. Let's suppose that when b's value is 0 he acts as if it is 1 with probability 1/2 and as if it is 0 with probability 1/2. So in effect bidder b sees value 0 with probability 1/4 and value 1 with 3/4 probability. Bidder a never makes mistakes about his value for the object, but he is aware of the mistakes that bidder b makes. Both bidders bid optimally given their perceptions of the value of the object. Assume that if there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.
  - Is bidding his true value still a dominant strategy for bidder a?